A VARIABLE STEP-SIZE PROPORTIONATE NLMS ALGORITHM FOR ECHO CANCELLATION

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The proportionate normalized least-mean-square (PNLMS) algorithms were developed in the context of network echo cancellation. They outperform the normalized least-mean-square (NLMS) algorithm only when the echo path is sparse. Unfortunately, real-world network echo path may not be that sparse sometimes, while the acoustic echo paths are usually less sparse. The improved PNLMS (IPNLMS) algorithm is less sensitive to the sparseness character of the echo path. In order to enhance the performance of this algorithm we propose a variable step-size (VSS) version of it, providing a feasible solution for the conflicting requirements of fast convergence and low misadjustment. The simulation results prove that the proposed algorithm performs very well despite of the character of the echo path, being suitable for both network and acoustic echo cancellation.

1. INTRODUCTION

Echo cancellation is one of the most popular applications of adaptive filtering [1]. In both network and acoustic echo cancellation contexts the basic solution is to build a model of the impulse response of the echo path using an adaptive filter, which provides at its output a replica of the echo. The fundamental difficulties facing the design of an adaptive filter for echo cancellation include the long duration and time-varying nature of the echo path as well as the highly non-stationarity character of the speech excitation signal. Even through various kinds of adaptive algorithms [2] are theoretically applicable for echo cancellation, in most cases a simple and robust algorithm outperforms more sophisticated solutions. Therefore, in many applications with limited precision and processing power, the normalized least-mean-square (NLMS) algorithm is preferred.

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In order to enhance the performance of the NLMS algorithm in terms of its convergence rate and tracking, a proportionate NLMS (PNLMS) algorithm has been proposed in [3], in the context of the network echo problem. It takes advantage of the fact that the network echo path is sparse in nature, *i.e.*, only a small portion of the coefficients is significantly different from zero (active coefficients). The idea behind PNLMS algorithm is to update each coefficient of the filter independently of the others by adjusting the adaptation step-size in proportion to the magnitude of the estimated filter coefficient. It redistributes the adaptation gains among all the coefficients and emphasizes the large ones in order to speed up their convergence, achieving a fast initial convergence rate. Unfortunately, after this initial phase, the convergence rate of the PNLMS algorithm slows down significantly, even becoming slower than NLMS. This is due to the fact that the equations used to calculate the step-size control factors are not based on any optimization criteria but are designed in an ad-hoc way. In order to deal with this problem, several versions of the PNLMS algorithm were proposed (see [4] and references therein). Nevertheless, they still assume the sparse character of the echo path impulse response, which could be proper for network echo path, but is far from true in the acoustic echo context, when the echo path may not be that sparse. Even in the case of a real-world network echo canceller, it is difficult to a priori know the sparseness "degree" of the echo path. Taking these considerations into account, an improved PNLMS (IPNLMS) algorithm was proposed in [5], based upon a rule that better exploits the "proportionate" idea. It behaves well even in the case when the echo path is nonsparse.

In this paper we propose a combination between the IPNLMS algorithm and a variable step-size NLMS (VSS-NLMS) algorithm [6]. The VSS approach provides a simple and efficient solution to the conflicting requirements of fast convergence and low misadjustment needed by any adaptive system. In addition, due to the IPNLMS features, the proposed algorithm is less sensitive to the character of the echo path, being suitable for both network and acoustic echo cancellation applications.

The remainder of the paper is organized as follows. Section 2 presents the backgrounds of PNLMS algorithms. The proposed variable step-size proportionate NLMS algorithm is derived in Section 3. Experimental results are shown in Section 4. Finally, Section 5 concludes this work.

