INFLUENCE OF THE TELLEGEN TYPE BI-ISOTROPIC SPHERE ON THE HOMOGENEITY OF A FIELD GENERATED BY TWO TOROIDAL ELECTRODES

ŽAKLINA J. MANČIĆ¹, ZLATA Ž. CVETKOVIĆ¹, BOJANA R. PETKOVIĆ², NIKOLA SIMIĆ³

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We have investigated the influence of the level of bi-isotropy and the size of the spherical body on the homogeneity of the field generated by two thin toroidal electrodes charged by two equal charges of opposite signs. We solve Poisson equation by the method of separation of variables. Our numerical experiments show that the increment of the level of bi-isotropy reduces the impact on the field homogeneity. However, when relative permittivity is large, influence of the level of bi-isotropy becomes negligible. Decrement of the size of the sphere decreases its effect on the field homogeneity.

1. INTRODUCTION

Biomedical engineering plays an important role in health of the modern society. Bioinstrumentation as a part of it applies electronics and measurement techniques to develop devices for diagnosis and treatment of various diseases. An essential part of some medical systems requires highly homogeneous electric fields in the region of interest, e.g. in vivo exposure medical systems [1], magnetic resonance imaging (MRI) [2], nuclear magnetic spectroscopy [3], electrophoresis in biochemistry [4] etc.

The simplest systems for generation of uniform electric and magnetic fields were described many decades ago [5, 6]. One pair of plan-parallel square electrodes can provide relatively high homogeneity of the electric field in the central area of the system. The degree of field homogeneity can be further increased using four square conductive contours, set equidistant in planes parallel to the plates [7, 8]. Homogeneous electric fields can be generated using other configurations, like biconical and toroidal electrodes [9–11]. Demands for increasing precision of medical instruments and extension of their use to different fields necessitated higher level and area of field homogeneity. This implied the systems for homogeneous field generation to be constantly improved.

Homogeneity of electric fields can be disrupted due to several reasons. For example, the occurrence of bubbles [12,13], thermally or electrically induced in the liquid medium or the presence of the imperfections in the vacuum medium, can significantly change the homogeneity of the field in the vicinity of the imperfections. Such imperfections are commonly referred to as external bodies and they can be modeled using various geometries, such as a sphere, ellipsoid or cylinder, and various mediums (conductor, bi-isotropic, isotropic and anisotropic dielectric). However, external bodies can be much larger, such as in a form of a conductive torroid immersed in the homogeneous electric field [14]. Analysis of the influence of an external body on the homogeneity of the field is important for a proper work of biomedical instrumentation and performing therapies.

Bi-isotropic media appear to be very important in the microwave and optical applications. A way to generate macroscopically magneto-electric effect was first suggested by Tellegen. He created a new passive circuit element, called gyror, and suggested relations to explain this effect in materials. The most well-known naturally occurring example of such material is chromium oxide.

Influence of an external cylindrical body made of isotropic, dielectric and bi-isotropic material on the field homogeneity is analyzed in [9]. System for homogeneous field generation consisted of four long linear conductors, with two of them of positive and other two of negative electric scalar potentials of equal absolute values.

The influence of a spherical conducting body on the achieved homogeneity of the field, generated using toroidal electrodes, was analyzed in [15]. The influence of an external body modeled as a prolate and oblate ellipsoidal electrode, was analyzed in [16].

In this paper, we consider effects of external spherical body, made of isotropic dielectric and bi-isotropic materials of Tellegen type, on the homogeneous electric field of toroidal electrodes. A homogeneous electrostatic field is obtained using two equal toroidal electrodes charged by equal charges of opposite signs, analogously to the theory of Helmholtz coils in homogeneous magnetic field generation. It is so-called primary cell, which represent a core and the simplest toroidal element within the modelling process of homogeneous field generation. Bi-isotropic materials of Tellegen type can be at the same time polarized and magnetized if they are placed into external electric or magnetic fields. This feature makes the analysis of the impact of these materials on the field homogeneity very important.

The paper is organized as follows: Section 2 describes Methodology, including model definition and method for solving Poisson’s equation. Results of numerical experiments for isotropic dielectric sphere and bi-isotropic sphere are presented in the Section 3. Section 4 contains conclusions.

