A NOVEL CONFIGURATION FOR STATIC PERMANENT MAGNET LEVITATION

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The paper presents a novel horizontal configuration of a permanent magnet static levitation stabilized by the presence of diamagnetic materials. The magnetomechanical interaction between the levitated magnet and the diamagnetic pieces was evaluated in order to investigate the restoring forces exerted on the floater when shifted from its equilibrium state. Thus, the equilibrium point coordinates of the suspended permanent magnet, and their stability areas are accurately established using both analytical and numerical procedures.

1. INTRODUCTION

Despite the well established Earnshaw’s theorem [1], that avoids the stable levitation of magnetic charges in the presence of static magnetic and gravitational fields, one shows that there are at least two situations where the existence of diamagnetic materials may provide a stable levitation configuration [2, 3]. These cases exploit the incidence of very large magnetic fields (up to 10 T) on diamagnetic objects, and also the incidence of conventional intensity magnetic fields (a few mT) on small magnetic dipoles in association to diamagnetic stabilizing sheets.

This phenomenon of stabilizing the intrinsically unstable equilibrium of the levitated permanent magnet (PM) by pieces of diamagnetic materials has awoken a great interest over the latest years due to the innumerous possibilities of technological applications in systems where it is extremely desirable to eliminate any kind of mechanic contact, friction or dissipation [4–8].

Previous works regarding the diamagnetic levitation of a permanent magnet concern mainly vertical configurations [9, 10]. This paper proposes a novel levitation setting, in which stabilization is done along the horizontal direction, allowing to the suspended magnet to freely move in the vertical plane.
The study accurately predicts, both analytically and numerically, the equilibrium point and its stability zone for a levitated permanent magnetic.

2. HORIZONTAL CONFIGURATION – EQUILIBRUM AND STABILITY

The simple model of the proposed levitation array – Fig. 1, consists in a magnetic field source – the two cylindrical coils, the levitated permanent magnet (PM) and two pieces of pyrolytic graphite (PG) as stabilizing diamagnetic material. Contrary to our intuition, for certain values of the current carried by the two coils ($I_0$), due to the high absolute value of the magnetic field gradient backwards on $z$-axis, the developed magnetic force can balance the gravity of the PM, and levitation may occur. The two pieces of diamagnetic material placed normal to the common axis of the coils act like a tiny servo-system providing the restoring forces needed for keeping the floater at the equilibrium point in a stability state.

![Fig. 1 – Cross section of the horizontal configuration for a permanent magnet levitation.](image)

Due to the cylindrical symmetry of the configuration, the coordinate of the equilibrium point ($z_0$) lies on the axis ($\Delta$) where the magnetic flux density exhibits only a horizontal $x$- component (Fig. 1):

$$B(z) = B_x(z)\hat{i},$$

(1)
where \( \mathbf{i} \) is the unit vector of the \( x \)-axis.

Stable static levitation of the permanent magnet requires a minimum of the total energy along with the equilibrium condition fulfillment [9]. Denoting \( \mathbf{M} \) the magnetic moment of the suspended PM of mass \( m \), the total potential energy \( U \) of the floater becomes:

\[
U = - \mathbf{M} \cdot \mathbf{B} + mgz = - MB_z(z) + mgz.
\]  (2)

The magnetic torque will align the levitated permanent magnet parallel to the local field direction; therefore, in the magnetic energy expression (2), only the \( x \)-component \( B_x(z) \) of the magnetic flux density is variable. Then the condition that point \((D, 0, z_0)\) is an equilibrium one requires a null value of the resulting force \( \mathbf{F} \) on the floater, which can be expressed in terms of energy gradient:

\[
\mathbf{F} = - \nabla U \bigg|_{x=D, y=0, z=z_0} = M \frac{\partial B_x(z)}{\partial z} \bigg|_{x=D, y=0, z=z_0} \mathbf{k} + mg \mathbf{k} = 0 \quad \Rightarrow \quad \frac{\partial B_x(z)}{\partial z} \bigg|_{x=D, y=0, z=z_0} = -\frac{mg}{M},
\]  (3)

where \( \mathbf{k} \) is the unit vector of the \( z \)-axis, which is downward oriented.

