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ACCURACY OF REFLECTING THE WAVEFORMS OF CURRENT AND VOLTAGE THROUGH THEIR SPECTRUM DETERMINED BY THE STANDARDS REGULATING MEASUREMENTS

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Key words: Harmonics, Inter-harmonics, Power quality analyzer, Measurements, Standards, Fourier transform.

The paper presents the questions of the accuracy of electric waveforms (current and voltage) by their spectrum, determined in accordance with the standards dedicated to the analysis of current quality. There has been also presented the voltage waveform, recreated from the spectrum that was obtained in accordance with the norm. It has been shown that for 3 different system frequencies, determined by the norm spectrum cutoff of up to 50 components may lead to the elimination of harmonics of significant values. Moreover, the volume of inter-harmonics and their impact on the voltage waveform has been analyzed. Exemplary results of measurements done with power quality analyzer in the laboratory have been presented.

1. INTRODUCTION

Several works have been devoted to the issue of the measurement of harmonics and inter-harmonics and to their use in the analysis of nonlinear circuits. These issues are discussed, among others, in articles [1–6]. The authors of works suggest the Discrete Fourier Transform (DFT) as an effective apparatus for analyzing harmonics/inter-harmonics. DFT is a convenient tool for the analysis of periodic waveforms in the frequency domain. However, the occurrence of inter-harmonics in waveforms complicates the selection of DFT parameters, and their incorrect values lead to inaccuracy [1]. Therefore, there are studies [2, 7] whose authors suggest algorithms that let obtain more accurate results than in case of the Fourier transform itself.

Legal regulations concerning the evaluation of the quality of current, rules and methods of measurement and selection of measuring apparatus [8–12] aim to normalize the procedures. Being familiar with these standards is essential to appropriately interpret results and formulate conclusions. At the same time, measurement standardization makes the results comparable. However, the analysis of particular definitions, requirements and specifications included in the norms let notice some ambiguities, and even inconsistences between them [13]. Moreover, the accuracy of reflecting the initial waveforms by the spectra of harmonics and inter-harmonics, restricted by norms to 50 harmonics, opens to doubt.

Analysis described in the article, refers to the particular case of the output voltage PWM powering the induction motor.

2. HARMONICS AND INTER-HARMONICS IN CURRENT AND VOLTAGE WAVEFORMS

Definitions of harmonics and inter-harmonics one can find in the norms that define the quality of electric current and electromagnetic compatibility of receiving devices [8, 9]. The harmonics are the current or voltage of which frequency is the integer multiple of the basic current supply frequency. Contribution of particular harmonics in the final signal shape was defined as (current and voltage) contribution factor of harmonics and is it calculated as follows:

$$w_k = \frac{X_k}{X_1} \cdot 100\% , \tag{1}$$

where: k – harmonic order,
X_k – rms of harmonic order k,
X_1 – rms of fundamental harmonic.

A total harmonicdistortion (THD) coefficient of waveform is defined as the ratio of the effective value calculated by excluding the first harmonic (it is assumed that stable component is zero) to the effective value of the first harmonic:

$$\mathrm{THD} = \frac{\sqrt{\sum_{k=2}^{n} X_k^2}}{X_1} , \tag{2}$$

where: k – harmonic order,
X_k – rms of k-harmonic of a particular signal $x(t)$,
X_1 – rms of fundamental harmonic,
n – number of harmonics considered in the analysis.

The inter-harmonics are the current or voltage of frequencies not being integer multiple of current supply frequency. The IEC-61000-2-1 standard [9] defines inter-harmonics as follows: „Between the harmonics of voltage and current there are components of frequencies that are not an integer multiple of the basic frequency. They can appear as discrete frequencies or as a wideband spectrum".

