A NEW METHOD FOR SOLVING THE PERIODIC STEADY STATE OF NONLINEAR CIRCUITS

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The periodicity condition of state variable ensures the circuit periodic steady state. Circuits with linear dynamic elements and nonlinear resistors can be solved iteratively using the method of equivalent sources, by replacing the nonlinear resistors with controlled sources having constant resistance throughout the iterations. The final values of the state variables appear as affine functions of their initial values and only the additive term changes iteratively. By imposing a periodicity condition the initial values of the state variables are obtained by solving a linear system of equations in which the system matrix remains unchanged during each iteration and only the right hand side is modified. This matrix is inverted only once before the iterations begin.

1. INTRODUCTION

The most popular method for solving periodic steady state of nonlinear circuits is the brute force, often used by the commercial simulators. By choosing arbitrary initial values for the state variables the transient analysis is performed until an asymptotic solution is reached. Even if various acceleration procedures are used [1–2] the computation time can become huge especially if the circuit has "large time constants". An important speed-up is obtained by handling the nonlinearity using the Equivalent Sources Method (ESM) [3] and applying the numerical form of the convolution integrals on time intervals. In [4] this technique was successfully applied for circuits having sources with large frequency differences.

The harmonic balance method can solve this problem also, but it leads to a large computation time if the nonlinearities are strong or if the sources have a broad spectrum of harmonics. Using ESM the circuit is solved for each harmonic [5]. The harmonics spectrum is progressively enlarged starting with a reduced number of harmonics and selecting the most relevant harmonics of the equivalent sources.

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In this paper a new procedure for finding the periodic steady state of nonlinear circuits is presented. The method is based on ESM and on imposing the state variable periodicity. The nonlinear resistive elements are replaced by linear generators for which the sources are nonlinearly controlled by the resistor voltage or current. Except the equivalent sources, the parameters of all other electrical elements remain unchanged for each iteration, allowing superposition. The state variables are determined by convolution integrals represented as invariant vectors for all iterations. These vectors are computed only once, before starting the iterations. The state variables are easily obtained by multiplying them with the source vectors. The periodic steady state is imposed by enforcing the initial values of the state variables be equal with their final values at the end of the period. A system of equations results, having the same matrix for all iterations and the right hand side (r. h. s.) changing. This matrix is inverted only once, before starting the iterations. The controlling quantities of the equivalent sources are then determined and the sources voltages are corrected for the next iteration.

2. ESM TREATMENT OF THE NONLINEARITY

The nonlinear resistive elements are grouped as a p-port and, for simplicity, it is assumed that the port voltages $\boldsymbol{u}$ are the controlling quantities $\boldsymbol{i} = \hat{F}(\boldsymbol{u})$ at any time $t \in [0,\ T]$, where T is the excitation period. The function $\hat{F} : R^p \to R^p$ is assumed Lipschitzian

$$\left\| \hat{F}(\boldsymbol{x}) - \hat{F}(\boldsymbol{y}) \right\| \le \Lambda \left\| \boldsymbol{x} - \boldsymbol{y} \right\| \ \forall\, \boldsymbol{x}, \boldsymbol{y} \in R^p , \tag{1}$$

and uniformly monotone

$$\left\langle \hat{F}(\boldsymbol{x}) - \hat{F}(\boldsymbol{y}), \boldsymbol{x} - \boldsymbol{y} \right\rangle \ge \lambda \left\| \boldsymbol{x} - \boldsymbol{y} \right\|^2 \ \forall\, \boldsymbol{x}, \boldsymbol{y} \in R^p , \lambda > 0 . \tag{2}$$

ESM replaces the nonlinear p-port with p voltage generators described by

$$u_k = r_k i_k + e_k , \tag{3}$$

where the source voltage e_k is a non-linear function of u [1]

$$e_k = u_k - r_k \hat{F}_k(\boldsymbol{u}) \equiv \hat{H}_k(\boldsymbol{u}) , \tag{4}$$

where $\hat{F}_k$ and $\hat{H}_k$ are the components of the functions $\hat{F}$ and $\hat{H}$. It can be proved [3] that if r_k is chosen as $r_k = r \in \left(0,\ 2\lambda / \Lambda^2\right)$, k = 1,2,...,p, then $\hat{H}$ is a contraction $\left\| \hat{H}(\mathbf{x}) - \hat{H}(\mathbf{y}) \right\|_g \le \theta \left\| \mathbf{x} - \mathbf{y} \right\|_g$, with the contraction factor

