ELECTRIC FLUX IN UNIPOLAR INDUCTION DEVICES

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Keywords: Electromagnetic quantities in moving media, Electric flux, Unipolar induction, Distribution of electric charge.

This paper is addressed to those who apply Hertz’s Electrodynamics to moving media, this being the only Galilean relativistic electrodynamics, similarly to the classical Mechanics. The purpose of this paper is to prove how the electric flux law must be applied in the study of the devices with moving media. In order to exemplify this, we have chosen the unipolar induction devices. If \( dS \) is an element on the surface \( \Sigma \) within a medium moving with respect to the laboratory system of reference at one point on \( dS \), the electric flux density is \( D \) or \( D^0 \), depending on how is defined or measured: by the observer \( \mathcal{R} \) within the laboratory system of reference, or by the local observer \( \mathcal{R}^0 \), on \( dS \) within a system moving together with the medium. There are differences between the electric fluxes \( \int_{\Sigma} D \cdot dS \) and \( \int_{\Sigma} D^0 \cdot dS \). The Maxwell’s Electrodynamics, extended for moving media, using only quantities defined by \( \mathcal{R} \), is the classical macroscopic Electrodynamics, which explains away all the experiments with moving media, including those of unipolar induction. In Maxwell-Hertz Electrodynamics, still used in engineering, the electric flux is \( \int_{\Sigma} D^0 \cdot dS \). This Electrodynamics cannot explain all the results obtained by unipolar induction experiments.

1. INTRODUCTION

In the system of reference of the laboratory, the observer \( \mathcal{R} \) examines the varied movement of a body, for example the movement of a body spinning around an axis. The physical-chemical properties of the body’s substance are randomly chosen. The electric flux law is the general law needed to determine the distribution of the electric charges within bodies. Let \( \Sigma \) be a closed surface within a body. In the laboratory system of reference, the surface is stationary and the medium is in motion. According to the electric flux law, the electric flux density is a vector that must be defined or measured by \( \mathcal{R} \), from within the system of reference in which the surface is defined. This is possible only if the measuring

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probes remain fixed relatively to the observer making the measurements. In the case of moving media, the measuring probes are attached to the body in motion at the points where the measurements are made, therefore they are stationary for another observer, the local observer $\mathcal{R}^0$. For example, in a meteorological balloon making its ascent, the temperature is measured with a thermometer placed inside the balloon, hence in motion. Between the electromagnetic quantities $\{E, B, D, H\}$, defined by $\mathcal{R}$ at the point of coordinates $(x_1, x_2, x_3)$ at the moment $t$, and the quantities $\{E^0, B^0, D^0, H^0\}$, measured at the same point $(x_1, x_2, x_3)$ and at the same moment $t$ by the observer (by the probes) $\mathcal{R}^0$, there are the following relations [9, 12, 16]:

$$E = E^0 - v \times B^0,$$

$$B = B^0 + \varepsilon_0 \mu_0 v \times E^0, \quad (2)$$

$$H = H^0 + v \times D^0, \quad (3)$$

$$D = D^0 - \varepsilon_0 \mu_0 v \times H^0. \quad (4)$$

$v$ is the instantaneous velocity of the point of coordinates $(x_1, x_2, x_3)$ of the body where the measuring probe is found, velocity measured by $\mathcal{R}$. Although the relations can be deduced on the basis of the special theory of relativity (for a uniform velocity, meaning for a translation of the measurement probes together with $\mathcal{R}^0$, relative to $\mathcal{R}$), if $v \ll c_0$, the relations are accurate, with reasonable approximation, for any movements, [5].

The electric flux law is written as:

$$\oint_{\Sigma} \mathbf{D} \cdot d\mathbf{S} = q_\Sigma, \quad (5)$$

when the closed surface $\Sigma$ is situated within a substance, in a continuity domain, or

$$\oint_{\Sigma} \mathbf{D} \cdot d\mathbf{S} - \int_{\Sigma} \left[ \mathbf{D} \right] \cdot d\mathbf{S} = q_\Sigma + q_S, \quad (6)$$

when the surface $\Sigma$ is intersected by a surface of discontinuity $S_f$; the points of the closed curve $\Gamma \subset \Sigma$; $q_S$ represents the charge distributed on this surface, $S_f$, [10], where the quantity within brackets means the variation of the respective quantity when crossing the discontinuity surface.

