ON THE TRANSIENT BEHAVIOR OF SATURATED SYNCHRONOUS MACHINES

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Key words : Synchronous machine, Saturation, Modeling and simulation.
In the design stage of synchronous machine is often necessary the predetermination of dynamic characteristics using modeling and simulation. The simulation results and their correlation with reality depend essentially on the way the synchronous machine characterized by high electric and magnetic stress is modeled. Significant data are presented in the following for comparative analyses of different ways to consider the saturation in synchronous machine in dynamic regime.

1. INTRODUCTION

The dynamic regime problems and the stability of the synchronous machine (actually the main source for obtaining electromagnetic energy) have generated numerous long term and wide researches [7–9]. Generally, the high precision evaluation of the behavior of the alternative current machines during dynamic processes (start, self-excitation) characterized by large variations of magnetic and electric stresses is closely determined by a correct mathematical modeling. The complicated couplings between the windings generate non linear equations systems, which can be solved numerically.

The mathematical models obtained by using representative phasors under controlled hypotheses, represent a coherent and efficient research base.

In [3] the general mathematical models of the saturated synchronous machine with salient poles for various combinations of the state variables are deduced, considering the cross-coupling inductance.

In the following, using [3], we present and interpret the results of significant simulations which allow the predetermination of the behavior of the synchronous machine, in a given dynamic process.

2. GENERAL EQUATIONS OF THE SATURATED SYNCHRONOUS MACHINE

The mathematical model of the synchronous machine saturated on the main flux way is considered to have the form:

$$\mathbf{U}_{dq} = A \frac{d\mathbf{X}_{dq}}{dt} + B \mathbf{X}_{dq}, \quad \mathbf{U}_{dq} = [u_d \quad u_q \quad u_E \quad 0 \quad 0]^T. \quad (1)$$

$\mathbf{X}_{dq}$ corresponds to diverse combinations of state variables and $A$, $B$ are matrices whose form is determined by the choice of the state variables (currents only, flux linkages only or some mixture of currents and flux linkages).

In the following we consider as state variables the currents of the machine windings. In this case, the state vector $\mathbf{X}_{dq} = [i_d \quad i_q \quad i_E \quad i_D \quad i_Q]^T$ correspond the matrices (see [3])

$$A = \begin{bmatrix} L_{m} + L_{sl} & m^2L_{sl} & L_{sl} & L_{sl} & m^2L_{sl} \\
L_{sl} & L_{m} + m^2L_{pq} & L_{pq} & L_{pq} & m^2L_{pq} \\
L_{sl} & m^2L_{pq} & L_{mn} + L_{s} & L_{mn} + L_{s} & m^2L_{pq} \\
L_{sl} & m^2L_{pq} & L_{mn} + L_{s} & L_{mn} + L_{s} & m^2L_{pq} \\
L_{pq} & m^2L_{pq} & L_{pq} & L_{pq} & L_{mn} + m^2L_{pq} \end{bmatrix},$$

$$B = \begin{bmatrix} R_s & -\omega(L_m + m^2L_m) & 0 & 0 & -\omega J_m \\
\omega(L_m + L_m) & R_s & \omega L_m & \omega L_m & 0 \\
0 & 0 & R_E & 0 & 0 \\
0 & 0 & 0 & R_D & 0 \\
0 & 0 & 0 & 0 & R_Q \end{bmatrix},$$

where $m^2 = L_{mq} / L_{md}$ is the saliency coefficient; the computational inductances are

$$L_{sd} = L_{md} \cos^2 \varphi + L_{m} \sin^2 \varphi, \quad L_{dq} = (L_{ml} - L_m) \sin \varphi \cos \varphi,$$

$$L_{sq} = L_{ml} \sin^2 \varphi + L_m \cos^2 \varphi, \quad \cos \varphi = \frac{i_{md}}{i_{mq}}; \quad \sin \varphi = \frac{i_{mq}}{i_m}. \quad (2)$$

At equation (1) we add the movement equation

$$m - M_r = \frac{J}{p} \frac{d\omega}{dt} = \frac{J}{p} \frac{d^2\beta}{dt^2}. \quad (3)$$
where

$$m = \frac{3}{2} p L_m \left[ (i_E + i_d)_q - m^2 i_d i_q + (1 - m^2) i_d q \right]$$

In the referential system $d, q$ we consider

$$u_d = \sqrt{2} U \cos(\omega_l t - \beta), \quad u_q = \sqrt{2} U \sin(\omega_l t - \beta),$$

where

$$\beta = \int \omega dt + \beta_0.$$

The speed of the main magnetic field $\psi_m$ during the dynamic processes will be

$$\omega_q = \frac{d\phi}{dt} + \omega,$$

where

$$\frac{d\phi}{dt} = \frac{1}{\psi_m} \left( \frac{d\psi_{mq}}{dt} \cos \phi - \frac{d\psi_{md}}{dt} \sin \phi \right) = \frac{1}{i_m} \left( \frac{di_{mq}}{dt} \cos \phi - \frac{di_{md}}{dt} \sin \phi \right).$$

The above notations are those of [3] and used by some authors.