$\theta = \sqrt{1 - 2r\lambda + r^2\Lambda^2} < 1$. In this case the scalar product and norm are defined as $\langle \boldsymbol{u}, \boldsymbol{v} \rangle_g = \boldsymbol{u}^{\mathrm{T}} \boldsymbol{G} \boldsymbol{v}$ and $\|\boldsymbol{u}\|_g^2 = \boldsymbol{u}^{\mathrm{T}} \boldsymbol{G} \boldsymbol{u}$, respectively, where $\boldsymbol{G}$ is the conductance diagonal matrix ($G_{kk} = g = 1/r$) defined for the nonlinear elements and T denotes the transpose. If the constitutive relations are time-invariant, then $\hat{H}$ is also a contraction on $R^p \times [0, T]$ and (1) becomes

$$\left\| \hat{H}(\boldsymbol{x}) - \hat{H}(\boldsymbol{y}) \right\|_{g,T} \le \theta \|\boldsymbol{x} - \boldsymbol{y}\|_{g,T}, \tag{5}$$

where $\langle \mathbf{u}, \mathbf{v} \rangle_{g,T} = \int_o^T \langle \mathbf{u}, \mathbf{v} \rangle_g \, \mathrm{d}t$, and $\|\mathbf{u}\|_{g,T}^2 = \int_0^T \|\mathbf{u}\|_g^2 \, \mathrm{d}t$.

In the case of a nonlinear one-port resistor, conditions (1) and (2) correspond to a strictly positive increasing bounded function. The conductance can be chosen as $\frac{1}{r} = g > \frac{g_{\min}}{2}$ and a contraction factor $\theta = \mathrm{Max}(1 - r \cdot g_{\min}, r \cdot g_{\max} - 1)$ is obtained [1, 2], where $g_{\min} = \operatorname*{Inf}_{u',u''} \frac{\hat{f}(u') - \hat{f}(u'')}{u' - u''}$, $g_{\max} = \operatorname*{Sup}_{u',u''} \frac{\hat{f}(u') - \hat{f}(u'')}{u' - u''}$, and $i = \hat{f}(u)$ is the u-i resistor relation. The smallest value of the contraction factor is $\theta_{\mathrm{opt}} = \frac{g_{\max} - g_{\min}}{g_{\max} + g_{\min}}$ and is obtained when $\frac{1}{r_{\mathrm{opt}}} = g_{\mathrm{opt}} = \frac{g_{\max} + g_{\min}}{2}$.

If the resistive elements are current or mixed controlled our method remains valid by using a dual formulation.

3. THE NEW METHOD USING ESM FOR SOLVING THE PERIODIC STEADY STATE

The steps for the periodic steady state computation of the nonlinear circuit are:

a) The matrix which gives the final values of the state variables from their initial values is computed. The final values are not known yet at this point. This matrix is evaluated only once and remains unchanged for all iterations.

b) Arbitrary initial values $\boldsymbol{e}^{(1)}$ for the equivalent sources voltages are chosen.

c) Having the $\boldsymbol{e}^{(i)}$ sources voltages and the independent sources voltages $\boldsymbol{s}$ their contribution to the final values of the state variables (at time t=T) are computed. Knowing these contributions and the matrix from a), the initial values of the state variables are computed as a result of the repetition condition of the state variable after one period.

d) Having the sources $\boldsymbol{s}$, $\boldsymbol{e}^{(i)}$ and the state variables, the control quantities $\boldsymbol{u}^{(i)} = \hat{L}(\boldsymbol{e}^{(i)})$ are computed by solving a linear circuit.

e) The sources voltages $\boldsymbol{e}^{(i+1)} = \hat{H}(\boldsymbol{u}^{(i)})$ are corrected.

If the error $\left\|\boldsymbol{e}^{(i+1)} - \boldsymbol{e}^{(i)}\right\|$ is not small enough the algorithm returns to step b).