In the field of electrodynamics of fixed bodies (Maxwell’s electrodynamics is considered this way in most handbooks) the electric flux law reads as follows [4, 6, 7, 11, 14, 15]:

$$E = E^0 - v \times B^0,$$
The electric flux through a closed surface is proportional to the electric charge of the interior of that surface. Therefore the relation:

$$\Psi_\Sigma = q_\Sigma, \quad \text{or} \quad \oint_{\Sigma} \mathbf{D} \cdot d\mathbf{S} = q_\Sigma$$

(7)

holds for any closed surface.

All the consequences of this law have been confirmed by experience, not only in electrostatic fields but also in variable fields.

In the area of the electrodynamics of the bodies in motion (Hertz’s electrodynamics is still employed in engineering) only local quantities are used in equations, meaning the electromagnetic quantities measured by observers $\Re^0$, and the integration domains (curves and surfaces) are considered to be attached to the bodies in motion.

At any moment in time, the electric flux through a closed surface attached to the moving medium is proportional to the electric charge of inside that surface. The following relation exists:

$$\oint_{\Sigma} \mathbf{D}^0 \cdot d\mathbf{S} = q_\Sigma,$$

(7′)

and also in local form, for domains of continuity:

$$\operatorname{div} \mathbf{D}^0 = \rho_v.$$

(7′′)

It should be noted that the equation and the motion of the surface $\Sigma$ are those recorded by the observer of the system of reference of the laboratory $\Re$, and not by the observer $\Re^0$ for which the surface and the medium remain fixed. Consequently, the relations (7) and (7′) are different because $\mathbf{D}(x_1, x_2, x_3, t) \neq \mathbf{D}^0(x_1, x_2, x_3, t)$ (the only identity which stands is: $\oint_{\Sigma} \mathbf{D} \cdot d\mathbf{S} = \oint_{\Sigma^0} \mathbf{D}^0 \cdot d\mathbf{S}^0$, since the electric charge is a relativist invariant).

The movement of the substance is important, due to the fact that the electric flux density can not generally be measured from the reference frame of $\Re$, but by the measurement probes of the local observer $\Re^0$.

The electric flux $\oint_{\Sigma} \mathbf{D} \cdot d\mathbf{S}$ can be calculated at the moment of time $t$ if we know, for points of coordinates $(x_1, x_2, x_3)$ on infinitesimal surface elements $d\mathbf{S}$, the electric flux densities $\mathbf{D}^0(x_1, x_2, x_3, t)$ and the magnetic field strengths $\mathbf{H}^0(x_1, x_2, x_3, t)$, measured by the observer $\Re^0$ and the velocities $\mathbf{v}(x_1, x_2, x_3, t)$ of the material points on $d\mathbf{S}$, measured from within the system of reference $\Re$. Then,
\[ \oint \mathbf{D} \cdot d\mathbf{S} = \oint \mathbf{(D^0 - e_0 \mu_0 v \times H^0)} \cdot d\mathbf{S} . \] 

(8)

It can be noted that

\[ \oint \mathbf{D} \cdot d\mathbf{S} \neq \oint \mathbf{D^0} \cdot d\mathbf{S} . \] 

(9)

The second flux is the electric flux from Hertz’s electrodynamics of the moving media. On the basis of Hertz’s electrodynamics, this inequality made impossible to justify some experimental results.

In electrodynamics, for domains of continuity, the electric flux law reads as follows, [16]:

*At any given moment in time, the electric flux through a closed surface \( \Sigma \) is proportional with the accurate electric charge of inside the surface, regardless of the properties of the medium in which \( \Sigma \) is found and of the status of motion of the medium in relation with the system of reference \( \mathcal{R} \), if the electric flux density vector is defined in the points of the surface \( \Sigma \), from the frame of reference of the laboratory.*

2. UNIPOLAR INDUCTION IN ISULATED MEDIA. EXPERIMENTAL INVESTIGATIONS

In the uniform magnetic field \( \mathbf{B} \) of a long, straight inductor, an insulated cylindrical tube (conductivity \( \sigma = 0 \)) spins with a rotation speed \( n \), the relative permittivity of the tube is \( \varepsilon_r \) and the relative permeability \( \mu_r \), the same as in Wilson’s experiment back in 1904 and in 1913, [1, 2]. A direct current flows through the inductor. The insulated tube is coaxial with the inductor. The concentric surfaces of the tube, having the radii \( r_1 \) and \( r_2 \), (the internal and external surfaces of the tube) are metallic. Two brushes P and R make contact with these surfaces. The motion induced voltage can be measured between the connection points A and C, connected through immobile wires with the brushes R and P. The following quantity has been measured ([1], experimentally, for this “source” with infinite internal resistance):

\[ U_{AC} = \left( 1 - \frac{1}{\varepsilon_r \mu_r} \right) n \Phi_e , \] 