2. METHODOLOGY

2.1. MODEL DEFINITION

A system of two very thin toroidal electrodes of radius $a$ and of $2h$ distance between them is presented in Fig. 1. This is a so-called toroidal primary cell for generating homogeneous electric field. The distance between the

¹ University of Niš, Faculty of Electronic Engineering, Aleksandra Medvedeva 14, Niš, Serbia, E-mail: zaklina.mancic@elfak.ni.ac.rs; zlata.cvetkovic@elfak.ni.ac.rs
² Institute of Biomedical Engineering and Informatics, Technische Universität Ilmenau, Ilmenau 98693, Germany, E-mail: bojana.petkovic@tu-ilmenau.de
³ University of Novi Sad, Faculty of Technical Sciences, Trg Dositeja Obradovića 6, Novi Sad, Serbia, E-mail: nikolasmimic@uns.ac.rs
center of the system and each toroidal electrode is
\( R = \sqrt{a^2 + h^2} \). We observe the field along the axis of the
system (z-axis).

Dimensions of a primary cell of the first order can be
obtained using Taylor series for modeling:
\( h/R = 0.77459667 \) and \( a/R = 0.63245552 \) [10]. After
generating homogeneous electric field, a spherical external
body of radius \( b \) is placed into the field. The center of the
external body coincides with the coordinate origin of the
coordinate system, Fig. 1.

\[ \begin{align*}
\text{Fig. 1} & \quad \text{A system of two toroidal electrodes generates a homogeneous}
\text{electric field. An external spherical body of bi-isotropic Tellegen material is placed at the center of the system.}
\end{align*} \]

2.2. METHODS

We observe the influence of the spherical body of
dielectric and bi-isotropic material of Tellegen type [17, 18]
on the field homogeneity. For solving similar problems in
electromagnetism, the method of images is the method of
choice whenever it is possible [9, 15], [19–21]. In contrast
to that, we apply the method of separation of variables in a
spatial coordinate system [22, 23]. As bi-isotropic
materials are polarized and magnetized at the same time
after they are placed into electrical or magnetic field, both
electric and magnetic scalar potentials appear.

In the case of bi-isotropic medium of Tellegen type,
constitutive relations between the electric field vector \( \mathbf{E} \),
vector of electric displacement \( \mathbf{D} \), magnetic flux density
vector \( \mathbf{B} \) and magnetic field vector \( \mathbf{H} \) are defined as [17]:
\[ \mathbf{D} = \varepsilon \mathbf{E} + \xi \mathbf{H}, \quad \mathbf{B} = \mu \mathbf{H} + \xi \mathbf{E}, \quad (1) \]
where \( \varepsilon = \varepsilon_r \varepsilon_0 \) ( \( \varepsilon_r \) is relative permittivity and \( \varepsilon_0 \) is permittivity of vacuum).

Similarly, \( \mu = \mu_r \mu_0 \), where \( \mu_r \) is relative magnetic
permeability, while magnetic permeability of free space is
\( \mu_0 = 4\pi \cdot 10^{-7} \text{ H/m} \). \( \xi \) is a bi-isotropic parameter.

Starting from Maxwell's equations:
\[ \begin{align*}
\text{div} \mathbf{D} & = \rho / \varepsilon_c, \quad (4) \\
\text{div} \mathbf{B} & = 0, \quad (5)
\end{align*} \]
where \( \rho \) is the volumetric density of free charges in the
observed environment and
\[ \varepsilon_c = \varepsilon_0 \left(1 - \frac{\xi^2}{\varepsilon_r} \right). \quad (6) \]

Now, by setting
\[ \mathbf{E} = -\text{grad}(\varphi), \quad (7) \]
where \( \varphi \) is electric scalar potential, the Poisson's equation
for the electric scalar-potential inside the volume of bi-
isotropic sphere is obtained:
\[ \Delta \varphi = -\rho / \varepsilon_c. \quad (8) \]

Divergence of magnetic field vector from (2) is:
\[ \text{div} \mathbf{H} = -\frac{\xi}{\mu} \text{div} \mathbf{E}. \quad (9) \]

By setting
\[ \mathbf{H} = -\text{grad}(\varphi_m), \quad (10) \]
one obtains the Poisson's equation for the magnetic scalar
potential \( \varphi_m \) in the bi-isotropic environment:
\[ \Delta \varphi_m = \frac{\xi}{\varepsilon_r \mu} \rho. \quad (11) \]

Poisson's equations (7) and (10) apply to points inside the
volume of bi-isotropic sphere. Calculation of the potential in
the air is based on the solution of the Laplace’s equation.