The convergence. The circuit is assumed to have N_R linear resistors, N_L inductors and N_C capacitors. Let $(\boldsymbol{u}',\boldsymbol{u}'')$ and $(\boldsymbol{i}',\boldsymbol{i}'')$ be two solutions of the linear circuit which correspond to the $(\boldsymbol{e}',\boldsymbol{e}'')$ sources voltages and let $\Delta\boldsymbol{u} = \boldsymbol{u}'-\boldsymbol{u}''$, $\Delta\boldsymbol{i} = \boldsymbol{i}'-\boldsymbol{i}''$ be the difference solution corresponding to the $\Delta\boldsymbol{e} = \boldsymbol{e}'-\boldsymbol{e}''$ difference. The voltage difference is zero for the independent sources. Having a linear circuit, Tellegen theorem gives

$$\begin{aligned}\langle \Delta\mathbf{u}, \Delta\mathbf{i} \rangle = \sum_{\rho=1}^{N_R} r_\rho (\Delta i_\rho)^2 + \sum_{k=1}^{p} g_k (\Delta u_k)^2 - \sum_{k=1}^{p} g_k \Delta u_k \Delta e_k + \\ + \sum_{\lambda=1}^{N_L} \Delta i_\lambda L_\lambda \frac{\mathrm{d}\Delta i_\lambda}{\mathrm{d}t} + \sum_{\gamma=1}^{\mathrm{N}_C} \Delta u_\gamma C_\gamma \frac{\mathrm{d}\Delta u_\gamma}{\mathrm{d}t} = 0.\end{aligned} \tag{6}$$

Integrating over a period and taking into account the periodicity condition of the state variables, the previous relation becomes

$$\int_0^T \sum_{\rho=1}^{N_R} r_\rho (\Delta i_\rho)^2 \,\mathrm{d}t + \int_0^T \sum_{k=1}^{p} g_k (\Delta u_k)^2 \mathrm{d}t - \int_0^T \sum_{k=1}^{p} g_k \Delta u_k \Delta e_k \mathrm{d}t = 0 \,. \tag{7}$$

Because the first term is positive one obtains

$$\int_0^T \sum_{k=1}^{p} g_k (\Delta u_k)^2 \mathrm{d}t - \int_0^T \sum_{k=1}^{p} g_k \Delta u_k \Delta e_k \mathrm{d}t \le 0 \;\text{ or }\; \|\Delta\boldsymbol{u}\|_{g,T}^2 \le \langle \Delta\boldsymbol{u}, \Delta\boldsymbol{e} \rangle_{g,T} \,. \tag{8}$$

Therefore $\|\Delta\boldsymbol{u}\|_{g,T} \le \|\Delta\boldsymbol{e}\|_{g,T}$. So, the function $\hat{L}$, that relates the $\boldsymbol{u}$ solution to the $\boldsymbol{e}$ sources voltages by solving the linear circuit, is non-expansive. Because $\hat{H}$ is a contraction, the iterative method described in steps a), ..., e) is a Picard-Banach procedure, that converges at the fixed point of the composed function $\hat{H} \circ \hat{L}$.

4. THE ANALYTIC SOLUTION OF THE LINEAR CIRCUIT

In order to simplify the presentation, a RC circuit is analyzed, the state variables being the capacitor voltages only.

The currents at the ports where the capacitors are connected are

$$\boldsymbol{i}_c = -\boldsymbol{G}_c\boldsymbol{u}_c + \boldsymbol{G}_{cs}\boldsymbol{s} + \boldsymbol{G}_{ce}\boldsymbol{e} = \boldsymbol{C}\frac{\mathrm{d}\boldsymbol{u}_c}{\mathrm{d}t}, \tag{9}$$

where $\boldsymbol{C}$ is the capacitors matrix (a diagonal matrix if the capacitors are not coupled), $\boldsymbol{G_c}$ is the positive defined symmetric matrix of the ports conductances where the capacitors are connected to, $\boldsymbol{G_{cs}}$ is the matrix of the transfer conductances between the ports where the independent sources are connected and the capacitor ports, $\boldsymbol{G_{ce}}$ is the matrix of the transfer conductances between the ports where the equivalent sources are connected and the capacitors ports

$$G_{c_{l,m}} = -i_{c_l}\Big|_{\substack{\mathbf{s}=\mathbf{0},\mathbf{e}=\mathbf{0}\\ u_{c_m}=1,\\ u_{c_k}=0 \ \text{for}\ k\neq m}}, \quad G_{cs_{l,m}} = -i_{c_l}\Big|_{\substack{\mathbf{u}_c=\mathbf{0},\mathbf{e}=\mathbf{0}\\ s_m=1,\\ s_k=0 \ \text{for}\ k\neq m}},$$

$$G_{ce_{l,m}} = -i_{c_l}\Big|_{\substack{\mathbf{u}_c=\mathbf{0},\mathbf{s}=\mathbf{0}\\ e_m=1,\\ e_k=0 \ \text{for}\ k\neq m}}. \tag{10}$$

