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Dedicated to the memory of Academician Andrei Ţugulea

OVERVIEW OF OUR RECENT RESULTS IN SIGNAL ANALYSIS USING RECURRENCE PLOTS

ALEXANDRU SERBANESCU [1], CORNEL IOANA [2], ANGELA DIGULESCU [1]

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This paper highlights the main results of our research in the field of recurrence plot analysis in the last five years. It briefly discusses results in the study of the method itself, as well as in some applications of it, *i.e.* characterization of speech signals, detection and localization of partial discharges in electric cables, representation of transient signals, estimation of signal parameters, reduction of additive noise, and representation of digitally modulated signals amplitude shift keying (ASK), frequency shift keying (FSK), phase shift keying (PSK) and orthogonal frequency division multiplex (OFDM).

1. INTRODUCTION

Most researchers would agree that a candidate complex system should have most or all of the following "ingredients":

- The system contains a collection of many interacting objects.
- The objects can adapt their strategies according to their history.
- The system is typically "open".
- The system appears "alive".
- The system exhibits emergent surprising phenomena and even extreme.
- The emergent phenomena typically arise in the absence of any sort of "invisible hand or central controller".
- The system shows a complicated mix of ordered and disordered behavior. (Neil Johnson – "Simply Complexity", 2010).

Numerous systems, especially those in the animate realm, possess complexities that are at best nonlinear and non-predictable. Common ground between living and nonliving systems resides in their shared property of recurrence. As signals grow in complexity, however, recurrences become rarer, and efficient compressibility is resisted. But the lesson is clear: Insofar as natural patterns are found in all dynamical systems, the degree to which those systems exhibit recurrent patterns speaks volumes regarding their underlying dynamics. That is, within the dynamical signals expressed by living and non-living signals there are stretches, short or long, of repeating patterns. And, on reflection, it should be appreciated that the entire scientific enterprise is based upon the concept of recurrence: To be accepted as valid, experimental results must be repeatable in the hands of the original investigator and verifiable by independent laboratories. Patterns of recurrence in nature necessarily have mathematical underpinnings, which will be given by recurrence quantification analysis (Eckmann, Kamphorst, and Ruelle, 1987). The paradigm of deterministic chaos has influenced thinking in many fields of science. New mathematical concepts, such as Lyapunov exponents, recurrences and fractal dimensions have been brought out. Nonlinear and especially chaotic systems show rich dynamical structures and sometimes provide an explanation for irregular fluctuations in "real life" systems which do not seem to be inherently stochastic. Now, chaos theory has taught us that random input is not the only possible source of irregularity in the output of a system. Nonlinear, chaotic systems can produce very irregular data with purely deterministic equations of motion in an autonomous way, *i.e.* without time dependent inputs. Of course, a system which has both nonlinearity and random input, will most likely produce irregular data as well.

Recurrence plots, a rather promising tool of data analysis, have been introduced by Eckman *et al.* in 1987. They visualize recurrences in phase space and give an overview of the system dynamics. Two features have made the method rather popular. Firstly, they are rather simple to compute and secondly they are putatively easy to interpret. However, the straightforward interpretation of recurrence plots for some systems yields rather surprising results. Besides, the attractor can be reconstructed from the recurrence plot (Thiel *et al.*, 2004). This means that it contains all topological information of the system under question in the limit of long time series (Bandt *et al.*, 2002). It has been proven mathematically that one can recreate a topologically equivalent picture of an original multidimensional system behavior by using the time series of a single observable variable (F. Takens – "Detecting strange attractors in turbulence", 1981). The basic idea is that the effect of all others (unobserved) variables is already reflected in the series of the observed output. Furthermore, the rules that govern the behavior of the original system can be recovered from its output. When we are trying to understand an irregular (which essentially means here non-periodic) sequence of measurements an immediate question is what kind of process can generate such a series. In the deterministic picture, irregularity can be autonomously generated by the nonlinearity of the intrinsic dynamics. The two paradigms, *nonlinear deterministic* and *linear stochastic* behavior, are the extreme positions in the area spanned by the properties: "nonlinearity" and "stochasticity". Now, let us suppose that we have a dynamical system which is both stationary and deterministic. We must now examine this system. Suppose that we can measure some single scalar quantity at any time. That is, we have an observation function $g: M \rightarrow R$. This observation function provides us with a way to