Remark: When saturation is taken into account in the conventional way, the cross-coupling inductance vanishes ($L_{dq} = 0$). The classical approach form of the mathematical model is obtained by introducing $L_{mt} = L_m = L_m \left( i_m \right)$. For the simplified computation (with $L_m = L_{mt} = const$) the matrix $A$ becomes invariant and the solving of the system describing the dynamical process is much simpler. In this case computational inductances become $L_{dd} = L_{qq} = L_m$ and $L_{dq} = 0$

3. SIMULATION RESULTS

We study in the following the effects of dynamic saturation ($L_{dq} \neq 0$) and saliency on the behavior of synchronous machine with star winding and nominal voltage $U_n = 220V$ (see Appendix) at the dynamic asynchronous start and synchronization considering the above theory.

The curves $m(\omega), \omega_d(t), \omega(t)$ were obtained considering various levels of magnetic saturation (different $U$) and resistive torques $M_r$. The proposed model, the
simplified model and the classical model are used for comparison. The simulations were performed in Matlab using a fifth order Runge-Kutta with variable step.

With small supplying voltage ($U = 100$ V) when the saturation is not present and for the same resistive torque ($M_r = 0$), the three computing methods lead to practically identical results. This was to be expected and validates the general system of equations (1).

For an average saturation level ($U = U_n = 220$ V) some small differences of waveform and amplitude are to be remarked. Repeated simulations showed that synchronization is possible till $M_r \approx 2$ Nm. In the permanent regime the results are practically identical, which confirm that the three mathematical models are correct. This results were those expected and are not presented.

To underline the necessity of consideration of dynamic saturation in formulation of the mathematical models, in the sequel the characteristics for $U = 300$ V and different values of the resistive torques $M_r$ given by the three models are shown.

![Fig. 1a – Characteristic $\omega_p(t)$, $\omega(t)$ for $U = 300$ V, $M_r = 8$ Nm.](image1)

![Fig. 2a – Characteristic $\omega_p(t)$, $\omega(t)$ for $U = 300$ V, $M_r = 7.5$ Nm.](image2)

![Fig. 1b – Characteristic $m(\omega)$ for $U = 300$ V, $M_r = 8$ Nm.](image3)

![Fig. 2b – Characteristic $\omega_p(t)$, $\omega(t)$ for $U = 300$ V, $M_r = 7.5$ Nm (detail).](image4)
Fig. 2c – Characteristic $m(\omega)$ for $U = 300$ V, $M_r = 7.5$ Nm.

Fig. 2f – Characteristic $L_{mt}(t)$ for $U = 300$ V, $M_r = 7.5$ Nm.

Fig. 2d – Characteristic $m(\omega)$ for $U = 300$ V, $M_r = 7.5$ Nm (detail).

Fig. 3a – Characteristic $m(\omega)$ for $U = 300$ V, $M_r = 4$ Nm.

Fig. 2e – Characteristic $L_{mt}(t)$ for $U = 300$ V, $M_r = 7.5$ Nm.

Fig. 3b – Characteristic $m(\omega)$ for $U = 300$ V, $M_r = 4$ Nm (detail).
Fig. 3c – Characteristic $L_m(t)$ for $U = 300$ V, $M_r = 4$ Nm.

Fig. 4b – Characteristic $m(\omega)$ for $U = 300$ V, $M_r = 7.5$ Nm, classical model.

Fig. 3d – Characteristic $L_{mt}(t)$ for $U = 300$ V, $M_r = 4$ Nm.

Fig. 5a – Characteristic $\omega_\psi(t)$, $\omega(t)$ for $U = 300$ V, $M_r = 7.5$ Nm, simplified model ($L_m = 0.4$ H).

Fig. 4a – Characteristic $\omega_\psi(t)$, $\omega(t)$ for $U = 300$ V, $M_r = 7.5$ Nm, classical model.

Fig. 5b – Characteristic $m(\omega)$ for $U = 300$ V, $M_r = 7.5$, Nm simplified model ($L_m = 0.4$ H).
Figs 1, 2, 3 show the characteristics for $M_r = 8$ Nm, 7.5 Nm and 4 Nm using the proposed model. They suggest an asynchronous operation with a large slip, one practically synchronous operation, and respectively a synchronous operation. Fig. 2a offers interesting information regarding the characteristic $\omega_{\psi}(t)$: the rotating field speed grows from $\omega_{1n}/2$ to $\omega_{1n}$ in a time comparable with the duration of the mechanical transient regime. A reversal of the direction may intervene (in our analyses for $U = 100$ V, 220 V) and almost always over synchronous oscillations exist, finally damped. This observation is a general characteristic for alternating current machines. One can observe the close connection between the mechanical and electromagnetic transient processes, in fact one electro-mechanic process. One can observe the effect of the Gorges phenomenon on the characteristic $m(t)$ at approximate $\omega_{1n}/2$. Finally, the synchronous operation is not obtained. Fig. 2b details the asynchronous sustained operation, where the oscillations of $\omega(t)$ are closely correlated with those of $\omega_{\psi}(t)$.