(10)

where

\[ \Phi_e = \pi (r_2^2 - r_1^2) \mathbf{B} \] 

(11)
is the magnetic flux of “excitation”.

In order to justify this experimental result it is necessary to apply the law of electromagnetic induction to the contour $\Gamma$, (ACPRA), a fixed contour in relation to the system of reference of the laboratory:

$$\oint_{\Gamma} E \cdot d l = - \int_{\partial \Gamma} \frac{\partial B}{\partial t} \cdot d S. \tag{12}$$

The magnetic flux density is constant and, therefore,

$$\oint_{\Gamma} E \cdot d r = U_{AC} + \oint_{\Gamma} E \cdot d r = 0. \tag{13}$$

The electric voltages along the conductor wires are zero, the intensity of the current flowing through them being zero. $E$ is a quantity that can only be defined by the observer $\Re$, from the system of reference of the laboratory. It is dependent upon the quantities measured by the probes of $\Re^0$, attached to the tube in rotation, according to the relation, [9, 12, 16]:

$$E = E^0 - \mathbf{v} \times \mathbf{B}^0. \tag{14}$$

The electric voltage can be calculated as follows:

$$U_{AC} = - \oint_{\Gamma} E^0 \cdot d r + \oint_{\Gamma} \mathbf{v} \times \mathbf{B}^0 \cdot d r;$$

$$U_{AC} = - \oint_{\Gamma} E^0 \cdot d r + \pi n (r_2^2 - r_1^2) \mathbf{B}^0. \tag{15}$$

The electric flux law is to be applied for the surface $\Sigma$, a cylindrical surface, coaxial with the insulated tube, enclosed in the insulate medium. Inside the surface, the insulate medium is electric uncharged, quite from the beginning of the rotation, hence:

$$\oint_{\Sigma} D \cdot d S = 0. \tag{16}$$

The electric flux density $D$ is not accessible to measure, it is a quantity belonging to a body in motion; it is expressed depending on the quantities measured by the measurement probes $\Re^0$, hence [9, 12, 16]:

$$D = D^0 - \varepsilon_0 \mu_0 \mathbf{v} \times \mathbf{H}^0. \tag{17}$$
Since the medium is linear,

\[ \mathbf{D}^0 = \varepsilon_0 \varepsilon_r \mathbf{E}^0, \]
\[ \mathbf{B}^0 = \mu_0 \mu_r \mathbf{H}^0. \]  

(18)

The electric flux is written as:

\[ \oint_{\Sigma} \mathbf{D}^0 \cdot d\mathbf{S} = \oint_{\Sigma} \varepsilon_0 \varepsilon_r \mathbf{E}^0 \times \mathbf{H}^0 \cdot d\mathbf{S}, \]
\[ \oint_{\Sigma} \varepsilon_0 \varepsilon_r \mathbf{E}^0 \cdot d\mathbf{S} = \oint_{\Sigma} \varepsilon_0 \varepsilon_r \mathbf{E}^0 \cdot \left( \frac{B^0}{\mu_0 \mu_r} \right) \cdot d\mathbf{S}, \]
\[ \oint_{\Sigma} \mathbf{E}^0 \cdot d\mathbf{S} = \oint_{\Sigma} \mathbf{E}^0 \cdot \left( \frac{B^0}{\varepsilon_0 \varepsilon_r} \right) \cdot d\mathbf{S}. \]  

(19)

Therefore, the electric field strength inside the insulate tube measured by the probes attached to the tube will be

\[ \mathbf{E}^0 = \frac{2\pi n}{\varepsilon_0 \varepsilon_r} \frac{B^0}{r}. \]

Introducing this last relation into (15), we obtain:

\[ U_{AC} = -\oint_{\Gamma} 2\pi n \frac{B^0}{\varepsilon_0 \varepsilon_r} r \cdot d\mathbf{r} + \pi n \left( r_2^2 - r_1^2 \right) B^0 = \left( \frac{1}{\varepsilon_0 \varepsilon_r} - 1 \right) n\Phi, \]  

(20)

because, in the expression of the excitation magnetic flux,

\[ \mathbf{B} = B^0 + \varepsilon_0 \mu_0 \mathbf{E}^0 \times \mathbf{B}^0 = B^0 - \left( \frac{\nu^2}{c_0^2 \varepsilon_0 \varepsilon_r} \right) \frac{B^0}{\mu_0 \mu_r} \approx B^0, \]  

(21)

for velocities much lower than the speed of light in empty space.