2. **BACKGROUNDS OF PNLMS ALGORITHMS**

Let us consider a single-talk echo canceller configuration, where we try to model an echo path with impulse response $\boldsymbol{h} = [h_0, h_1, \ldots, h_{L-1}]^{\mathrm{T}}$ using an adaptive filter, $\hat{\boldsymbol{h}}(n) = [\hat{h}_0(n), \hat{h}_1(n), \ldots, \hat{h}_{L-1}(n)]^{\mathrm{T}}$. Superscript T denotes transposition, L is the length of the systems, and n is the time index. The desired signal for the adaptive filter is $y(n) = \mathbf{h}^{\mathrm{T}}\mathbf{x}(n) + v(n)$, where $\mathbf{x}(n) = [x(n), x(n-1), \ldots, x(n-L+1)]^{\mathrm{T}}$ is a real-valued vector containing the L most recent samples of the input signal (*i.e.*, far-end) and $v(n)$ is the system noise, assumed to be stationary, real-valued, and independent of the input signal $x(n)$.

The NLMS algorithm [2] can be resumed by the following equations:

$$e(n) = y(n) - \hat{\mathbf{h}}^{\mathrm{T}}(n-1)\mathbf{x}(n), \tag{1}$$

$$\hat{\mathbf{h}}(n) = \hat{\mathbf{h}}(n-1) + \frac{\mu \mathbf{x}(n) e(n)}{\mathbf{x}^T(n)\mathbf{x}(n) + \delta_{NLMS}}, \tag{2}$$

where μ is the adaptation step ($0 < \mu < 2$) and δ_{NLMS} is the regularization factor, i.e., a positive constant that prevents division by small numbers.

The PNLMS algorithm [3] assigns an individual step-size to each filter coefficient, in such a way that a larger coefficient receives a larger increment, thus increasing the convergence rate of that coefficient. Consequently, the active coefficients are adjusted faster than non-active coefficients (i.e. small or zero coefficients), so that the PNLMS algorithm converges faster than NLMS for sparse impulse responses, when only a small percentage of coefficients is significant. The update equation (2) is modified as follows:

$$\hat{\mathbf{h}}(n) = \hat{\mathbf{h}}(n-1) + \frac{\mu \mathbf{G}(n-1)\mathbf{x}(n) e(n)}{\mathbf{x}^T(n)\mathbf{G}(n-1)\mathbf{x}(n) + \delta_{PNLMS}}, \tag{3}$$

where $\mathbf{G}(n-1)$ is a diagonal matrix that adjusts the step-sizes of the individual taps of the filter and $\delta_{PNLMS} = \delta_{NLMS}/L$ is the regularization factor of the PNLMS algorithm. The diagonal elements of $\mathbf{G}(n)$, denoted by $g_l(n)$ with $0 \le l \le L-1$, are calculated using the following procedure:

$$\gamma_l(n) = \max\left\{\rho \max\left\{\delta_p, \left|\hat{h}_0(n)\right|, \left|\hat{h}_1(n)\right|, \ldots, \left|\hat{h}_{L-1}(n)\right|\right\}, \left|\hat{h}_l(n)\right|\right\}, \tag{4}$$

$$g_l(n) = \frac{\gamma_l(n)}{\sum_{i=0}^{L-1} \gamma_i(n)}. \tag{5}$$

Parameters ρ and δ_p are positive numbers with typical values $\rho = 5/L$ and $\delta_p = 0.01$. The constant ρ prevents the very small coefficients from stalling and the parameter δ_p regularizes the updating when all coefficients are zero at initialization.

The main limitation of the PNLMS algorithm is that it assigns too much adaptation gains to large coefficients to speed up their convergence at the cost of small coefficients. Accordingly, after an initial fast convergence phase, this algorithm slows down and become even slower than NLMS. A version of the PNLMS algorithm, called PNLMS++, was proposed in [7]. It alternates between the PNLMS and NLMS algorithms during adaptation based on the following simple rule. For odd-numbered time steps it computes $\boldsymbol{G}(n)$ as in PNLMS, while for even-numbered steps it uses $\boldsymbol{G}(n) = \boldsymbol{I}$, where $\boldsymbol{I}$ is the identity matrix. The PNLMS++ algorithm improves the performance of PNLMS after the initial convergence, making it to converge at least as fast as the NLMS algorithm. Nevertheless, it does not exploit the structure of the estimated impulse response. The algorithm is efficient for the extreme cases, when the impulse response is sparse or highly dispersive. Otherwise, when the impulse response is between sparse and dispersive, it performs almost like the NLMS algorithm because the rule used in PNLMS become less important. A similar type of algorithm, called composite PNLMS (CPNLMS), was developed in [8]. It switches from the PNLMS algorithm to the NLMS algorithm when slow convergence is detected. Nevertheless, its limitations are the same.