The equilibrium condition is not self-sufficient, stability having to be achieved. That goal requires a positive concavity of the energy surface around the equilibrium point for all possible directions. Therefore, the stability condition read:

\[
D_x = \frac{\partial^2 U}{\partial x^2} \bigg|_{x=D, y=0, z=z_0} > 0; \quad D_y = \frac{\partial^2 U}{\partial y^2} \bigg|_{x=D, y=0, z=z_0} > 0; \quad D_z = \frac{\partial^2 U}{\partial z^2} \bigg|_{x=D, y=0, z=z_0} > 0.
\]  (4)

The left-hand side quantities of the inequalities (4), denoted by \( D_x, D_y \) and \( D_z \) are called discriminants of stability. For a stable static levitation of the floating permanent magnet, all three stability discriminants at the equilibrium point must be positive simultaneously.

As we already explained in the previous section, two pieces of diamagnetic material placed in the proximity of the levitated magnet along the \( x \)-axis, stabilize the intrinsically unstable equilibrium of the floater for this direction. Consequently, a new term \( Cx^2 \) is added to the right-hand side of (2), which represents the diamagnetic material contribution to the potential energy \( U \).

The expression for the term \( C \) (called the diamagnetic influence factor) can be calculated using the dipole approximation of the floater and the corresponding mirror images within the two diamagnetic stabilizing pieces. An analytical expression for \( C \) is given in [10]:

\[
C = \frac{6|x| \mu_0 M^2}{\pi(2c + w)^3},
\]  (5)
where: $\mu_0 = 4\pi \times 10^{-7}$ H/m is the magnetic permeability of vacuum, $\chi$ is the magnetic susceptibility of the diamagnetic plates, $c$ is the separating distance between the floater and each diamagnetic sheet, and $w$ and $h$ are the dimensions of the cylindrical floater, as depicted in Fig. 2.

Due to the geometry of this configuration, without diamagnetic materials, setting $C = 0$, the curvature of floater potential energy along the $x$-axis, at the equilibrium point, becomes negative and thus, the system is unable to provide horizontal stability: $D_x < 0$.

### 3. MAGNETIC FLUX DENSITY COMPUTATION

According to (3) and (4) the problem of determining the coordinates of the equilibrium point and its stability area for the suspended permanent magnet leads to the evaluation of the magnetic flux density, produced by the two coils at different points located in the vicinity of the equilibrium point. This goal is achieved by using the analytical expression for the off-axis magnetic flux density of a single circular current loop [11] and then by summing up the contribution of all the turns of the two solenoids. Then first and second order spatial derivatives of the flux density may be also determined.

![Fig. 2 – Suspended permanent magnet between diamagnetic pieces.](image1)

![Fig. 3 – Geometrical data for the coils.](image2)
Fig. 3 shows the geometrical dimensions of one coil along with the other technical data: $I_0$ the intensity of the current through the coils, $d$ the diameter of the winding conducting wire, $N_v$ and $N_h$ number of turns on the vertical and horizontal direction respectively, $a_1$ the inner radius of the coils and $D$ is the distance from the origin of coordinates to the symmetry axes of the system.

Assuming the above geometrical data one can write the expression of the magnetic field $B_z(z)$ on the symmetry axis $\Delta$ as follows:

$$B_z(z) = \sum_{i=1}^{N_v} \sum_{j=1}^{N_h} B_{x,y}(z),$$

$$\frac{dB_z(z)}{dz} = \sum_{i=1}^{N_v} \sum_{j=1}^{N_h} \frac{dB_{x,y}(z)}{dz},$$

$$\frac{d^2B_z(z)}{dz^2} = \sum_{i=1}^{N_v} \sum_{j=1}^{N_h} \frac{d^2B_{x,y}(z)}{dz^2},$$

with:

$$B_{x,y}(z) = \frac{\mu_0 I_0}{\pi \sqrt{(a_i + z)^2 + (D - l_j)^2}} [K(k_y) + A_y E(k_y)],$$

$$A_y = \frac{a_i^2 - z^2 - (D - l_j)^2}{(a_i - z)^2 + (D - l_j)^2};$$

$$k_y = \sqrt{(a_i + z)^2 + (D - l_j)^2},$$

where, $K$ and $E$ are the complete elliptic integrals of the first and second kind respectively.