[1] Military Technical Academy Bucharest, Romania, E-mail: alexe1serbanescu@yahoo.com, angela_digulescu@yahoo.com;
[2] GIPSA-lab, Universite Grenoble Alpes, France, E-mail: Cornel.Ioana@gipsa-lab.grenoble-inp.fr.

measure the current state of the system: $g(z_n)$. Since $g(\bullet)$ gives us only a scalar value, it cannot offer a complete description of the system. But, observing $x_n = g(z_n)$ at many successive times will! *i.e.* the evolution of (x_n) will be in many ways similar to that of (z_n) – according to the celebrated Taken's embedding theorem.

Our work in the last years showed that recurrence plot analysis (RPA) can at least be a viable alternative to classic signal processing tools. The following sections present an overview of some interesting results spanning various fields, from speech and electric cables to digital modulations.

2. OVERVIEW OF THE MAIN RESULTS IN RPA STUDY

2.1. RESULTS IN PHASE SPACE RECONSTRUCTION

We showed in [1] that a rescaling of the time axis of the analyzed signal modifies the phase space trajectory when the embedding parameters are not changed. More precisely, the parameter that matters here is τ, *i.e.* the time delay. However, if we perform an appropriate scaling of this parameter the trajectory remains basically the same (but with a different density or "drawing" speed). The effect on the recurrence plot is dilation or contraction of the patterns it contains.

In [1], we showed as well that when dealing with rescaling of the amplitude axis an appropriate scaling of the recurrence threshold, ε, is required in order for the structures contained by the recurrence plot to remain the same. Hence, a time-varying ε would be required when analyzing a signal that contains at different time instants components of different amplitudes. We showed that this inconvenient is removed if we replace the Euclidean distance with an angular distance. By using the latter, the resulting recurrence matrix becomes practically immune to slow amplitude variations in the analyzed signal [2]. (However, we have not obtained yet similar results concerning the rescaling of the time axis. If the analyzed signal contains at different time instants the same basic signal dilated or contracted, a fixed τ analysis will not work well.)

In [3], we showed that phase space trajectory reconstruction by time-delay embedding is essentially equivalent to an analysis of the signal on time overlapped windows. While the size of these windows is $1+(m-1)\cdot\tau$, they contain the entire information that can be found in the phase space vectors (and even more, if τ is greater than 1). We named these signal slices vector samples. We also showed that the distance between two points on the trajectory is in fact the result of computing a function of the two vector samples that define the two points. If instead of choosing a distance in the mathematical sense we choose an arbitrary function, instead of recurrences we obtain a broader class, *i.e.* generalized recurrences. Therefore, we showed that from a signal processing point of view the effect of the (time-delay embedding) RPA method is similar to the effect of an analysis of the similarity between all pairs of two signal windows (similarity which is computed in the sense of a chosen distance). Besides, we showed in [4] that such an analysis is at the bottom of the computation of several frequently used signal processing measures.

2.2. RESULTS IN PHASE SPACE TRAJECTORY ANALYSIS

We already mentioned the angular distance [1, 2], that helped us perform a recurrence plot analysis that was independent of the slow amplitude variations in the signal. Besides it we also introduced in [4] other pseudo-distances, *i.e.* the dynamic range distance and the scalar product distance. We made use of them in defining generalizations for the autocorrelation function and for the signal derivative. Both of them proved to be more noise robust than the classic counterparts.
Another RPA method parameter that we studied is the recurrence threshold, ε. As the value of this parameter decides is a recurrence is recorded or not, it is desirable that ε adapts to the level of noise in the signal. We managed to obtained that by computing ε as the mean distance between all successive pairs of points on the trajectory [5, 4].