2.1. FOURIER ANALYSIS

An elementary tool that is used for determining the content of harmonics and inter-harmonics in waveforms is the spectral analysis basing on the Fourier transform. Each continuous, periodic waveform $x(t)$ can be denoted by the form of the discrete Fourier series:

$$x(t) = \sum_{k=0}^{\infty} \left(A_k \cos(k\omega t) + B_k \sin(k\omega t) \right) , \tag{3}$$

$$A_k = \frac{2}{T} \int_0^T x(t) \cos(k\omega t) \mathrm{d}t , \tag{4}$$

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$$B_k = \frac{2}{T}\int_0^T x(t)\sin(k\omega t)\mathrm{d}t\,,$$

where: $T = 1/f_1$ – period,
f_1 – waveform frequency,
ω – pulse repetition.

Waveforms almost periodic and aperiodic can be approximated with this series.

In order to obtain a discrete signal, analyzed the waveform of $x(t)$ is sampled at regular intervals T_s, with the conditions specified in the Kotielnikow-Shannon theorem. In one period waveform are registered N samples:

$$Ts = T / N. \tag{5}$$

Considering also that

$$T = 2\pi / \omega\,, \tag{6}$$

m samples of discrete signal $x(m)$ can be written as a series of Fourier:

$$x(n) = \sum_{k=0}^{N-1}\left(A_k \cos\left(k\frac{2\pi}{N}m\right) + B_k \sin\left(k\frac{2\pi}{N}m\right)\right)\,, \tag{7}$$

$$A_k = \sum_{n=0}^{N-1} x(m)\cos\left(k\frac{2\pi}{N}m\right)\,,$$

$$B_k = \sum_{n=0}^{N-1} x(m)\sin\left(k\frac{2\pi}{N}m\right)\,. \tag{8}$$

Writing coefficients of the Fourier series in the complex form, we obtain a formula for the discrete fourier transform (DFT):

$$X_k = A_k + B_k$$

$$X_k = \sum_{n=0}^{N-1} x(m)\exp\left(-\mathrm{j}\,k\frac{2\pi}{N}m\right) \tag{9}$$

$$k = 0,1,....$$

Fourier transform, or used for digital signal processing its discrete version (DFT) allows to quickly and accurately assess which components the signal consists of.

A traditional, discrete Fourier transform (DFT), as well as its fast variation (FFT) make an easy and effective tool that can be used for analyzing waveform distortion. However, when in a waveform occur inter-harmonics, of which frequencies are not the frequency multiple of current supply, some difficulties appear [1, 2, 13]: minimal sampling time can be long and amount of samples high, basic Fourier frequency is not equal to current supply frequency and can be hard to determine. Border Nyquist frequency can be high and its breaking can lead to aliasing. Incorrect use of the Fourier transform algorithms can lead to false interpretations of spectrum [1, 2]. In case of spectral leakage – spilling energy of one frequency onto other frequencies or to aliasing waveform distortion in the sampling process because of failure to meeting the assumptions of Kotielnikow-Shannon theorem. Spectrum leakage is the consequence of necessity (from practical reasons) of reducing the amount of samples, consequently reducing the measurement time as well. Setting the measurement time which is different from the basic Fourier period causes the waveform to break the continuity at the edges of the measurement window (Fig.1.); artificial discontinuities will appear as high frequencies in the waveform spectrum. This problem is solved by using the so-called time-weighted windows, most often a rectangular window or the Hanning window, but other windows like Hamming's or Kaiser's may be used as well, etc.

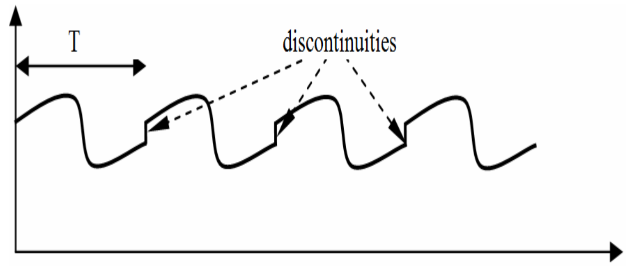

Fig. 1 – Periodic waveform obtained as a result of reducing the measurement time.

2.2. SPECTRUM RESOLUTION

During the measurement of the harmonics, a sufficient spectrum resolution equals the resolution of system (f_1), as all measured components will have the frequencies being their multiple. The inter-harmonics measurement requires higher resolution of spectrum in order to have „access" to the components which are not a factor of basic system resolution.