For $M_r = 8$ Nm (Fig. 1a) the operation is far from synchronous; the permanent oscillations are present in the curves $\omega_{\psi}(t)$ and $\omega(t)$ with a clear underlining of the Gorges phenomenon which does not allow synchronization.

The characteristics of the Figs. 1b and 2c present the correspondent dynamical torques, also affected by the Gorges phenomenon which finally leads to an asynchronous oscillating operation, and respectively practically synchronous. Fig. 2c also shows the large oscillations of the asynchronous torque in the vicinity of the synchronous speed, and in the detailed Fig. 2d the closed limit cycle represented in bold gives the limits in which $m$ and $\omega$ oscillate permanently. When using the proposed method we take into account $L_m$ and $L_{mL}$ (Figs. 2c, 2f) which also oscillate in large limits. Fig. 2e underlines the effect of sustained asynchronous oscillations on $L_m$. 
Fig. 3 presents the same dynamic characteristics as above for \( M_r = 4 \) Nm, for which the synchronization is achieved. The detailed representation from Fig. 3b different of Fig. 2d, confirms machine synchronization through oscillations that damp in the well defined point of synchronism. Also, taking into account that synchronization is finally achieved, the damped oscillations of \( L_m \) and \( L_{m'} \) disappear and are replaced by constant values corresponding to the level of magnetic stress of the machine (Figs 3c and 3d).

Many other simulations show that for the high saturation level considered \( (U = 300 \) V), the synchronous operation is achieved for \( M_r < 5 \) Nm and asynchronous one (with \( \omega \) a little bigger than \( \omega_{1n/2} \)) for \( M_r > 7.8 \) Nm. In the interval \( (5-7.8) \) Nm a practically synchronous operation is achieved.

Finally the results of classical and simplified methods are presented. The Figs 4a and 4b show the same characteristics when the classical model is used for \( M_r = 7.5 \) Nm. One can see the previous results are not confirmed. We get an asynchronous operation with a large slip, torque oscillations and repeated reversals of \( \omega_{\psi}(t) \). The classic model gives also inconclusive results for values sensibly smaller than \( M_r = 7.5 \) Nm.

The method with constant parameters offers results comparable with the proposed method if one considers for \( L_m = \text{const} \), the final saturated value of the given dynamic regime (in our case \( L_m \approx 0.15 \) H); if we take \( L_m = \text{const} \) corresponding to the case of no saturation \( (L_m \approx 0.4H) \), the constant mathematical model fails (Figs 5a, 5b).

For \( M_r = 4 \) Nm the classical method suggest a practically synchronous operation (compare Fig. 6 with Fig 3b). As above, the simplified method offers results comparable with the proposed method if one considers the saturated value of \( L_m = \text{const} \) (in our case \( L_m = 0.12 \) H). For \( L_m \approx 0.4 \) H we obtain a practically synchronous operation (Fig. 7) with the same comment as for Figure 2 b.

For \( M_r = 8 \) Nm the results and interpretations corresponding to classical and simplified methods are as above and similar to Fig. 4–Fig. 7.

4. CONCLUSIONS

Many simulations allow the following general conclusions.

For usual voltages \( (U \approx U_n) \) all three models offer close results and can be equally used.

For high voltages the effect of \( L_{m'} \) (of \( L_m \)) becomes important. One can obtain different characteristics when using the three models. Close characteristics are obtained by the enhanced model and the simplified model for \( L_m = \text{const} \) (if we
take $L_m$ of the end of the dynamic regime given by the proposed model). This is an indirect method for the validation of the proposed model. The classic method offers inconclusive results.

The enhanced model is recommended for the analysis of dynamic regimes with high solicitations, electric and magnetic. It allows valuable pre-determinations for convenient design of a synchronous machine due to operation into a given dynamical regime.

The classic method is the most inaccurate. Even the method with $L_w = \text{const}$ that assumes a small volume of calculations is preferable.

The equation (4) contains valuable information about the velocity $\omega_{\psi}(t)$ of the main rotating magnetic field during the transient processes. A close connection between $\omega_{\psi}(t)$ and $\omega(t)$ is observed in all simulations shown above.

Finally we insist on the computational character of the inductances $L_{d}^{\sigma}, L_{q}^{\sigma}, L_{d}^{\sigma}$ which depend on saturation, reference frame and state variables.

APPENDIX

The motor parameters used for simulation are

- $R_s = 2 \, \Omega$, $L_{is} = 0.017 \, \text{H}$, $L_{Es} = 0.0221 \, \text{H}$, $L_{Ds} = 0.0204 \, \text{H}$,
- $L_{Qs} = 0.0187 \, \text{H}$, $L_{DEs} = 0.00884 \, \text{H}$, $R_E = 0.6 \, \Omega$,
- $R_D = 0.6 \, \Omega$, $R_Q = 6 \, \Omega$, $p = 2$, $J = 0.024 \, \text{kgm}^2$, $m^2 = 0.6$

The characteristics $\psi_m(i_m'), L_m(i_m'), L_{md}(i_m')$ are given in Fig. 8.

![Fig. 8 – Characteristics $\psi_m(i_m'), L_m(i_m'), L_{md}(i_m')$.](image)

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