The IPNLMS algorithm proposed in [5] uses a smother choice as compared to (4). In this case, the elements of $\boldsymbol{G}(n)$ will be computed as:

$$g_l(n) = \frac{1-\alpha}{2L} + (1+\alpha)\frac{\left|\hat{h}_l(n)\right|}{2\sum_{i=0}^{L-1}\hat{h}_i(n)+\xi}, \text{ for } 0 \le l \le L-1, \tag{6}$$

where $-1 \le \alpha \le 1$, and ξ is a very small positive number to avoid division by zero, especially at the beginning of the adaptation where all the taps of the filter are initialized to zero. It can easily be checked that for $\alpha = -1$ the IPNLMS and NLMS algorithms are equivalent, while for α close to 1 the IPNLMS algorithm behaves like PNLMS algorithm. In practice, good choices for the parameter α are 0 or –0.5. The regularization parameter of the IPNLMS algorithm should be taken as $\delta_{\mathrm{IPNLMS}}= = \delta_{\mathrm{NLMS}}(1-\alpha)/2L$. The IPNLMS algorithm outperforms PNLMS for both sparse and nonsparse impulse responses.

3. VARIABLE STEP-SIZE PNLMS ALGORITHM

The VSS-NLMS adaptive algorithms provide an efficient solution to the conflicting requirements of fast convergence and low misadjustment needed by any adaptive system [9]. Recently, a nonparametric VSS-NLMS (NPVSS-NLMS) was proposed [6]. It is very simple and easy to control in real-world applications. As compared to other VSS-NLMS algorithms, this algorithm is derived with almost no assumptions and it requires a minimum a priori parameters. The basic idea is to define the a posteriori error signal as

$$\varepsilon(n) = y(n) - \hat{\boldsymbol{h}}^{\mathrm{T}}(n)\boldsymbol{x}(n) = \boldsymbol{x}^{\mathrm{T}}(n)\left(\boldsymbol{h} - \hat{\boldsymbol{h}}(n)\right) + v(n), \text{ for } 0 \le l \le L-1, \tag{7}$$

and to find the step-size parameter such that

$$E\left\{\varepsilon^2(n)\right\} = \sigma_v^2, \tag{8}$$

where $E\{\bullet\}$ denotes the mathematical expectation and $\sigma_v^2 = E\left\{v^2(n)\right\}$ is the power of the system noise. Imposing (8) it results that [6]

$$\mu_{\mathrm{NPVSS}}(n) = \frac{1}{\mathbf{x}^T(n)\mathbf{x}(n) + \delta_{\mathrm{NPVSS}}}\left[1 - \frac{\sigma_v}{\xi + \sigma_e(n)}\right], \tag{9}$$

where $\sigma_e^2 = E\left\{e^2(n)\right\}$ is the power of the error signal, $\delta_{\mathrm{NPVSS}} = \delta_{\mathrm{NLMS}}$ is the regularization factor, and the small positive number ξ avoids division by zero. Therefore, the NPVSS-NLMS update is

$$\hat{\boldsymbol{h}}(n) = \hat{\boldsymbol{h}}(n-1) + \mu_{\mathrm{NPVSS}}(n)\boldsymbol{x}(n)e(n). \tag{10}$$

The a priori parameter needed by the algorithm is the power of the system noise, σ_v^2. In echo cancellation context it can be easily estimated during silences. The power of the error signal is estimated as follows:

$$\sigma_e^2(n) = \lambda\sigma_e^2(n-1) + (1-\lambda)e^2(n), \tag{11}$$

where λ is an exponential window. Its value is chosen as $\lambda = 1 - 1/KL$, with $K \ge 2$. The initial value is $\sigma_e^2(0) = 0$. Theoretically, it is clear that $\sigma_e(n) \ge \sigma_v$, which implies that $\mu_{\mathrm{NPVSS}}(n) \ge 0$. Nevertheless, the estimation from (11) could result in a lower magnitude than σ_v^2 (especially in the initial phase of the algorithm), which would make $\mu_{\mathrm{NPVSS}}(n)$ negative. In this situation, the problem is solved by setting $\mu_{\mathrm{NPVSS}}(n) = 0$.