If the measurement time (T_w) equals the period, then the spectrum resolution equals the frequency of the basic harmonic:

$$f_w = f_1 = \frac{1}{T}\,. \tag{10}$$

If the measurement time (measuring window) includes a integer p of the waveform periods $T_W = pT_1$, the spectrum resolution increases (bars density in the spectrum increases):

$$f_w = \frac{1}{pT} = \frac{f_1}{p}\,. \tag{11}$$

By extending the measuring window the spectrum resolution improves, which in turn makes the inter-harmonics measurement possible. The width of window can be extended by increasing the number of N samples or by decreasing the sampling frequency f_S:

$$T_W = NT_s = \frac{N}{f_s}\,. \tag{12}$$

When selecting an amount of samples and the sampling frequency, the measuring window must be of length being the multiple of basic period and the sampling frequency must meet the rules of the sampling theorem:

$$f_s > 2f_g\,, \tag{13}$$

where: f_S – sampling frequency,
f_g – highest frequency in a waveform.

2.3. NORMALIZATIONS RELATED TO MEASUREMENT

The questions of the harmonics and inter-harmonics measurement of current and voltage are regulated by the IEC 61000-4-7 [10]. In accordance with the norm, the harmonics and inter-harmonics ought to be measured in a rectangular window which is synchronized with the measured period, of 10 (50 Hz) or of 12 (60 Hz) periods width. As a result of such a measurement we obtain a spectrum of 5 Hz resolution. Regarding the norm the so-called groups of harmonics and inter-harmonics should be created from single bars.

The IEC 61000-4-30 [12] standard defines the measuring methods of electric current quality rates for current supply systems of alternating current of 50/60 Hz resolution. It also gives requirements concerning the interpretation of measurement results. The Fourier analysis, in accordance with the standard [12], is conducted in the time window covering 10 periods, that is 200 ms at the network resolution of 50 Hz or for 12 periods for the supplying system of the rated resolution of 60 Hz (analogically as in [10]). The norm also suggests performing the averaging of selected values in a very short (3 s), short (10 min) and long (2 h) measuring time.

2.4. THE CONCEPT OF GROUPING

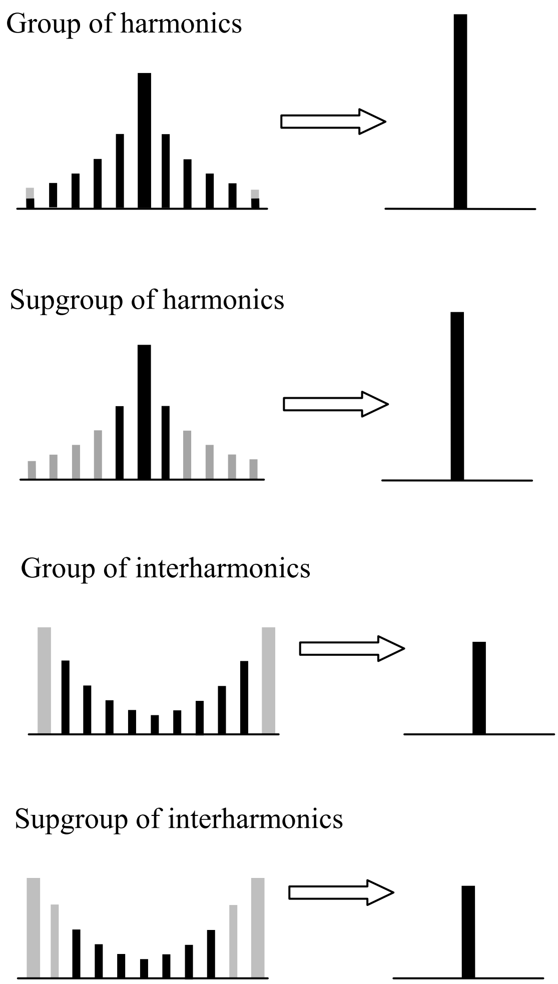

Fig. 2 – The way of creating groups of harmonics and inter-harmonics.