2.3. RESULTS IN RECCURENCE PLOT IMAGE PROCESSING

As the recurrence plot is in fact an image, the idea of using image processing to reduce noise came naturally. We obtained promising results in this area both by using a combination of several nonlinear filters applied sequentially in order to reconstruct the diagonal structures through interconnection of points spread by noise [6], and by using a single (nonlinear, as well) filter designed to reconstruct the diagonal structures based on the distribution of black points along the diagonals in the recurrence plot [7].

3. CHARACTERIZATION OF SPEECH SIGNALS

In [8], it was shown that the recurrence rate computed on time sliding windows can be successfully used to obtain a real-time segmentation (in vowels, consonants, and transient parts) of the speech signal. The segmentation signal thus obtained can be also used as an excitation control signal in order to obtain a good quality in speech synthesis.

The recurrence rate computed on time sliding windows was also involved in obtaining a (time-varying) measure that was able to separate the fundamental periods in vowel segments. It should be mentioned that the recurrence plot was previously filtered by removing from it all diagonal lines whose length was below a certain threshold. The signal obtained by computing from this recurrence plot the recurrence rate on time sliding windows was used in extracting the basic vowel structure on a fundamental period. This basic structure is important in tasks such as speech recognition and compression.

4. DETECTION AND LOCALIZATION OF PARTIAL DISCHARGES IN ELECTRIC CABLES

In [9], the RPA methodology was used in estimating the pulse width in a system for localizing the partial discharges in electric cables. The system was based on the fact that the time dilatation of the signal is proportional to its propagation time. The estimation of pulse widths was performed by computing a local recurrence rate for the analyzed signals. (This local recurrence rate was discussed in more detail in [5].)

5. REPRESENTATION OF TRANSIENT SIGNALS

By transient signal we understand a signal portion whose duration is finite and much smaller that the observation duration of the entire signal. Such a signal has, generally, the shape of a pulse or of a time-localized oscillation.

We proposed in [3] two new recurrence quantification analysis (RQA) measures $-\sigma_c$, and σ_d. They are computed by performing a normalized summing of the elements of the recurrence matrix column-wise and diagonal-wise, respectively. Then, we showed in [4] that σ_d is very similar to the signal autocorrelation function, while σ_c is very similar to a time-distributed histogram of the signal.

The computation of these two measures requires two parameters: w (the vector sampling window size), and ε (the recurrence threshold). If we maintain one of them fixed while sweeping with the other one the whole range of values, we obtain two types of bidimensional representations: the $t-\varepsilon$, and the $t-w$ representations.

We showed [4] that the representations based on σ_c allow the time localization of the transient signal, while the representations based on σ_d allow the identification of its fundamental period (after it was first localized). Likewise, from the analysis of these bi-dimensional representations one can qualitatively estimate the influence that the w and ε parameters have on the computation of the σ_c and σ_d RQA parameters. Thus, this analysis showed that the best results are obtained for values of w smaller than half of the fundamental period of the signal and for values of ε around the mean distance between all successive points on the phase space trajectory (that we mentioned in Section 2.2).

In addition to the representations mentioned here, in [4] we introduced another two $t-w$ representations: a $t-w$ representation of the dynamic range distance from the zero point (which proved to be useful in estimating the apparition time and duration of a transition between two stable states in a signal), and a $t-w$ representation of the dynamic range distance between successive vector samples, multiplied by the sign of the difference between these vector samples (which proved to be much more noise robust than the classic signal derivative). (The dynamic range "distance", as well as this generalization of the signal derivative, were already mentioned in Section 2.2.)

6. ESTIMATION OF SIGNAL PARAMETERS

When analyzing a transient signal, the main parameters of interest are its time of arrival and its duration. We showed [3–5] that by using the representations discussed in the previous section these parameters can be easily obtained (from a time detection curve computed by using the σ_c RQA measure). Compared to the classic energy detector, our detector proved to be more efficient in terms of high probability of detection at low false alarm probability. Its disadvantage is (at least for the moment) its computational complexity, as it requires N^2 steps for a signal of N samples.

