FREQUENCY OFFSET RESISTANT RECEPTION ALGORITHM FOR ORTHOGONAL FREQUENCY DIVISION MULTIPLEXING SYSTEMS

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Performances of the orthogonal frequency division multiplexing (OFDM) systems are very much sensitive to the frequency offset. In the system with differential detection at the receiver, if the phase reference is not stable, due to Doppler effect, fading, or the difference between the oscillators frequencies at the transmitter and the receiver, the performance improvement may be obtained by using more than two consecutive symbols within the multiple-symbol differential detection (MSDD). A novel approach, based on fast MSDD algorithm with frequency offset hypotheses (HFMSDD), that provides reception of OFDM/MDPSK signal with much higher carrier frequency offset than MSDD-OFDM receiver, is proposed in this paper.

Performance of proposed receiver and MSDD-OFDM receiver, are very close only for low frequency offsets. However, proposed receiver operates well in a significantly wider frequency offset range, without the need for any pilot symbols, therefore without any bandwidth efficiency loss. Numerical results are presented to illustrate the effectiveness of the proposed scheme.

1. INTRODUCTION

The orthogonal frequency division multiplexing (OFDM), introduced by Chang in his innovative paper in 1966 [1], is a modulation scheme used in modern communication systems such as wireless local area network (WLAN), digital video broadcasting (DVB), digital audio broadcasting (DAB) or 4G long term evolution (LTE), [2]. The OFDM systems are popular due to the fact that they support broadband communications in multipath fading channels [3].

All the good properties of the OFDM arise from the orthogonality between the subcarriers. In order to maintain the orthogonality of the OFDM subcarriers there has to be good synchronization between the received and locally generated carriers. The carrier frequency offset synchronization is also very important for other modern communication technologies, such as code division multiple access (CDMA). Therefore, the problem of the frequency offset is often considered in the literature, together with the problems of Doppler shift and phase noise [4–11]. Paper [4] considers OFDM $M$-ary phase shift keying (MPSK) and OFDM $M$-ary quadrature amplitude modulation (MQAM) systems in an additive white Gaussian noise (AWGN) channel in the presence of carrier frequency offset and carrier phase noise, where $M$ is the number of modulation levels. It was shown that the OFDM system is for an order of magnitude more sensitive to the carrier frequency offset than a single carrier system of the same bit rate, in terms of the bit error probability. OFDM system with space-time and space-frequency diversity at the transmitter and maximal ratio combining receive diversity is analysed in [5] in the presence of carrier frequency offset, phase noise and channel estimation errors, in frequency selective Rayleigh fading channels. The analysis showed that the phase noise does not have a significant influence on the system performance in case of the perfect channel estimation. However, the transmit diversity is more sensitive to the channel estimation errors than the receive diversity. Papers [6–8] consider blind carrier frequency offset estimation without training sequences in OFDM system with in-phase/quadrature (I/Q) imbalance [6, 7] and for multiuser OFDM uplink with large number of receive antennas [8]. The problem of the CDMA signal reception is considered in [9–11]. The results showed that the proposed algorithm performed better than the existing algorithms at that time.

Algorithms for the frequency detection and synchronization, depending on the characteristics of the transmitted signal (pilot-based or not), may be classified into three groups: algorithms based on the analysis of special synchronization symbols embedded in the OFDM frame [12–16], algorithms based on the analysis of the received data at the output of the fast Fourier transform (FFT) (non-pilot-aided) [17], and the algorithms based on the analysis of guard time redundancy [18, 19].