In order to approach the goal of finding a variable step-size PNLMS algorithm suitable for any type of echo path impulse response, we will apply the idea of NPVSS-NLMS algorithm to the IPNLMS algorithm. Let us write the update relation of the IPNLMS algorithm as follows:

$$\hat{\boldsymbol{h}}(n) = \hat{\boldsymbol{h}}(n-1) + \mu(n)\boldsymbol{G}(n-1)\boldsymbol{x}(n)e(n), \tag{12}$$

where $\boldsymbol{G}(n)$ is the diagonal matrix with the elements given by (6). The issue is to find an expression for the step-size parameter $\mu(n)$ according to (8). Taking into account (1), (7), and (12) we may write

$$\varepsilon(n)=e(n)\left[1-\mu(n)\boldsymbol{x}^{\mathrm{T}}(n)\boldsymbol{G}(n-1)\boldsymbol{x}(n)\right]. \tag{13}$$

Squaring the previous equation, then taking the expectations at both sides, imposing (8) and following the same procedure as in derivation of NPVSS-NLMS algorithm [6] it will result that

$$\mu_{\text{NPVSS-IPNLMS}}(n)=\frac{1}{\boldsymbol{x}^{\mathrm{T}}(n)\boldsymbol{G}(n-1)\boldsymbol{x}(n)+\delta_{\text{NPVSS-IPNLMS}}}\left[1-\frac{\sigma_v}{\xi+\sigma_e(n)}\right] \tag{14}$$

where $\delta_{\text{NPVSS-IPNLMS}} = \delta_{\text{IPNLMS}}$ is the regularization factor and ξ (small positive constant) avoids division by zero. The power of the error signal is estimated as in (11), and $\mu_{\text{NPVSS-IPNLMS}}(n)$ is set to 0 when $\sigma_e(n) < \sigma_v$. We will refer to this algorithm as the nonparametric VSS improved PNLMS (NPVSS-IPNLMS) algorithm. Looking at (14) it is obvious that before the algorithm converges, $\sigma_e(n)$ is large compared to σ_v and consequently the adaptation step is close to 1, which provides the fastest convergence. When the algorithms starts to converge to the true solution, $\sigma_e \approx \sigma_v$ and $\mu_{\text{NPVSS-IPNLMS}}(n) \approx 0$. In fact, this is the desired behaviour for the adaptive algorithm, leading to both good convergence and low misadjustment.

4. **SIMULATION RESULTS**

For the experiments we have considered five impulse responses [Fig. 1(b)–(f)] of length $L = 512$ (64 ms using an 8 kHz sampling rate), with different degrees of sparseness. The first one [Fig. 1(b)] and the third one [Fig. 1(d)] are network echo paths, according to ITU-T G.168 [10]; the second one [Fig. 1(c)] is a multireflection hybrid. In Fig. 1(e) is plotted a measured acoustic echo path (quasi-sparse), while in Fig. 1(f) is depicted a dispersive impulse response. The input signal $x(n)$ is a speech signal [Fig. 1(a)]. An independent white Gaussian noise signal $v(n)$ is added to the output of the echo path, resulting the desired signal $y(n)$. We consider a 25 dB signal-to-noise ratio (SNR) and we assume that the power of the system noise, σ_v^2, is known. In order to test the tracking capabilities of the adaptive algorithms, in all the simulations we consider an abrupt change of the echo path after 5 seconds, shifting the impulse response on the right by 12 samples. The measure of performance is the normalized misalignment (in dB), defined as $20\log_{10}(\|\boldsymbol{h} - \hat{\boldsymbol{h}}(n)\|_2/\|\boldsymbol{h}\|_2)$, where $\|\bullet\|_2$ denotes the l_2 norm. The choices for the parameters of the analyzed adaptive algorithms are as follows. The NLMS and IPNLMS algorithms use an adaptation step μ = 0.2. For the IPNLMS algorithm we