When it comes to estimating the fundamental period of a signal, the σ_d RQA measure (introduced in the previous section) proved to be more efficient than the classic averaged magnitude difference function (AMDF), as we showed in [10]. The measure proposed by us accentuates the desired maxima and removes the local maxima introduced by the higher frequency components in the signal. We also showed that the results can be improved even more in terms of signal-to-noise ratio (SNR) by first applying a nonlinear filter on the recurrence plot (as already mentioned in Section 2.3).

The σ_d measure proved to be useful also as a mean to distinguish sounds produced by beaked whales from sounds produced by dolphins [11]. This distinction can be made on the basis that beaked whales sounds have a slight frequency modulation. Although the slope of these chirp signals is very low, it causes a significant change of σ_d. As we showed in [4], not even the fractional Fourier transform allows such an easy distinction between a sinusoidal signal and a signal with a very small frequency modulation.

In the previous section we already mentioned the use of the dynamic range distance in estimating the time instant and the duration of the transition between two stable states in the signal. We also showed in [4] how it can be used for making a fast estimation of the signal time of arrival in an ultrasonic water pipe investigation system. In [4] we showed as well that obtaining the signal maxima envelope is essentially the result of processing the trajectory obtained by vector sampling the signal with a window of size w. More precisely, this processing consists of computing the maximum distance between every point on the trajectory and the origin.

7. REDUCTION OF ADDITIVE NOISE

Regarding additive noise reduction, we mentioned in Sections 2.3. and 6 that by filtering the recurrence plots we can obtain a reduction of the effect noise that was present in the analyzed signal. In fact, this is not a reduction of noise in the analyzed signal, but a reduction of the effect that noise has on the analysis of the signal.

In some particular cases, as we showed in [3], the signal can be partially cleaned by using the σ_c measure (introduced in Section 5). The method consists of multiplying the signal with the computed σ_c, and then recomputing σ_c and so on.

For relatively low levels of noise the method gives good results after only few such iterations. The disadvantage is that it works only for signals in which transients are rare. Besides, only the noise in the transients-free areas of the signal is reduced.

8. REPRESENTATION OF DIGITALLY MODULATED SIGNALS

8.1. ASK, FSK, AND PSK

In [10], we showed the results of applying the RPA method on digitally modulated signals (amplitude shift keying, frequency shift keying, and phase shift keying). We showed that when represented as phase space trajectories these signals become: a series of concentric ellipses having the same eccentricity (for the ASK signal), a series of concentric ellipses having different eccentricities (for the FSK signal), and a series of similar overlapped ellipses (for the PSK signal).

In the case of the ASK signal, the areas corresponding to different amplitudes can be distinguished in the recurrence plot based on the thickness of the diagonal lines. Given that the recurrence threshold, ε, is fixed for the entire signal, the recurrence ball contains more points when the sinusoid amplitude is smaller, which leads to a thicker diagonal line.