The problem of the frequency offset may be mitigated by using of the ordinary differential detection, where the phase of the current symbol is compared with the phase of the previous one [20, 21]. However, there is the differential detection performance loss compared to the ideal coherent detection. To deal with this problem, multiple-symbol differential detection (MSDD) may be used [22–26]. A multiple-symbol differential detection technique for MPSK which is based on maximum-likelihood sequence estimation (MLSE) of the transmitted phases was proposed in [22] and further analysed in [23]. It was shown that the proposed technique approaches the performance of the differentially coherent detection at a price of increased complexity. The gap between the MSDD and the coherent detection is smaller for higher number of symbols in MSDD. The improvement of the MSDD algorithm, in terms of complexity, is proposed in [24] with the introduction of the fast MSDD algorithm. The complexity of the fast MSDD algorithm is of order $N \log_2 N$ operations, compared to $N^2$ operations in the original algorithm. Paper [25] analyses differential PSK (DPSK) system with MSDD in the presence of carrier frequency offset. It was shown that the conventional MSDD algorithm performance degrades in case of nonzero frequency offset. The degradation is higher for higher number of symbols in MSDD. The performance is improved with the introduction of double DPSK modulation scheme and the results showed that the performance of an optimum MSDD.
receiver with double DPSK modulation and an infinite number of received samples, approaches that of a conventional DPSK demodulator with two received samples. An extension of the original, non-fast MSDD algorithm for OFDM systems was proposed in [26]. The results indicated that the MSDD in OFDM systems behaves similarly to the MSDD in single carrier systems.

In this paper, we propose an OFDM M-ary differential phase shift keying (MDPSK) receiver. We started with fast MSDD (FMSDD) algorithm [25] and adapted it to work in OFDM system, in a similar manner as the original MSDD algorithm was adapted in [26] to work with OFDM system. Also, the concept of the frequency offset hypotheses was introduced in the proposed algorithm and it is named HFMSDD (Fast MSDD with Hypotheses). We introduced the frequency offset hypotheses, i.e. assumptions regarding the frequency offset. The receiver chooses the most likely frequency offset hypothesis. If the frequency offset is correctly predicted, then it may be successfully compensated. The analysis of the proposed receiver is performed in AWGN channel with the respect of the error probability. The AWGN channel was chosen in order to show what is the maximum performance improvement of the proposed algorithm with regards to the carrier frequency offset.

The rest of the paper is organized as follows. The system model is defined in Section 2, and the proposed algorithm is described in Section 3. Numerical results are given in Section 4 and concluding remarks in Section 5.

2. SYSTEM MODEL

Analysis and simulations in this paper are performed in the digital complex baseband domain. The i-th sample of the OFDM symbol generated by the inverse fast Fourier transformation (IFFT) at the transmitter is:

\[ s(i) = \frac{1}{N} \sum_{n=0}^{N-1} A_n(k) e^{j2\pi \frac{ni}{N}}, \quad 0 \leq i \leq N-1. \]  

\( A_n(k) \) is the amplitude of the n-th subcarrier in the k-th OFDM frame, defined as:

\[ A_n(k) = A_n(k-1) e^{j\Delta \phi_n}, \]  

where \( \Delta \phi_n \) represents the symbol, which is transmitted in the n-th OFDM channel and k-th OFDM frame. N is the number of OFDM data channels and M is the number of different phases in MDPSK modulation.

The block diagram of the OFDM/MDPSK signal receiver used in this paper is shown in Fig. 1. The received signal is down converted, low-pass filtered, and sampled with the period \( T_f = T_{GI} + T_s \), where \( T_{GI} \) is the guard interval duration, and \( T_s \) is the symbol interval duration. The guard interval (GI) is defined as a sum of zero padding (ZP), as defined in [27], and the cyclic prefix (CP) as defined in [28], i.e. \( T_{GI} = T_{ZP} + T_{CP}. \) \( N_{VC} \) is the number of virtual channels (Fig. 1). S/P represents serial to parallel converter and it requires timing synchronization. After removing the cyclic prefix, a discrete Fourier transform (DFT) of length \( N \) is performed. In this case, we use OFDM demodulator with \( N \) subcarriers and discrete Fourier transform.