Maxwell's electrodynamics confirms the result obtained experimentally.

Hertz's classical electrodynamics cannot explain how this experimental device works. Indeed, the laws of Faraday and the electric flux law are in this case written (for contour \( \Gamma \) and surface \( \Sigma \), considered attached to the rotating tube):
For symmetry reasons, the closed surface $\Sigma$, being coaxial with the rotating tube and because no electric charge exists on the insulated rotating tube (even if $\Sigma$ is intersected by a discontinuity surface):

$$\oint_{\Sigma} D^0 \cdot dS = \oint_{\Sigma} E^0 \cdot dS = 0.$$

or $E^0 = 0$. Then,

$$U_{AC} + \oint_{\Gamma_2} E^0 \cdot d\mathbf{r} = \oint_{\Gamma_1} \mathbf{v} \times B^0 \cdot d\mathbf{r};$$

$$U_{AC} = \pi n (r_2^2 - r_1^2) B^0 \approx n \Phi_e.$$

The quantity obtained experimentally is not verified by the calculated electrical voltage, if the Hertz's electrodynamics is used.

If the contact points A and C are short-circuited and disconnected during the motion, and the tube stops rotating (A and C disconnected), we obtain in the end, a cylindrical capacitor with unequally charged plates. The electrical charge, lower in absolute value, can be measured by integrating the current, when the capacitor discharges.

Let us determine these electric charges: The insulating medium is being polarized by the electric field induced by motion. The polarization charges on the surfaces $r_1$ and $r_2$ of the tube are opposed by true electric charges which appeared following the electrical induction in the metallic cylindrical surfaces. Next, we shall compute the electric charge discharged by the capacitor, which can be measured in laboratory.

On the discontinuity surface $r_2$, the local form of the electric flux law is written:
According to Ohm’s law, in copper $E_{\text{Copper}} = 0$ and, using (19)

\[
E_{\text{Insulator}}^0 = 2\pi n \frac{B^0}{\varepsilon_r} \ .
\]

Then,

\[
-\varepsilon_0 2\pi n r_2 B^0 - \varepsilon_0 2\pi n r_2 \frac{B^0}{\mu_r} + \varepsilon_0 2\pi n r_2 \frac{B^0}{\mu_r} = \rho_s ,
\]

Similarly,

\[
\rho_s (r_2) = -\varepsilon_0 2\pi n r_2 B^0 .
\]

If the connecting points A and C are short-circuited when the rotating speed became zero (tube stopped spinning) the charge flowing through the conductor from C to A is the charge on the surface $r_1$, lower in absolute value than the charge on the surface $r_2$:

\[
\Delta Q = 4\pi^2 \varepsilon_0 n l r_1^2 B^0 ,
\]

$l$ represents the length of the insulated tube. This result can be verified experimentally and is independent of the material properties (permittivity and permeability of the insulated tube).

Another example which shows that the electric flux law must be written in function of the electric flux density defined in the laboratory system of reference (and not in the frame of the local observer) is offered by the unipolar induction in conductive media.

### 3. UNIPOLAR INDUCTION IN CONDUCTIVE MEDIA

In Wilson’s experimental device we replace the insulating material tube with a copper tube. The experiment now resembles the experiment with the Faraday disc. The lateral surfaces of the cylindrical copper tube, of radii $r_1$ and $r_2$, will be found at different electrical potentials due to the motion induced electric field. Motion induced phenomena are well explained for conductive media in classical
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Thus the voltage measured between the connection points A and C

\[ U_{AC} = \pi n\left(r_2^2 - r_1^2\right)B^0 = n\Phi_e, \]  