When the harmonics and inter-harmonics occur in the time waveform, it is practically impossible to accurately determine its spectrum, and it often results in smudging [1, 2]. The spectrum smudging is the reason why determining the amplitudes of particular components gives unclear results, thus the harmonics power of a specified frequency disperses onto neighboring frequencies.

The norm [10] suggests grouping of the neighboring harmonics/inter-harmonics. After introducing of the grouping, sought harmonic "absorbs" neighboring bars and becomes their representative. Grouping in case of harmonics can cover one neighbor both sides (3 bars) or 4,5 neighbors both sides (10 bars) (Fig.2).

3. **LABORATORY RESEARCH AREA**

The studies have been conducted in a lab research area comprising of:

- Emerson Commander SK variable frequency drive and induction motor,
- Elspec Portable Blackbox G4500 power quality analyzer,
- Laptop with data bases MS SQL and specialized software: PQSCADA Management Studio and PQInvestigator.

Fig. 3 – Power analyzer Elspec Blackbox G4500.

Variable frequency drive was the supply source of a regulated frequency. Output voltage and current of the drive were measured as well as their spectral analysis with Elspec G4500 was conducted. Elspec G4500 is a class A microprocessor current quality analyzer. The device has 11 measurement channels enabling the voltage and current measurement in each phase. Voltage is sampled 1024 times in each period which allows analyzing up to 511 harmonics components. The current channels are sampled 256 times in a period, and the harmonics analysis is possible up to 127 components. Moreover, the current quality analyzer enables the analysis of inter-harmonics. The analyzer also enables to register current quality parameters in accordance with the EN 50160 and IEC 61000-4-15 standards, as well as with the standards of multiple countries. Going further, clients can pre-define a particular standard. However, further analysis of the gathered data in the SCADA system, using PQInvestigator, allows obtaining any matching and tabular changes of particular electric amounts in the previously set averaging times.

4. MEASUREMENT

Measurements were conducted for a few input frequencies of inverter analyzing the content of harmonics and inter-harmonics as well as high fidelity of the initial current and voltage waveforms.

In Figs. 4–7 one can see the results of measurements and analyses for the output frequency of an inverter equal 49.7 Hz (system frequency). The current waveform is practically undistorted (Fig. 4a), its spectrum verifies it (Fig. 4b).

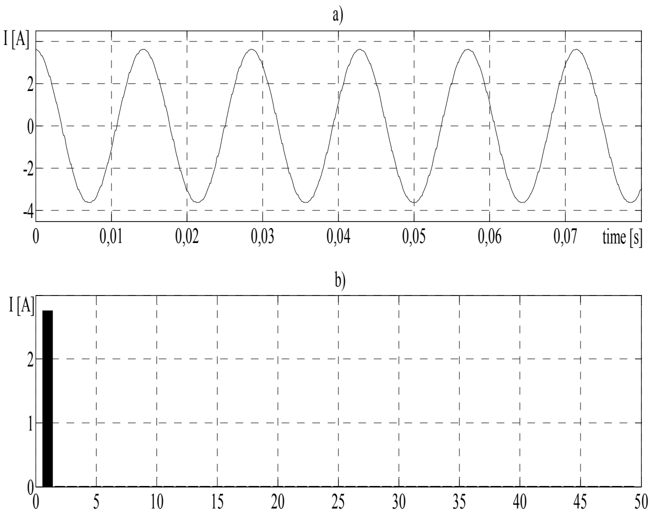

Fig. 4 – Current waveform a) and its spectrum b) at the 49.7 Hz frequency.

High distortions occur in the voltage waveform (Fig. 5a), in the waveform spectrum obtained in accordance with the IEC 6100-4-7 [10] standard the harmonics 1 and 3 are dominating; values of other components are practically minor (Fig. 5b).