From (9) the following equation is obtained

$$\frac{\mathrm{d}\boldsymbol{u}_c}{\mathrm{d}t} + \boldsymbol{\alpha}\boldsymbol{u}_c = \boldsymbol{P}\boldsymbol{s} + \boldsymbol{Q}\boldsymbol{e} \equiv \boldsymbol{f}, \tag{11}$$

where $\alpha = \boldsymbol{C}^{-1}\boldsymbol{G}_c$, $\boldsymbol{P} = \boldsymbol{C}^{-1}\boldsymbol{G}_{cs}$, $\boldsymbol{Q} = \boldsymbol{C}^{-1}\boldsymbol{G}_{ce}$. The solution of equation (11) is

$$\boldsymbol{u}_c(t) = \mathrm{e}^{-\boldsymbol{\alpha}t}\int_0^t \mathrm{e}^{\boldsymbol{\alpha}\tau}\boldsymbol{f}(\tau)\mathrm{d}\tau + \mathrm{e}^{-\boldsymbol{\alpha}t}\boldsymbol{u}_c(0), \tag{12}$$

where $\mathrm{e}^{-\boldsymbol{\alpha}t}$ is a matrix function. From the capacitor voltage periodicity condition $\boldsymbol{u}_c(T) = \boldsymbol{u}_c(0)$, the initial value is obtained as

$$\boldsymbol{u}_c(0) = (1-\mathrm{e}^{-\boldsymbol{\alpha}T})^{-1}\mathrm{e}^{-\boldsymbol{\alpha}T}\int_0^T \mathrm{e}^{\boldsymbol{\alpha}\tau}\boldsymbol{f}(\tau)\mathrm{d}\tau \ . \tag{13}$$

The voltages at the ports of the nonlinear resistor are

$$\boldsymbol{u}(t) = \boldsymbol{A}\boldsymbol{s}(t) + \boldsymbol{B}\boldsymbol{e}(t) + \boldsymbol{D}\boldsymbol{u}_c(t), \tag{14}$$

where $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{D}$ are the transfer matrices between the ports where the independent sources, the equivalent sources and the capacitors, respectively are connected and the ports l where the nonlinear resistors are connected

$$A_{l,m} = u_l\Big|_{\substack{\mathbf{u}_c=\mathbf{0},\mathbf{e}=\mathbf{0}\\ s_m=1,\\ s_k=0 \ \text{for}\ k\neq m}}, \quad B_{l,m} = u_l\Big|_{\substack{\mathbf{u}_c=\mathbf{0},\mathbf{s}=\mathbf{0}\\ e_m=1,\\ e_k=0 \ \text{for}\ k\neq m}}, \quad D_{l,m} = u_l\Big|_{\substack{\mathbf{s}=\mathbf{0},\mathbf{e}=\mathbf{0}\\ u_{c_m}=1,\\ u_{c_k}=0 \ \text{for}\ k\neq m}}. \tag{15}$$

The ***P***, ***Q***, ***A***, ***B***, ***D***, $e^{-\alpha t}$ and $(1-e^{-\alpha t})^{-1}$ matrices are computed only once, before the first iteration.

The numerical solution. The previous method can be easily applied only in the case of at most 2 x 2 matrices. Unfortunately, in the case of circuits with many state variables, the computational effort required for the computation of the matrix function $e^{-\alpha t}$ is significant.

However, the proposed method can be easily applied for the case of complicated circuits, by employing a numerical method for solving the linear circuit.

The $[0, T]$ interval is divided in N equal subintervals $[t_n, t_{n+1}]=[(n-1)\Delta t, n\Delta t]$, where $\Delta t = t_{n+1} - t_n$. By assuming on the $[t_1, t_2]$ interval the following excitations

$$e'_m = 1 - t/\Delta t \ \text{ for } \ t \in [0,T], \ e'_m = 0 \ \text{ elsewhere}, \tag{16}$$

and

$$e''_m = t/\Delta t \ \text{ for } \ t \in [0,\Delta t], \ e''_m = 2 - t/\Delta t \ \text{ for } \ t \in [\Delta t, 2\Delta t], \\ e''_m = 0, \ \text{ elsewhere}, \tag{17}$$

for the independent and controlled sources, the linear circuit response at any linear excitation on the $[t_n, t_{n+1}]$ interval can be obtained. Thus, only the computation of the circuit response vectors to the excitations in (16) and (17) is required. The transient response to the (16) and (17) excitations could be obtained numerically using the companion circuit. Each $[t_n, t_{n+1}]$ interval is divided in equal intervals. When employing nodal analysis, the system matrix remains unchanged and only the r. h. s. modifies as a function of the considered sources. The circuit response at the initial unitary values of the state variables is obtained with the same matrix. The initial values of the state variables are obtained by imposing the periodicity condition. Then the values of the control quantities are evaluated. The iteration is finished by correcting the voltages of the controlled sources with (4).