![Fig. 1 – Proposed OFDM/MDPSK receiver model.](image)

If we assume that the correct frame and timing synchronization is achieved, then the received sequence in \( n \)-th OFDM channel and \( k \)-th OFDM frame, after stripping the cyclic prefix, can be expressed as:

\[ X_n(k) = \frac{1}{\sqrt{N}} \sum_{i=kN}^{(N+1)k-1} r(i) e^{-j2\pi \frac{ni}{N}}, \quad 0 \leq n \leq N-1, \]

where

\[ r(i) = s(i) + w(i) \]

and \( w(i) \) is the AWGN with power spectral density \( N_0 / 2. \)

3. FAST HFMSDD ALGORITHM DESCRIPTION

The proposed receiver uses Fast MSDD algorithm, improved by adding some frequency offset hypotheses, i.e. assumptions regarding the frequency offset. In the first part of the algorithm, Fast MSDD algorithm is executed for each OFDM channel and for each frequency offset hypothesis. After that, these results are used to determine the most likely frequency offset hypothesis and, finally, to make a decision on the transmitted symbol. The operation of the algorithm may be described with the following set of equations.

The signal processing block chooses the phase vector \( \Phi_{n,n_H} = \{\phi_{n,n_H}(k-1),...\phi_{n,n_H}(k-N_S + 1)\} \) that maximizes \( S_{n,n_H}(k) \), for each OFDM channel and for each hypothesis, where:

\[ S_{n,n_H}(k) = \sum_{n_S=0}^{N-1} X_n(k-n_S) e^{-j\phi(k-n_S)} e^{j\nu \phi_n} \]

\( X_n(k-n_S) \) is the OFDM channel and \( n_S \)-th received MDPSK symbol in \( n \)-th OFDM channel and \( n_c = N_{VC} / 2,...,N-(N_{VC} / 2 + 1), \)

\( n_H = 0,...,N_H - 1 \), and \( k \) is the discrete time. \( N_S \) represents the number of symbols in multiple symbol differential detection, and \( N_H \) represents the number of hypotheses. The received signal is multiplied by the signal with the frequency (phase) offset \( \phi_n \):

\[ \phi_n = n_H \Delta \phi - \frac{(N_H - 1) \Delta \phi}{2}, \]

where \( \Delta \phi \) is the algorithm parameter that represents a phase step between the two consecutive predicted phase offsets.
The algorithm, for one hypothesis, is basically fast MSDD algorithm and may be described as follows. To find $\Phi_{n,n_H}$, first we need to remodulate the $X_n(k-n_S)$ so that it is translated to MDPSK sector $[0,2\pi/M]$, for each $n_S = 0,...,N_S-1$. In order to remodulate $X_n(k-n_S)$ let

$$
\Phi_{n,n_H} = \Phi_{n,n_H}(k), \Phi_{n,n_H}(k-1),...,\Phi_{n,n_H}(k-N_S+1)
$$

be the unique $\Phi_{n,n_H}$ for which

$$
\arg \left[ X_n(k-n_S) e^{-j\Phi(k-n_S)} e^{j\pi a_0 k} \right] \in \left[ 0, \frac{2\pi}{M} \right].
$$

Let us define

$$
Z_{n,n_H}(k-n_S) = X_n(k-n_S) e^{-j\Phi(k-n_S)} e^{j\pi a_0 k}.
$$

Then we calculate and sort the values $\arg \left[ Z_{n,n_H}(k-n_S) \right]$ in order from largest to smallest. The sorted list is defined by a new function $I_{n,n_H}(\hat{i})$ as follows:

$$
0 \leq \arg \left[ Z_{n,n_H}(k-l_{n,n_H}(N_S-1)) \right] \leq \arg \left[ Z_{n,n_H}(k-l_{n,n_H}(N_S-2)) \right] \leq \cdots \leq \arg \left[ Z_{n,n_H}(k-l_{n,n_H}(0)) \right] < \frac{2\pi}{M}.
$$

The points $Z_{n,n_H}(k-l_{n,n_H}(i))$, $i = 0,...,N_S-1$, are the remodulations of $X_n(k-n_S)$, $n_S = 0,...,N_S-1$, in $[0,2\pi/M]$, ordered by the value of their angle.