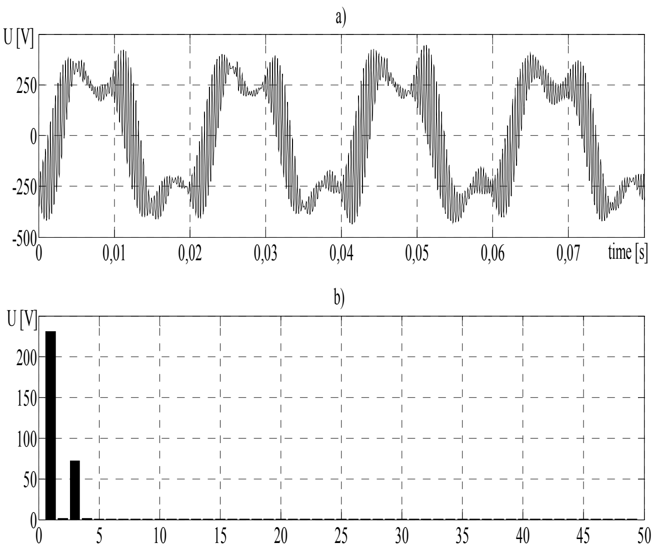

Fig. 5 – Voltage waveform (a) and its spectrum (b) at the 49.7 Hz frequency.

Recreating the voltage on the basis of the gained amplitude and phase spectrum we achieve the process that reflects the character of the initial waveform (Fig. 6). However, it is necessary to assure that the waveform has no characteristic high-frequency oscillations, which have a significant impact on the voltage amplitude.

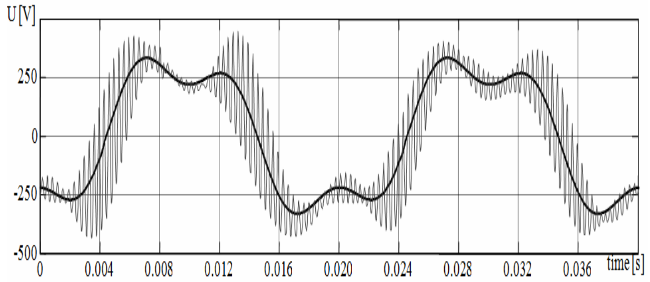

Fig. 6 – Voltage waveform at the 49.7 Hz frequency. Original waveform – thin, grey line; waveform recreated from harmonics – thick, black line.

Analyzing a complete spectrum of voltage (Fig. 7a) one can notice that the harmonics 60, 62, and 58 have high values, and harmonics 64, 61, and 59 (from the highest to the lowest) have lower values. However, in the case of instruments, which measuring according to [10] these components will not be included. By the comparison of THD value for the harmonics 50 (the norm [10]) and 511 (full spectrum), we achieve respectively: $THD_{50} = 24.5$ % and $THD_{511}= 50.3$%. Other elements which can affect the shape of voltage waveform are the inter-harmonics. Their values are lower than the ones of the harmonics mentioned earlier however they also influence the voltage waveform.

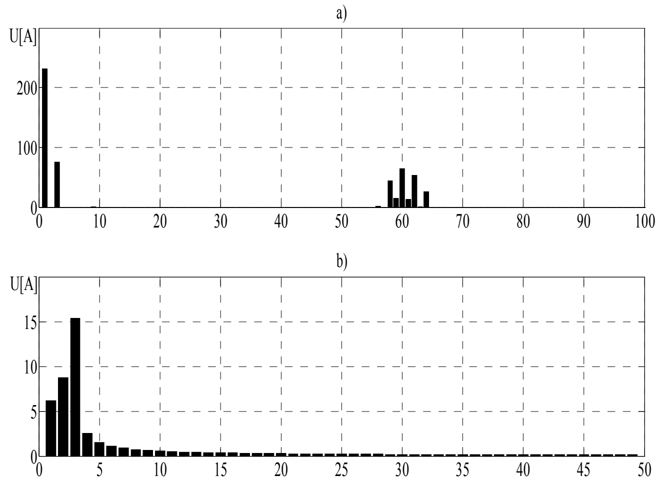

Fig. 7 – Voltage waveform at the 49.7 Hz frequency: extended spectrum of harmonics (a), spectrum of inter-harmonics (b).