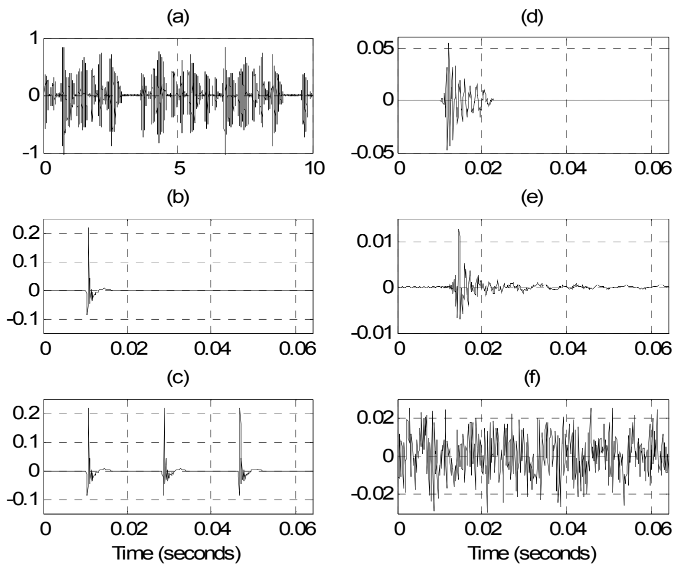

Fig. 1 – a) Speech input signal; b) sparse network echo path according to ITU-T G.168; c) multireflection hybrid; d) network echo path (quasi-sparse) according to ITU-T G.168; e) measured acoustic echo path (quasi-sparse); f) dispersive impulse response.

fix $\alpha = 0$. In the case of NPVSS-NLMS and NPVSS-IPNLMS algorithms the exponential window λ uses $K = 6$. The regularization factor for NLMS and NPVSS-NLMS algorithms was set to $20\sigma_x^2$, and for IPNLMS and NPVSS-IPNLMS algorithms it is considered $20\sigma_x^2/2L$ (where σ_x^2 is the input signal variance). The results of the experiments are presented in Figs. 2–6.

In the case of the very sparse impulse response from Fig. 1b, the NLMS algorithm is outperformed by all the other algorithms (Fig. 2). In terms of the convergence rate and tracking capability, the NPVSS-IPNLMS algorithm behaves similarly to the IPNLMS algorithm. The situation starts to be different when we deal with the multireflection hybrid from Fig. 1c. The NPVSS-IPNLMS algorithm provides the best performance (Fig. 3). Its superiority becomes more apparent for the echo paths from Figs. 1d and 1e, as it can be noticed from Figs. 4 and 5. Finally, when the impulse response is dispersive [as in Fig. 1f] the NPVSS based algorithms rule, while the NLMS and IPNLMS algorithms perform similarly (Fig. 6). Summarizing these results, the proposed NPVSS-IPNLMS algorithm performs very well despite of the character of the echo path impulse response, being a suitable candidate for both network and acoustic echo cancellation systems.

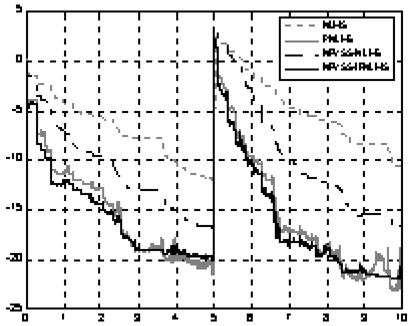

Fig. 2 – Misalignment of the algorithms for the impulse response from Fig. 1b.

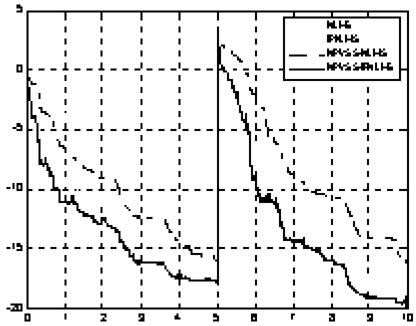

Fig. 3 – Misalignment of the algorithms for the impulse response from Fig. 1c.