The next step is to determine:

$$
q' = \max \left\{ q + \sum_{i=q}^{q+N_S-1} Z_{n,n_H}(k-l_{n,n_H}(i)) \right\},
$$

$$
q = 0,...,N_S-1.
$$

Based on $q'$ the new phase values are determined:

$$
\Phi_{n,n_H}(k-l_{n,n_H}(i)) = \begin{cases} 
\Phi_{n,n_H}(k-l_{n,n_H}(i)), & q' \leq i \leq N_S-q \\
\Phi_{n,n_H}(k-l_{n,n_H}(i)+1) + \frac{2\pi}{M}, & q' < q + N_S - 2 
\end{cases}
$$

The evaluation of eq. (11) gives elements $\Phi_{n,n_H}(k-l_{n,n_H}(i))$ for $i = 0,...,N_S-1$. By arranging the elements $\Phi_{n,n_H}(k-l_{n,n_H}(i))$, in order of value $l_{n,n_H}(i)$, we form the sequence $\Phi_{n,n_H}(k), \Phi_{n,n_H}(k-1),...,\Phi_{n,n_H}(k-N_S+1)$ which represents the vector $\Phi_{n,n_H}$ for one hypothesis.

After calculating of vector $\Phi_{n,n_H}$ for each hypothesis, we perform low-pass filtering in order to suppress the influence of noise:

$$
\hat{S}_{n_H}(k) = (1-A)\hat{S}_{n_H}(k-1) + A\sum_{n=N_{VC}/2+1}^{N_{VC}-1} S_{n,n_H}(k), n_H = 0,...,N_H-1,
$$

and determine the hypothesis with maximum $\hat{S}_{n_H}(k)$:

$$
n_{H\_max} = \arg \max_{n_H} \hat{S}_{n_H}(k).
$$

Since the data is differentially encoded, the final decision on the transmitted symbol is made using the following equation:

$$
\Delta \phi_n(k) = \Phi_{n,n_H\_max}(k) - \Phi_{n,n_H\_max}(k-1)
$$

$$
k = 1,...,N_S-1, n = \frac{N_{VC}}{2},...,N - \frac{N_{VC}}{2} + 1.
$$

Complexity wise, OFDM modification of the fast MSDD algorithm implements FMSDD in each OFDM channel, i.e. in $N-N_{VC}$ channels. Therefore, the complexity is increased $N-N_{VC}$ times. However, the operations in every channel are independent of each others, and they may be executed in parallel. Also, if we consider OFDM and non-OFDM systems with the same bit rate, the bit rate in each OFDM channel is much lower than the total bit rate. Therefore, the processing power needed for each OFDM channel is much lower than the processing power needed for non-OFMD Fast MSDD algorithm. So, the execution time and the power consumption are approximately the same as in non-OFDM fast MSDD algorithm. With the introduction of the frequency offset hypotheses, the complexity increases. As already explained, during the first part of the proposed HFMSDD algorithm, the fast MSDD is repeated in each OFDM channel for each of $N_H$ hypotheses. Hence, the complexity is $(N-N_{VC}) \times N_H$ times higher than in the original fast MSDD algorithm. Again, all these operations may be executed in parallel. The second part of HFMSDD algorithm begins with the smoothing in eq. (12), finding maximum in eq. (13), and making the decision in eq. (14).

There are $(N-N_{VC}+1) \times N_H$ additions and $2 \times N_H$ multiplications in eq. (12), $N_H - 1$ comparisons in eq. (13) and $(N-N_{VC}) \times N_S$ additions in eq. (14). One can conclude that the complexity of the second part of HFMSDD algorithm is much lower than the complexity of the first part and may be neglected. Consequently, the proposed algorithm has complexity that is proportional to the number of hypotheses.