Using spherical coordinates and the method of separation
of variables, electric and magnetic scalar potential can be
presented by expressions (11) and (12), respectively:

\[ \varphi = \begin{cases} 
\sum_{n=0}^{b} \left( A_n r^n + \frac{B_n}{r^{n+1}} \right) [P_n(\cos \theta) - P_n(-\cos \theta)], & r \leq b \\
\sum_{n=0}^{b} \left( C_n r^n + \frac{D_n}{r^{n+1}} \right) [P_n(\cos \theta) + P_n(-\cos \theta)], & b \leq r < R \\
\sum_{n=0}^{R} \left( C_n - \frac{q[P_n(\cos \theta) + P_n(-\cos \theta)]}{4\pi \varepsilon_0 R^{n+1}} \right) r^n + \left[ D_n + \frac{q[P_n(\cos \theta) - P_n(-\cos \theta)]}{4\pi \varepsilon_0 R^{n+1}} \right] 1 / r^{n+1} [P_n(\cos \theta) - P_n(-\cos \theta)], & R \leq r
\end{cases} \quad (11) \]
\[ \varphi_m = \begin{cases} 
\sum_{n=0}^{b} \left( A_{n1} r^n + \frac{B_{n1}}{r^{n+1}} \right) [P_n(\cos \theta) - P_n(-\cos \theta)], & r \leq b \\
\sum_{n=0}^{b} \left( C_{n1} r^n + \frac{D_{n1}}{r^{n+1}} \right) [P_n(\cos \theta) + P_n(-\cos \theta)], & b \leq r \leq R \\
\sum_{n=0}^{R} \left( C_{n1} - \frac{q[P_n(\cos \theta) + P_n(-\cos \theta)]}{4\pi \varepsilon_0 R^{n+1}} \right) r^n + \left[ D_{n1} + \frac{q[P_n(\cos \theta) - P_n(-\cos \theta)]}{4\pi \varepsilon_0 R^{n+1}} \right] 1 / r^{n+1} [P_n(\cos \theta) + P_n(-\cos \theta)], & R \leq r
\end{cases} \quad (12) \]
Piecewise solutions (11) and (12) take the Poisson’s and Laplace’s equation into account. To ensure that electric and magnetic scalar potential has finite values at zero, coefficients $B_n$ and $B_{nl}$ have to be equal to zero:

$$B_n = 0 \quad \text{and} \quad B_{nl} = 0.$$  \hfill (13)

In order to ensure that magnetic scalar potential has a finite value when $r$ goes to infinity, coefficient $C_{nl}$ has to be zero as well:

$$C_{nl} = 0.$$  \hfill (14)

Using the condition of the potential continuity on the boundary surface between two mediums (bi-isotropic sphere and air), the following constants are obtained:

$$D_n = \frac{q n A_{n B} (2 n + 1) b^{2 n + 1}}{4 \pi R^{n+1}}$$  

$$A_n = \frac{-q n \zeta^2 (2 n + 1)}{4 \pi R^{n+1}}$$  

$$\times \left\{ P_n (\cos \theta_0) - P_n (-\cos \theta_0) \right\}$$  \hfill (18)

$$D_n = \frac{-q n^2 \xi^2 (2 n + 1)^2}{4 \pi R^{n+1}}$$  

$$\times \left\{ P_n (\cos \theta_0) - P_n (-\cos \theta_0) \right\}$$  \hfill (19)

where:

$$\theta_0 = \arctan \left[ \frac{a}{h} \right],$$  \hfill (20)

and $P_n (\cos \theta)$ are Legendre polynomials.

3. NUMERICAL RESULTS AND DISCUSSION

3.1. STUDY I: INFLUENCE OF THE RELATIVE PERMI TIVITY ON ELECTRIC FIELD HOMOGENEITY

In this study we consider a toroidal system presented in the Fig. 1, of dimensions $b/R = 0.2$, values of relative permittivity 2, 5 and 30 in the cases of a strong bi-isotropy ($\frac{\xi^2}{\epsilon \mu} = 0.9$) and a weak bi-isotropy ($\frac{\xi^2}{\epsilon \mu} = 0.2$).