Analogically, measurements and analyses have been conducted for the frequencies of 42.5 Hz (Fig. 8) and 57.5 Hz (Fig. 9). The results were similar to the ones in case of the 49.7 Hz frequency.

In case of the 42.5 Hz frequency, in the spectrum obtained in accordance with the standard [10] (Fig. 8b) only the harmonics 1 and 3 and minimally harmonic 4 have practical impact on the shape of voltage waveform. The registered temporal voltage values (Fig. 8a) show that it contains the components of high frequencies and amplitudes. In a complete spectrum occur the harmonics 71, 70, 74, 140, 142, 212, 4, 69, 72 (from the highest to the lowest) of values from 97.5 V to 22.7 V, so carrying large amounts of energy (Fig.8c). A THD_{511}=76.17 % coefficient is definitely higher than THD_{50} = 21.0 %. In case of this frequency, one cannot skip the inter-harmonics which achieve values up to 30 V (Fig. 8d).

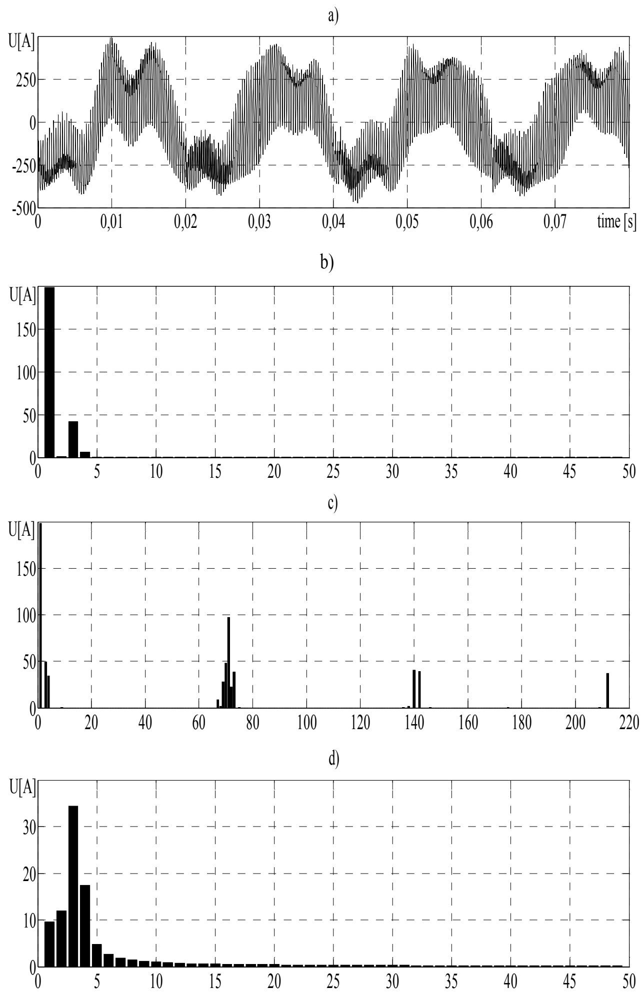

Fig. 8 – Voltage waveform (a), its spectrum (b), extended spectrum (c) and spectrum of inter-harmonics (d) at the 42.5 Hz frequency.