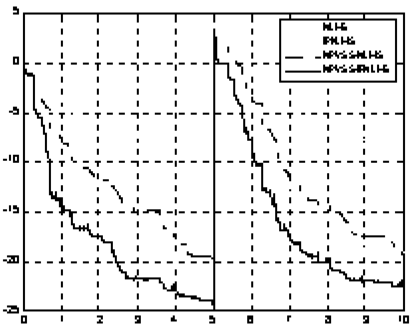

Fig. 4 – Misalignment of the algorithms for the impulse response from Fig. 1d.

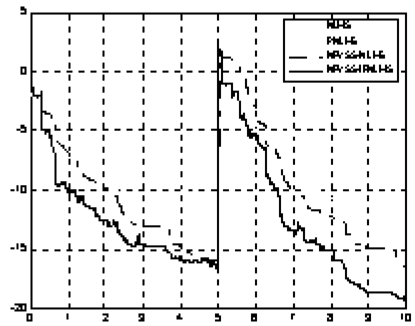

Fig. 5 – Misalignment of the algorithms for the impulse response from Fig. 1e.

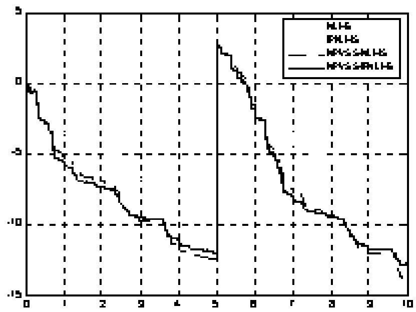

Fig. 6 – Misalignment of the algorithms for the impulse response from Fig. 1f.

5. **CONCLUSIONS**

In echo cancellation context, most of the PNLMS algorithms are in general very sensitive to the character (*i.e.*, sparse or nonsparse) of the echo path impulse response. Among these algorithms, the IPNLMS algorithm is less sensitive to this aspect. On the other hand, the NPVSS-NLMS algorithm offers a good solution to the conflicting requirement between fast convergence rate and low final misadjustment in a very simple manner. In this paper we have combined these two techniques, obtaining an NPVSS-IPNLMS algorithm. The simulation results show that this algorithm offers a very good performance for both sparse and non-sparse echo paths. Consequently, it is a very good candidate for real-world echo cancellers.

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REFERENCES

1. J. Benesty, Y. Huang, Eds., *Adaptive Signal Processing–Applications to Real-World Problems*, Berlin, Springer-Verlag, 2003.
2. S. Haykin, *Adaptive Filter Theory*, Fourth ed., Upper Saddle River, NJ: Prentice-Hall, 2002.
3. D. L. Duttweiler, *Proportionate normalized least mean square adaptation in echo cancellers*, IEEE Trans. Speech Audio Processing, **8**, *5*, pp. 508–518 (2000).
4. H. Deng, M. Doroslovacki, *Proportionate adaptive algorithms for network echo cancellation*, IEEE Trans. Signal Processing, **54**, *5*, pp. 1794–1803 (2006).
5. J. Benesty, S. L. Gay, *An improved PNLMS algorithm*, IEEE ICASSP 2002, Orlando, Florida, U.S.A., May 13–17, pp. II-1881–II-1884.
6. J. Benesty, H. Rey, L. Rey Vega, S. Tressens, *A nonparametric VSS NLMS algorithm*, IEEE Signal Process. Lett., **13**, *10*, pp. 581–584 (2006).
7. S. L. Gay, *An efficient, fast converging adaptive filter for network echo cancellation*, Asilomar Conf. Signals, Systems, Comput. 1998, Pacific Grove, U.S.A., Nov. 2–5, pp. 394–398.
8. M. Nekuii, M. Atarodi, *A fast converging algorithm for network echo cancellation*, IEEE Signal Process. Lett., **11**, *4*, pp. 427–430 (2004).
9. H.-C. Shin, A. H. Sayed, W.-J. Song, *Variable step-size NLMS and affine projection algorithms*, IEEE Signal Process. Lett., **11**, *2*, pp. 132–135 (2004).
10. * * *, *Digital Network Echo Cancellers*, ITU-T Rec. G.168, 2002.