Distributions of the electric field strength $E$ along the $z$-axis normalized by the electric field $E_0$ at the point $r = 0$ for the system without a sphere, for the values of relative permittivity equal to 2, 5 and 30 and levels of bi-isotropies 0.2 and 0.9 are presented in the Fig. 2.

Figure 2 shows that an increase in relative permittivity increases a distortion of the electric field outside the sphere, when keeping dimensions of the sphere and a level of bi-isotropy constant. This is valid for the cases of a weak bi-isotropy ($\frac{\xi^2}{\epsilon \mu} = 0.2$) as well as for the cases of a strong bi-isotropy ($\frac{\xi^2}{\epsilon \mu} = 0.9$).

Electric field inside a sphere of a material with high bi-isotropy is higher comparing to the field of a sphere with low bi-isotropy. Electric field $E$ along the $z$-axis normalized by the field $E_0$ is presented by black solid line.

![Image](image-url)

Fig. 2 – Distribution of the electric field along the $z$-axis normalized by the field $E_0$ at the point $r = 0$, for $b/R = 0.2$, levels of bi-isotropy 0.2 and 0.9 and relative permittivity 2, 5 and 30.

Coefficients $A_n$, $A_{nl}$, $C_n$, $D_n$ and $D_{nl}$ in expressions for electric scalar potential (11) and magnetic scalar potential (12), where $\epsilon_r = 2$, $\xi^2 / \epsilon \mu = 0.9$, $b/R = 0.2$, are presented in the Fig. 3 a–Fig. 3 e, respectively. The coefficients are equal to zero when $n$ is an even number. Numerical experiments have shown that first fifteen nonzero terms provide a satisfactory convergence of the series.
3.2. STUDY II: INFLUENCE OF A LEVEL OF BI-ISOTROPY ON ELECTRIC FIELD HOMOGENEITY

We investigate an influence of a level of bi-isotropy on distortion of the homogeneity of the electric field. We consider the same system as in the Study I.

Figure 2 shows that a decrement of a level of bi-isotropy increases a distortion of the field homogeneity, keeping a value of relative permittivity and size of the sphere constant. However, this only applies when the relative permittivity of the sphere is low. When relative permittivity of the material of the sphere is high ($\varepsilon_r = 30$), the influence of the level of bi-isotropy on the field distortion can be neglected.

Equipotential lines for electric scalar potential in a system of toroidal electrodes and a sphere $b/R = 0.2$, relative permittivity $\varepsilon_r = 2$, and without bi-isotropy, $\xi^2/\varepsilon\mu = 0$ (a), with a weak bi-isotropy, $\xi^2/\varepsilon\mu = 0.2$ (b) and strong bi-isotropy, $\xi^2/\varepsilon\mu = 0.9$ (c).

Equipotential lines for magnetic scalar potential in a system of toroidal electrodes and a sphere $b/R = 0.4$, relative permittivity $\varepsilon_r = 2$, for the case of a weak bi-isotropy ($\xi^2/\varepsilon\mu = 0.2$) and strong bi-isotropy ($\xi^2/\varepsilon\mu = 0.9$) are presented in the Fig. 5a and 5b, respectively. Fig. 5a shows the central part of the system, while Fig. 5b represents potential lines for the over the system in 2D.
along the sphere surface itself when spheres are small, e.g. 0/ and the field distortion at the surface of the sphere where $b/R = 0.5$.

4. CONCLUSIONS

We have investigated an influence of the Tellegen type bi-isotropic sphere on a homogeneous field produced by two toroidal electrodes. This impact was analyzed through the observation of the relative permittivity of the sphere, level of material bi-isotropy and a size of the sphere. Numerical results have been obtained using the method of separation of variables.

The results show that increment of the relative permittivity increases a distortion of the homogeneous electric field. On the other hand, lower level of bi-isotropy has a larger impact on the field homogeneity. This applies only when relative permittivity is low. High value of relative permittivity suppresses the influence of bi-isotropy. In that case, level of bi-isotropy can be neglected. As the dimensions of the sphere decrease, its influence on the surrounding field decreases, too.

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