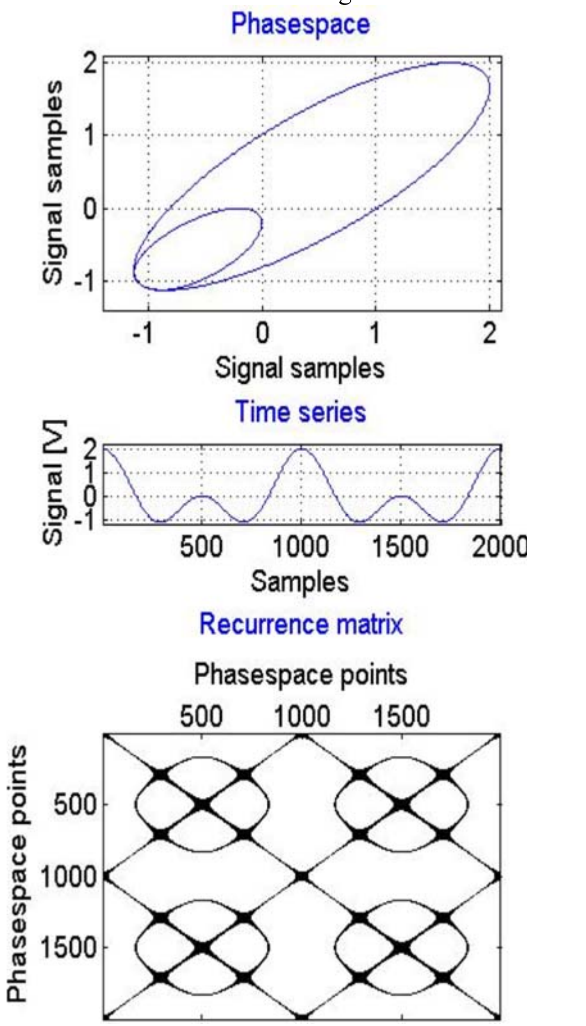

Fig. 1 – OFDM signal with two subcarriers: the phase space trajectory of the signal; the time series of the signal; the recurrence matrix of the signal.

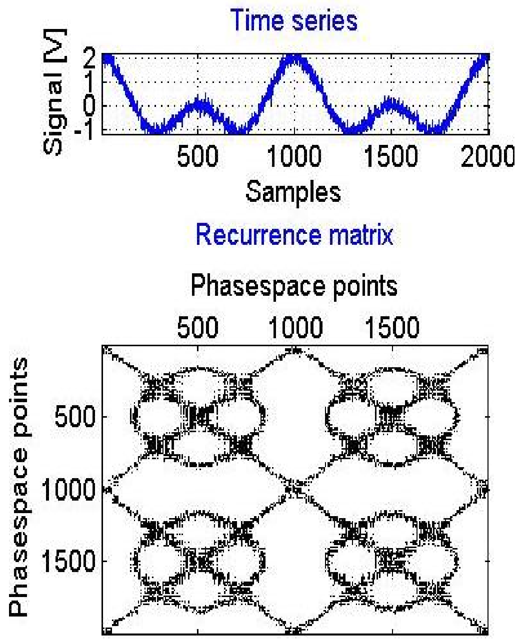

Fig. 2 – Noisy OFDM signal: the time series of the signal; the recurrence matrix of the signal.

The recurrence plot obtained for the FSK signal, however, contains diagonal lines having the same thickness (because the amplitudes of the sinusoids are the same). What differs in this case is the distance between succesive diagonal lines (which is normal, if we consider that this distance is in fact the distance between the time instants when the signal repeats itself – hence, it is the fundamental period of the signal).

For the PSK signal the recurrence plot contains neither different thicknesses of the diagonal lines, nor different distances between them. Nevertheless, the different areas of the signal are identifiable through the “ruptures” caused by the phase jumps on the recurrence plot. Although the blocks located in the recurrence plot along the main diagonal are identical, between such a block and the block that succeeds it horizontally there is an offset between the diagonal lines. This offset can then be easily related to the phase difference between the two corresponding portions of the signal.

Therefore, the recurrence plots that correspond to signals having one of the three digital modulation types described here allows a quick identification of the modulation type and of the time instants when the transmitted symbol changes. We proposed in [10] two methods for identifying these transition moments between two symbols. The recurrence plots also allow us to obtain the parameters that correspond to the signal segments where the transmitted signal remains constant.

8.2. OFDM

A new challenge for this approach is the detection of the OFDM modulation. Firstly, we have considered the simplest case: transmission with two subcarriers. When transmitting {1,1,1,1} the results can be seen in Fig. 1. Because the cyclic prefix brings the same information on the recurrence matrix, it has been removed in order to simplify the image.

Although the time series of the signal does not exactly say what it contains, in phase space it can be clearly seen that the signal contains two sine waves. In its evolution on one period it can be observed that the two tangent ellipses are described in only one period, afterwards the process being continued. These ellipses do not have the same center, but they are tangent, meaning that one period of the signal contains the two ellipses, hence the two sine waves.