In a following paper we will present in detail the numerical method for solving the periodic steady state of the linear circuit and the important advantages that follow from applying the presented method.

5. NUMERICAL RESULTS

The independent pulsed voltage source s in Fig. 1 has amplitude s_0, period T and pulse width t_c (Fig. 4), being described by

$$s(t) = \begin{cases} s_0, & \text{for } t \in [0, t_c) \\ 0, & \text{for } t \in [t_c, T) \end{cases}. \tag{18}$$

The diode nonlinearity is given by the slopes of the half-lines of the *u*-*i* characteristic from Fig. 3.

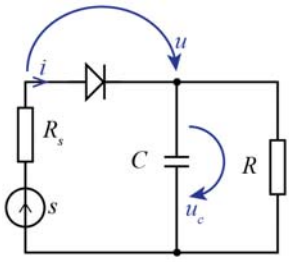

Fig. 1 – The nonlinear circuit.

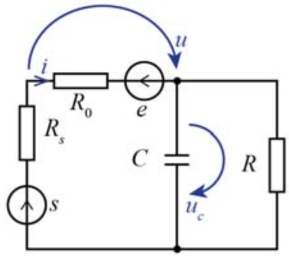

Fig. 2 – The linear circuit with the nonlinear voltage controlled *e* source.

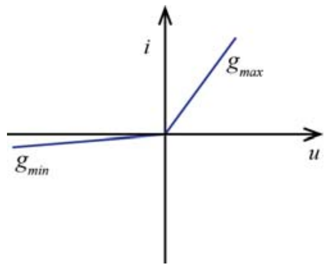

Fig. 3 – The current-voltage characteristic of the nonlinear element.

Figure 2 shows the "linear" circuit in which the nonlinear element has been replaced by the voltage generator containing the voltage source $e(u)$ controlled by voltage u at the generator terminals. The generator resistance was chosen $r_0=1/g_{\max}$, giving a contraction factor of $\theta=1-g_{\min}/g_{\max}$. Due to the simple nature of the chosen circuit the linear circuit can be solved analytically at each iteration. The interval $[0, T]$ is divided in N equal intervals Δt, so that $t_c/T = n_c/N$. The equivalent sources are assumed to have a linear variation on the intervals.

$$e(t) = e_k + \frac{1}{\Delta t}(t-t_k)(e_{k+1}-e_k)\,,\; k = 1,2,\ldots,N, \tag{19}$$

where $e_k = e(t_k)$. Due to the simple nature of the circuit the matrices $\boldsymbol{P}$, $\boldsymbol{Q}$, $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{D}$, $e^{-\alpha t}$, $(1-e^{-\alpha t})^{-1}$ become numbers:

$$G_c = -i_c\big|_{\substack{s=0,e=0\\ u_c=1}} = \frac{1}{R} + \frac{1}{R_s+R_0}\,, \qquad G_{cs} = i_c\big|_{\substack{u_c=0,e=0\\ s=1}} = \frac{1}{R_s+R_0}\,,$$

$$G_{ce} = i_c\big|_{\substack{u_c=0,s=0\\ e_m=1}} = -\frac{1}{R_s+R_0}\,, \qquad \alpha = \frac{G_c}{C}\,, \qquad P = \frac{G_{cs}}{C}\,, \qquad Q = \frac{G_{ce}}{C}\,,$$

$$A = u\big|_{\substack{u_c=0,e=0\\ s=1}} = \frac{R_0}{R_0+R_s}\,,\; B = u\big|_{\substack{u_c=0,s=0\\ e=1}} = \frac{R_s}{R_0+R_s}\,,\; D = u\big|_{\substack{s=0,e=0\\ u_c=1}} = -\frac{R_0}{R_0+R_s}\,.$$