### 4. NUMERICAL RESULTS

Performance of the described system is analysed using Monte-Carlo simulation with one million simulation steps. The carrier frequency is 2.4 GHz, the sampling period before DFT block is $T_s = 100$ ns, where $T_s$ represents system sampling interval at the input of the receiver. OFDM simulation parameters are $N=32$, the number of virtual channels is $N_{VC} = 4$. The zero padding and the cyclic prefix are chosen to be $T_{zp} = T_{cp} = 4T_s$, which does not limit the generality of the results. Based on the above:

$$
T_i = T_{zp} + T_{cp} + T_s = 4T_s + 4T_s + 32T_r = 40 T_r = 4 \mu s
$$

The analyzed modulation formats are BDP5K, 4DPSK and 8DPSK.

We tested the system for different values of the following parameters: the number of hypotheses ($N_H$), the number of symbols in fast multiple-symbol differential detection algorithm ($N_S$) and the ratio between the phase step and the bandwidth of one OFDM channel ($\Delta \phi / B_r$). The proposed system is compared with the ordinary differential detection
OFDM system (DD-OFDM) [29] and fast MSDD-OFDM (FMSDD-OFDM), analysed in [26].

Figure 2 shows the symbol error probability as a function of the energy per symbol to noise power spectral density ratio \( E_s / N_0 \) if there is no frequency offset in an AWGN channel. It may be seen that the proposed algorithm has the same performance as the original FMSDD-OFDM algorithm, for any signal to noise ratio and for all analyzed modulation formats. Therefore, the introduction of the frequency offset hypotheses does not impair performance in case of zero frequency offset. The behaviour of the proposed algorithm in the presence of the frequency offset will be shown in the following figures.

In Figs. 3, 4, 5, and 6 signal to noise ratio is chosen to be \( E_s / N_0 = 6, 10, \) and \( 15.5 \) dB for BDPSK, 4DPSK, and 8DPSK, respectively, in order to have a similar error probability range.

Figure 3 shows symbol error rate versus normalized frequency offset \( \Delta f \times T_s \), with \( \Delta f / B_c \) as a parameter, for \( N_U = 3 \) and \( N_H = 7 \). The positions of the frequency offset hypotheses (predicted offsets) are also shown in Fig. 3. It may be noticed that there is a significant improvement in the system performance, introduced by the proposed algorithm, in the presence of frequency offset. It can also be seen that the proposed receiver’s performance drops beyond certain value of the frequency offset. In the case of BDPSK, the normalized frequency offset threshold is \( \Delta f T_s = 0.25 \). This frequency offset is equivalent to the phase offset of \( \pi / 2 \) radians. In this case the receiver does not know what symbol is transmitted, since both symbols are equally likely transmitted.

If the normalized frequency offset is higher than 0.25, for example 0.30, the receiver estimates the offset to be 0.05 and detects the opposite symbol. Therefore, the error probability is high.

Similar conclusion stands for 4DPSK and 8DPSK, but due to the number of constellation points, the maximum tolerated frequency offset is 0.125 and 0.0625, respectively. The figure also shows the influence of the frequency step \( \Delta f / B_c \) on the error probability. The error probability is higher if the distance between the actual frequency offset and the closest one of the predicted offsets is higher. Therefore, the frequency step for the predicted offsets should be smaller. However, a smaller step leads to the narrower frequency offsets range the receiver is capable to deal with, as can be seen in Fig. 3. Also, the predicted frequency offset values have to be within \( |\Delta f| \leq 0.25 \) range for BDPSK, \( |\Delta f| \leq 0.125 \) for 4DPSK, and \( |\Delta f| \leq 0.0625 \) for 8DPSK. This is the reason why the set of frequency steps \( \Delta f / B_c \) is different for Figs. 3a, b, and c. Namely, since the number of hypotheses, in Fig. 3, is the same for all the considered modulation formats, the frequency step is smaller for higher level modulation.

The previous analysis shows that the proposed receiver behaves similarly for all the considered modulation formats. Therefore, without loss of generality, the further analysis will be performed only for 4DPSK modulation.