Moreover, the representation of the recurrence matrix is given by the unique representations of the ellipses in phase space, which are in turn given by the parametric curves resulted from the time series. Hence, the relationship between the time series and the recurrence matrix is unambiguous.

The recurrence matrix confirms that there are two oscillations at a time. Accordingly, the periodic pattern is a sign of periodicities in the time series, the distance between the patterns corresponding to a period.

Moreover, it is clear that the first frequency is $1/T$ and the second frequency is $2/T$, that is f_0, respectively 2 f_0. This fact is confirmed by the two lines parallel with the main diagonal which are divided by one short line (that means that the frequency ratio is 2).

If the signal is affected by noise (SNR = 15 dB, Fig. 2), the main characteristics are kept. The phase space still has the two ellipses visible (although, if the SNR decreases, they become harder to distinguish) and the recurrence matrix has the same patters, but fuzzier with the declining of SNR.

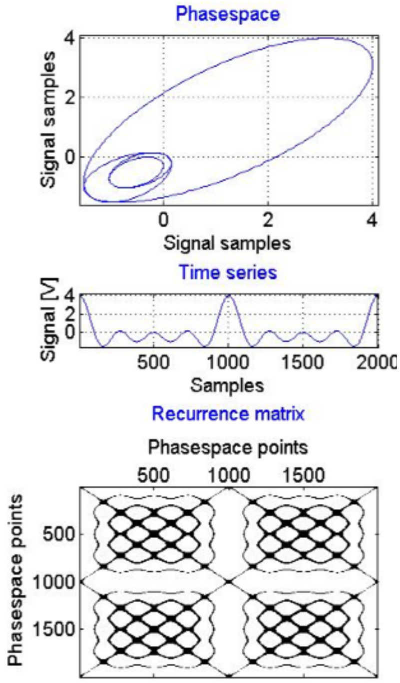

Fig. 3 – OFDM signal with 4 subcarriers: the phase space trajectory of the signal; the time series of the signal; the recurrence matrix of the signal.

In a similar way, if the number of subcarriers increases, for example if there are 4 subcarriers, then the recurrence matrix changes (Fig. 3).

The phase space of the signal reveals the simultaneous existence of four sine waves in one period of the time series. Also, the structures from the recurrence matrix divide the lines parallel with the main diagonal in four, meaning that the frequency ratio is 4. These short segments fraction the diagonal line into equidistant segments, meaning that the frequencies are f_0, 2 f_0, 3 f_0 and 4 f_0.

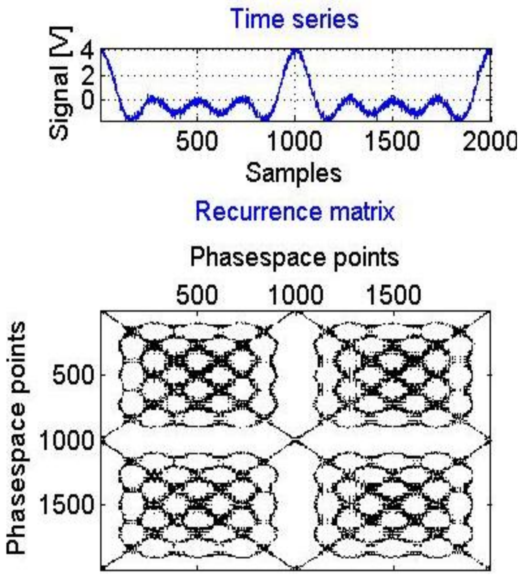

Fig. 4 – Noisy OFDM signal (4 subcarriers): the time series of the signal; the recurrence matrix of the signal.

When affected by noise (SNR = 15 dB), the time series succeeds to maintain its characteristics, although the phase space trajectory and the recurrence plot are not that clear any more (Fig. 4).