For time $t_n+1 = n\Delta t$, (12) becomes

$$u_c(t_{n+1}) = u_c(n\Delta t) = u_{c_{n+1}} = u_{c_{s;n+1}} + u_{c_{e;n+1}} + \Gamma_{n+1}u_c(0)\,,\; n = 1,2,\ldots, N, \tag{20}$$

where $\Gamma_{n+1} = e^{-\alpha n\Delta t}$, $n = 1,2,\ldots,N$, $\Gamma_1=1$ and

$$u_{c_{s;n+1}} = \frac{G_{cs}}{G_c} \times \begin{cases} (\Gamma_{n+1-n_c} - \Gamma_{n+1}) s_0, & \text{for } n > n_c \\ (1 - \Gamma_{n+1}) s_0, & \text{for } n \le n_c \end{cases}, \quad u_{c_{s;1}} = 0, \tag{21}$$

$$u_{c_{e;n+1}} = \frac{G_{ce}}{G_c}\left[(e_{n+1} - \Gamma_2 e_n) - \frac{1}{\alpha \Delta t}(1-\Gamma_2)(e_{n+1} - e_n)\right] + \Gamma_2 u_{c_{e;n}}, \quad u_{c_{e;1}} = 0. \tag{22}$$

From the periodicity condition $u_{c_{N+1}} = u_c(0)$ the initial value of the capacitor voltage is obtained

$$u_c(0) = \frac{u_{c_{s;N+1}} + u_{c_{e;N+1}}}{1 - \Gamma_{N+1}}. \tag{23}$$

The voltage $u_{c_{n+1}}$ can be calculated with (20) and the voltage u_{n+1} with (14)

$$u_{n+1} = As_{n+1} + Be_{n+1} + Du_{c_{n+1}}, \quad n = 0,1,2,\ldots, N. \tag{24}$$

Replacing (24) in (4) the new voltage e of the equivalent source is obtained.

Example 1. The circuit parameters are chosen as $R = 10\ \Omega$, $s_0 = 2$ V, $T = 10^{-3}$ s, $t_c/T = 0.2$, $R_s = 10\ \Omega$, $C = 10\ \mu$F and $g_{\min} = 10^{-6}\ \Omega^{-1}$, $g_{\max} = 1\ \Omega^{-1}$ which gives a contraction factor $\theta = 0.999999$. The contraction factor is very close to, but smaller than 1 and thus, is an assurance of the convergence.

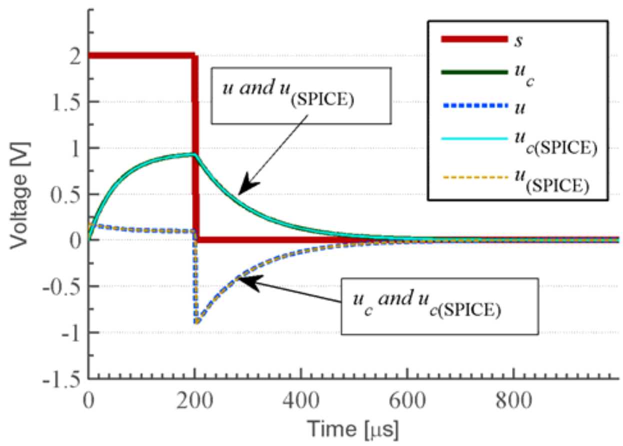

Fig. 4 – The transient response of the circuit to the voltage source s. Voltages u and u_c are obtained with the proposed method. Voltages $u_{c(\text{SPICE})}$ and $u_{(\text{SPICE})}$ are obtained with SPICE.

The problem was solved with the proposed method and with SPICE. The variation with time of the voltages from Fig. 1 are shown in Fig. 4.

For the proposed method the period was divided in 4 000 equal intervals and a relative error of $\mathrm{er} = \frac{1}{s_0}\left\| e^{(i+1)} - e^{(i)} \right\| < 0.96 \cdot 10^{-7}$ was reached after 148 iterations. It can be said that the solution is located in a sphere of the exact solution e^* of radius [3] $\frac{1}{s_0}\left\| e^* - e^{(i)} \right\| < \frac{er}{1-\theta} = 0.096$. The computing time was 0.06 s on a 2.128 GHz Intel processor notebook.

There is good agreement with the results obtained with SPICE.