In order to examine the stability of levitation for small displacement of the floater around the equilibrium point on the $y$- and $x$- axes, similar expression of the magnetic field flux density components $B_y(y)$ and $B_x(x)$ are obtained.

### 4. NUMERICAL EXAMPLE

In order to achieve a quantitative analyze for the above proposed configuration – Fig. 1, specific geometrical data and material properties to the model were set up. Thus, for the cylindrical symmetric coils we assume: $N_v = 20$ and $N_h = 10$ turns carrying the direct current $I_0 = 2.75$ A, a winding conductor diameter $d = 1.29$ mm (standardised Cu value with isolation layer), the coil thickness bounded by radii $a_1 = 50$ mm and $a_2 = a_1 + N_v d = 75.80$ mm and the value $50$ mm for the distance $D$. The floater was chosen flat as a tiny NdFeB cylinder (diameter $2w = 8$ mm and $h = 2$ mm height) with magnetic momentum $M = 0.104$ Am² and mass $m = 0.79$ g. The two pieces of pyrolitic graphite...
(magnetic susceptibility $\chi = 450 \cdot 10^{-6}$), placed initially at $c = 2$ mm, symmetrically to the levitated magnet, stabilize the equilibrium.

Assuming the above numerical data this levitation configuration asures an stable equilibrium point located at $x_0 = D = 50$ mm, $y_0 = 0$ mm $z_0 = 38.67$ mm and a value of $C = 1.5$ for the influence factor.

Fig. 4 – Equilibrium point and the magnetic flux density variation on symmetry axis $\Delta$.

Fig. 4 presents the coordinate of the equilibrium point determination and the magnetic flux density variation $B_x(z)$ – scaled by a factor of 10 for a better graphical representation – on the equal distant axis $\Delta$ from both solenoids.

Fig. 5 – Discriminant of stability variation on $y$-axis.

Fig. 6 – Discriminant of stability variation on $z$-axis.
The discriminant of stability variation along y- and z- axes, denoted with $D_y$ and $D_z$ are shown in Fig. 5 and Fig. 6, respectively. One can notice that their positive value around the equilibrium point assures stability in these directions.

Fig. 7 – Horizontal stability discriminant variation with and without diamagnetic material presence.

Fig. 7 shows the horizontal stability discriminants variation at the equilibrium point along x-axis with and without the presence of diamagnetic stabilizing materials denoted with $D_x(C)$ and $D_x$ respectively. In the absence of diamagnetic pieces the value of discriminant $D_x$ is negative and the system becomes unable to keep stability along x-axis.

The maximum value of the gap (stability area) is $c_{\text{max}} = 2.72$ mm. That corresponds to $C = 0.51$ which still ensures a positive value for the horizontal discriminant of stability $D_x(C)$ at the levitation point. Thus, the stability area is restricted by $c = 0 \ldots 2.72$ mm.

5. CONCLUSIONS

The issue of stabilizing the static levitation of a permanent magnet by the usage of diamagnetic materials is treated. The study examines a novel geometrical levitation configuration, in which the diamagnetic materials pieces are horizontally placed in the proximity of the suspended permanent magnet. This configuration, properly adjusted, allows several equilibrium and stability points of the PM in the vertical plane.
The potential energy of the suspended magnet, which provides the static equilibrium equations and stability restrictions, was stated in terms of diamagnetic and magnetomechanical interactions. That enabled one to accurately predict the location and quantify the stability of levitation.

For specific geometrical and material data the analytical and numerical computations results were in a very good match, validating the assumed theoretical predictions.

This kind of static stable levitation could provide simple and inexpensive solutions for a wide area of applications, where frictionless or non-contact is imperiously needed (high-sensitive gravity sensors, magnetic bearings or different geo-physical devices).

In microsystems, the scale reduction favors the diamagnetic effect by enhancing the volume forces and it is therefore desirable to apply diamagnetic levitation mechanisms to ensure accuracy and robustness of microdevices which contains moving components.

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