is theoretically obtained regardless of whether we apply Hertz equations or Maxwell's equations for moving media. But the electric charges distribution in these devices is calculated with Maxwell's Electrodynamics alone, not with Hertz's Electrodynamics. Let \( \Gamma \) be the closed contour ACPRA. Applying the law of Faraday from Hertz's electrodynamics we obtain:

\[ \oint_{\Gamma} E^0 \cdot dI = \oint_{\Gamma} (\nu \times B^0) \cdot dI, \]
\[ \oint_{\Gamma} E^0 \cdot dI = U_{AC} + \oint_{\eta} E^0 \cdot d\mathbf{r}, \]  

\[ \oint_{\Gamma} (\nu \times B^0) \cdot dI = \pi n\left(r_2^2 - r_1^2\right)B^0 = n\Phi_e. \]  

If we apply the electric flux law to the closed surface \( \Sigma \), it follows that the accurate electric charge inside this surface is zero, because, from Ohm's law,

\[ J^0 = \sigma E^0 = 0, \]  

therefore

\[ \oint_{\Sigma} D^0 \cdot d\mathbf{S} = 0. \]  

Only in this case one obtains \( U_{AC} = n\Phi_e \), the result confirmed by the experiment.

Within Maxwell’s electrodynamics, Faraday’s law applied to the closed contour \( \Gamma \) is written,

\[ \oint_{\Gamma} E \cdot d\mathbf{l} = -\int_{S_t} \frac{\partial B}{\partial t} \cdot d\mathbf{S} = 0. \]  

\( \mathbf{B} \) is a constant vector. Then,

\[ U_{AC} + \oint_{\eta} E \cdot d\mathbf{r} = 0. \]
But $E$ may not be defined only by $\mathcal{R}$, because the quantities are measurable only by the observer $\mathcal{R}^0$, from within a moving medium. Then,

$$U_{AC} + \int_{n}^{r} (E^0 - \nu \times B^0) \cdot \mathrm{d}r = 0,$$

(35)

$E^0$ being zero, according to Ohm's law,

$$U_{AC} = n\Phi_e.$$

(36)

The electric flux law is written for the surface having the radius $r_2$ like this:

$$\int_{S_2} [\mathbf{D}] \cdot \mathrm{d} \mathbf{S} = q_2,$$

(37)

$$\rho_{s2} = \varepsilon_0 2\pi r_2 nB^0,$$

$$q_2 = 4\pi^2 r_2^2 nB^0.$$

On the surface $S_1$,

$$q_1 = -4\pi^2 r_1^2 nB^0.$$

(38)

The observer from the laboratory's frame explains the compensation of the electric field induced by motion with a repartition of accurate electric charges inside the copper tube.

As the quantities measured by the probes in $\mathcal{R}^0$ are functions having coordinates $(x_1, x_2, x_3)$, just as the quantities measured from $\mathcal{R}$, we can apply the divergence operator to the relation between $\mathbf{D}$ and $\mathbf{D}^0$:

$$\text{div} \mathbf{D} = \text{div}(\mathbf{D}^0 - \varepsilon_0 \mu_0 \mathbf{v} \times \mathbf{H}^0),$$

$$\rho_v = \text{div} \varepsilon_0 E^0 - \varepsilon_0 2\pi nB^0 \text{div} \mathbf{r}.$$

(39)

$E^0 = 0$ and in cylindrical coordinates $\text{div} \mathbf{r} = 2$, thus,

$$\rho_v = -\varepsilon_0 4\pi nB^0.$$

(40)

The electric charge inside the volume bounded by the lateral surfaces is:

$$q_{\Sigma} = \int_{r_2}^{r_1} \rho_v \mathrm{d}V = -\varepsilon_0 4\pi^2 nB^0 (r_2^2 - r_1^2) I.$$

(41)

The conservation of electric charge is verified (the tube, in motion or not, has overall electric charge zero):
\[ q_{s1} + q_{s2} + q_{\Sigma} = 0. \]  

\( (42) \)

4. CONCLUSIONS

1. The results of Wilson's experiment cannot be explained if the electric flux is calculated from the laboratory system of reference, with the local electric flux density \( D^0 \).
2. This result can be verified measuring the electric charge discharged from the capacitor to the device with insulated medium, when \( n = 0 \).
3. The electric field induced through the movement of the conductor is opposed by the electric field of the charges distributed in the conductor, so that \( E^0 = 0 \).

Received on October 30, 2008

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