Symbol error probability versus normalized frequency offset for different carrier frequencies and frame rates is shown in Fig. 4. The results show that the normalization in the simulations was implemented correctly and the system performance does not depend on the actual values, but only on the normalized values.
The influence of the number of hypotheses, for the fixed frequency step $\Delta \phi / B_c = 2.5 \%$, is shown in Fig. 5. It is clearly visible that the higher number of hypotheses the wider the operating frequency offset range in the receiver. Also, in order to have good performance for the case of zero frequency offset, the number of hypotheses has to be an odd integer. In case of even number of hypotheses, on half of the predicted frequency offset values will be on the negative part of the frequency offset axis, and the other half on the positive part. Therefore, there will be no predicted frequency offset for $\Delta f T_s = 0$, and the error probability will be higher in that case, compared to the receiver with an odd number of frequency offsets. The time needed for one simulation point with $N_H = 5, 7, 9$ was 9:00, 9:10, 15:00 minutes, respectively, on an i7-2700 processor with 8 threads. As can be seen, the simulation time goes up if $N_H$ is higher than the number of parallel threads supported by the processor.

Figure 6 shows symbol error rate as a function of the normalized frequency offset, with the number of symbols in MSDD, $N_S$, as a parameter, for $\Delta \phi / B_c = 2.5 \%$ and 7 hypotheses. The error probability is lower for higher $N_S$. However, the highest improvement may be observed for $N_S = 3$ and $N_S = 5$. For $N_S > 5$ the gain in error probability is lower, and the complexity is significantly higher. For example, at zero frequency offset, the difference between symbol error probabilities for $N_S = 3$ and $N_S = 5$ and $N_S = 7$, $N_S = 7$ and $N_S = 9$ is $(16, 5, 2) \times 10^{-4}$, respectively.

5. CONCLUSIONS

This paper proposes a new algorithm for the reception of the OFDM/MDPSK signal. The algorithm is based on fast MSDD algorithm, which is first adapted to work in OFDM system, and then the frequency offset hypotheses were introduced. The algorithm determines the most likely hypothesis, i.e. the most likely value of the actual frequency offset and compensates it. The proposed algorithm is meant to be used in the presence of the significant frequency offset. However, it was shown that it does not impair the symbol error probability at the receiver even in case of zero frequency offset. The proposed algorithm has two main parameters: the number of the frequency offset hypotheses ($N_H$) and the frequency step between the hypotheses ($\Delta \phi / B_c$). The performance of the algorithm is better if the actual frequency offset is closer to the nearest predicted frequency offset. The algorithm may expand the frequency offset range with the acceptable reception quality in two different ways. The first way is to increase the number of the frequency offset hypotheses while keeping $\Delta \phi / B_c$ small. This approach has better performance than the second one but increases the complexity of the system. The other approach is to increase the frequency step and keep the number of hypotheses small. However, this approach causes higher performance loss if the actual frequency offset is between two predicted frequency offsets.

The number of symbols in MSDD has the same influence on the system performance as in the original MSDD algorithm. The error probability is lower for higher $N_S$, but the highest improvement may be observed for $N_S$ up to 5. For $N_S > 5$ the gain in error probability is lower, and the complexity is significantly higher.

Another advantage of the proposed OFDM receiver is that it maintains good performance in the presence of frequency offset without the need for any pilot symbols, therefore without any bandwidth efficiency loss.

Finally, the introduction of the frequency offset hypotheses increased the complexity of the receiver approximately $N_H$ times, but the execution time remained nearly the same, because almost all operations may be executed in parallel, for as long as the number of hypotheses is lower than or equal to the number of parallel threads supported by the processor. The energy consumption and the complexity may be further...
optimized if we adaptively change the number of hypotheses. For example, if the frequency offset is varying slowly, after we initially determine the value of the frequency offset, we may reduce the number of hypotheses, or turn them off completely for some period of time. However, this case was not analysed because the paper primarily focuses on the description and performance analysis of the proposed algorithm.

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