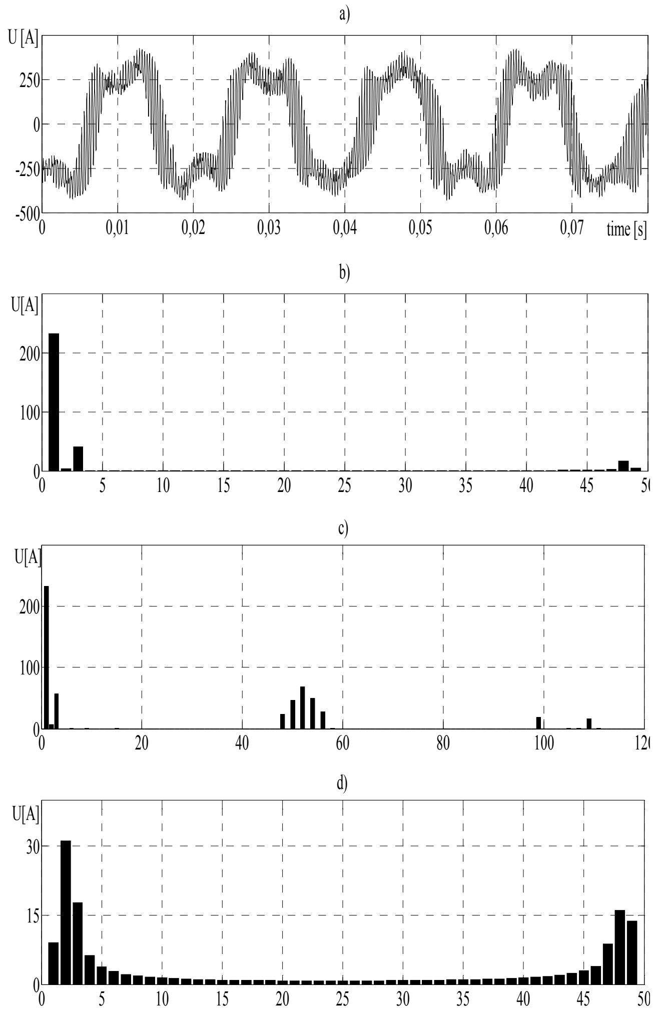

Fig. 9 – Voltage waveform (a), its spectrum (b), extended spectrum (c) and spectrum of inter-harmonics (d) at the 57.5 Hz frequency.

The analysis of voltage waveform at the 57.5 Hz frequency (Fig. 9a) leads to very similar conclusions. The spectrum obtained in accordance with the standard [10] (Fig. 9b) practically includes 1st and 3rd harmonic, although one can also notice a small amount of the harmonic 48. Extending the spectrum to higher frequencies (Fig. 9c) one can see high values of the harmonics: 52, 54, 50, 48, 95, and 109 (from the highest to the lowest). The highest of them has higher amplitude than the amplitude of the 3rd harmonic, reaching 67.9 V, while the lowest is 15.7 V, so they carry a large amount of energy.

The spectrum of inter-harmonics (Fig. 9d) contains a wide band of components; the second has amplitude of 31.2 V, 3–17 V, and 48–16 V. The spectrum waveform suggests that above the 50^{th} component occur the components of high values. It is confirmed by the comparison of THD coefficients, for the full spectrum (511 harmonics) and the spectrum consistent with the norm [10] (50 harmonics), $THD_{511} = 51.76$ %, $THD_{50} = 27.0$ %. Therefore, reducing the spectrum to 50th component may be the source of errors.

5. CONCLUSIONS

The presented question of the harmonics and inter-harmonics measurement and normative regulations also concerning the measurements, towards a common use of non-linear current receivers, became more significant. The conducted research allows formulating the following conclusions:

- standard [10] defines the measurement of harmonics and inter-harmonics for power supply systems, so use it, to measure the spectrum of voltage artificially jagged can cause significant inaccuracies. This observation is confirmed by the comparison of the THD coefficient of inverter voltage output (voltage artificially jagged) - set for the spectrum comprising 511 components with the coefficient set for the spectrum comprising 50 harmonics. The value of THD for 511 components is much higher than the value of THD achieved for 50 harmonics;
- in case of the frequency inverter which supplies the induction motor, the output current of inverter is practically undistorted, while the voltage is significantly distorted;

- in the output voltage of inverter there is a wide spectrum of harmonics. The components 1 and 3 occur notwithstanding the frequency of the output voltage. The remaining harmonics – of values higher than 50 and values comparable to the 3rd harmonic – depend on the frequency;
- inter-harmonics are significant in the process of output voltage of the frequency inverter;
- the harmonics and inter-harmonics terms make sense only in relations to current supply frequency. If the waveform is described regarding the Fourier frequency, then we only deal with the harmonics.

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