9. CONCLUSION

It is no wonder to anyone anymore that the world we live in is a large complex system having chaotic dynamics. Tools such as Taken's embedding theorem and the recurrence plot introduced by Eckmann *et al.* allow the study of this dynamics through the concept of recurrence by using a series of scalar observations of the system state. These tools formed together what is now known as recurrence plot analysis, or RPA plus recurrence quantification analysis (RQA).

While various applications of RPA were reported in the last years in various fields, our work was focused on studying the potential of the method in solving signal processing tasks (or at least bringing new insights on them). This paper brought together in a compact form the main results we obtained, as well as some work in progress.

In short, we studied RPA (+RQA) and used it in various signal processing problems. We studied its behavior for various signals and typical situations in signal processing. We proposed new methods to compute distances between phase space vectors, as well as new methods for choosing RPA parameters. We also showed the relationship between RPA and classic signal processing, we introduced two new RQA measures, and we proposed some filters for reducing the effect of noise on the recurrence plot. We obtained interesting results in speech signal analysis and synthesis, in partial discharge detection in electric cables, in the representation of transient signals, in the estimation of their parameters, as well as in the study of digitally modulated signals.

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REFERENCES

1. F.-M. Birleanu, C. Ioana, C. Gervaise, J. Chanussot, A. Serbanescu, and G. Serban, *On the Recurrence Plot Analysis method behaviour under scaling transform*, The 2011 IEEE Statistical Signal Processing Workshop (SSP), Nice, France, 28–30 June 2011, pp. 793–796.
2. F.-M. Birleanu, C. Ioana, C. Gervaise, A. Serbanescu, J. Chanussot, *Caracterisation des signaux transitoires par l'analyse des récurrences de phase*, Le 23eme Colloque GRETSI, Bordeaux, France, 5–8 September 2011.
3. F.-M. Birleanu, I. Candel, C. Ioana, C. Gervaise, A. Serbanescu, G. Serban, *A Vector Approach to Transient Signal Processing*, The 11th International Conference on Information Science, Signal Processing and their Applications, Montreal, Quebec, Canada, 2–5 July 2012, pp. 1174–1179.
4. F.-M. Birleanu, *Construction des espaces de representation RPA pour l'analyse des signaux transitoires*, PhD Thesis, Université de Grenoble, France, 2012.
5. F.-M. Birleanu, C. Ioana, A. Serbanescu, J. Chanussot, *A time-distributed phase space histogram for detecting transient signals*,

The IEEE International Conference on Acoustics, Speech and Signal Processing, pp. 3844–3847, Prague, Czech Republic, 22–27 May 2011.
6. F.-M. Birleanu, E. Sofron, G. Serban, *Using Image processing techniques for noise reduction in recurrence plots*, Proceedings of the IC ECAI (International Conference on Electronics, Computers and Artificial Intelligence), **4**, pp. 77–82, 2009.
7. F.-M. Bîrleanu, E. Sofron, *Improving recurrence-based fundamental frequency estimation by using image processing*, University of Pitesti Scientific Bulletin. Series: Electronics and Computers Science, **2**, *10*, pp. 13–16, 2010.
8. L.-A. Cernaianu, *Contributii la elaborarea metodelor de compresie a semnalului vocal*, PhD Thesis, Academia Tehnică Militară, Bucharest, România, 2009.
9. B. Gottin, *Analyse multi-capteurs de signaux transitoires issus de systèmes électriques,* PhD Thesis, Institut National Polytechnique de Grenoble, France, Septembre 2010.
10. A. Serbanescu, O. Stanasila, F.-M. Birleanu, *Analiza neliniara a seriilor de timp. Aplicatii in prelucrarea semnalelor*, Editura Academiei Tehnice Militare, Bucharest, 2011.
11. F.-M. Birleanu, C. Ioana, C. Gervaise, Y. Simard, *Recurrence plot analysis (RPA): a new tool for click detection and clustering clicks into trains*, The Fifth International Workshop on Detection, Classification, Localization, and Density Estimation of Marine Mammals using Passive Acoustics, Timberline Lodge, Mount Hood (Oregon, USA), 21–25 August 2011.