Example 2. The circuit proposed in [4] can be difficult to be solved numerically. The independent voltage source s is amplitude modulated as $s(t) = s_0 \sin(2\pi f_1)\sin(2\pi f_2)$ where s_0= 2 V, the signal frequency is f_1= 1 kHz and the carrier frequency is f_2= 0.1 GHz. The difficulty in solving this circuit is given by the fact that the source has very large time variations which requires a very small time step for the numerical calculations. Furthermore, the nonlinearity of the diode is strong. The other circuit parameters are R = 1 kΩ, R_s = 10 Ω, C = 1 μF. The slopes of the half-lines of the u-i diode characteristic are $g_{\min}= 10^{-6}\ \Omega^{-1}$ and $g_{\max}= 0.1\ \Omega^{-1}$. For the equivalent source a resistance of $r_0= 1/g_{\max}$ is chosen, which gives a contraction factor of $\theta = 0.999999$. The procedure presented in [4] combines ESM with brute force. In order to accelerate the computation, the time step was initially chosen to be $T_2/4$, than it was reduced to $T_2/12$, where $T_2= 1/f_2$. The iterative corrections of the equivalent sources stops at a relative error smaller than 10^{-3}. The same accuracy was employed for the capacitor voltage periodicity when checking the asymptotic solution. The computation time was 1287.22 s on the previous mentioned notebook.

In this paper a time step of $T_2/6$ was used. A relative error of $\mathrm{er} < 0.995 \cdot 10^{-7}$ was imposed to stop the iterative corrections of the equivalent sources. The solution is within a sphere of the exact solution of radius 0.00995. The number of iterations was 114 and the computing time was 7.32 s.

The variation with time of the capacitor voltage for a period $T_1= 1/f_1$ is plotted in Fig. 5 and a detailed variation is presented in Fig. 6.

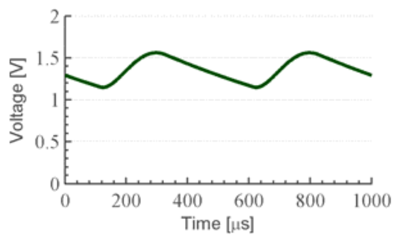

Fig. 5 – The variation with time of the capacitor voltage.

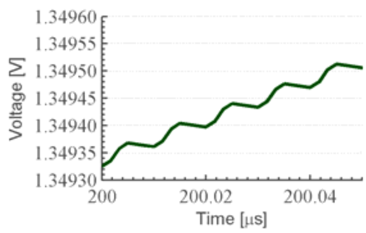

Fig. 6 – Detail of the variation with time of the capacitor voltage

Example 3. The advantages of our method when compared to brute force are spectacular when the “time constants” of the circuit are large.

In the case of brute force, state variable periodicity is achieved after many periods, leading to large execution times. If the parameters for the elements in the previous example are changed to C = 100 μF, f_1 = 10 kHz, f_2 = 1 GHz then the time constant of the linear circuit becomes $\tau = 1/\alpha = C/G_c = 2\cdot 10^{-3}$ s and $20T_1$ periods are required for the capacitor voltage to change e times. The relation (23) can be understood as a “over-relaxation” of the capacitor voltage. According to (20), when choosing an initial value of zero for the capacitor voltage, after a period, the value $u_c(T) = u_{c_{s;N+1}} + u_{c_{e;N+1}}$ is obtained, which is then amplified $\dfrac{1}{1-\Gamma_{N+1}} = \dfrac{1}{1-\mathrm{e}^{-\alpha T_1}} \cong 20$ times by (23).

A relative error of $0.94\cdot 10^{-7}$ was reached in a number of 116 iterations and thus the solution was located within a sphere of the exact solution of radius 0.0094. The computing time was 7.45 s.

6. **CONCLUSIONS**

The advantage of the presented method when compared with brute force is obvious in the case of linear circuits. Without retaining matrices of large dimensions, the voltages $u_{c_{s;N+1}}\ (u_{c_{e;N+1}} = 0)$ are computed through numerical integration. The initial value of the state variable can then be computed with (23). The periodic steady state solution is then obtained by integrating again in the time domain. In the case of complicated circuits relation (23) is replaced by a system of equations of dimension equal to the number of state variables.

The extension of this method to nonlinear circuits can be performed by applying ESM, but iterations are required for the correction of the equivalent sources.

When compared with brute force the procedure described in this paper has clear advantages in the case of circuits presenting strong nonlinearities and large “time constants”. The method of harmonic balance can be successfully used when only a few harmonics are required in describing the periodic steady state solution.

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