QUASI-VERTICAL PERMANENT MAGNET LEVITATION –
ANALYTICAL MODEL AND CHARACTERIZATION

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The recently discovered and reported quasi-vertical permanent magnet levitation phenomenon is revisited for a better characterization and modeling. The supposedly abnormal deviation of the floater magnet equilibrium point from the vertical symmetry axis of the levitation structure is further investigated. To achieve this goal, a more refined magnetic charge based model is proposed for determining the equilibrium point position. Finally, potential use of the levitation structure as highly sensitive tilt meter is identified and discussed.

1. INTRODUCTION

The classical vertical suspension of permanent magnets was first put in evidence by Boerdijk [1] who used diamagnetic materials for stabilizing their intrinsically unstable equilibrium. Diamagnetic stabilization of levitated permanent magnets principles were coherently stated along with the associated specific terminology [2], thus allowing for other levitation structures to be developed or stability area of the existing ones to be extended [3–6].

Figure 1a shows the classical vertical levitation setup, in which the magnetic moments of the two magnets are collinear and both pointing upward or downward for achieving the same suspension effect. This type of vertical structure offered Simon, Heffinger and Geim the context in which they introduced the main concepts and procedures for investigating and characterizing the diamagnetically stabilized permanent magnet levitation [2]. The present study is focused on a recently discovered antiparallel vertical levitation phenomenon with diamagnetic stabilization [7]. Its existence was reported in a conference paper, which has provided initial estimations for the equilibrium point position (along the x–axis), but only for the ideal structure depicted in Fig. 1b. In practice, for all realized physical models using different lifter magnets, the floater exhibits an apparently

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inexplicable shift from the vertical $x$-axis, where, by symmetry, the equilibrium point is expected to fall, as shown in Fig. 1c.

Following the same path proposed by Simon, Heflinger and Geim, the initial study reported in [7] also fully demonstrates the stability presented by the floating magnet at the equilibrium point, in ideal setups with one or two diamagnetic slabs. In Fig. 1a, b and c, any of the two stabilizing pieces may be removed, followed by a small shift of the floater’s equilibrium point toward the remaining one. In any of the single plate cases there is a single diamagnetic force present, the balance of forces being modified accordingly, in contrast to the initial configuration where the two diamagnetic forces cancel each other out. Supplementarily, for the tilted floater case (Fig. 1c) the effect of the torque(s) exerted by the diamagnetic plate(s) must also be estimated. A logical explanation for the floater shift from the vertical symmetry axis consists in the existence of an opposite $z$-axis resultant force component or and $y$-axis torque component acting on the levitated magnet, which are generated within the levitator (with no exterior intervention).

To account for this apparently counterintuitive phenomenon, three main factors (Fig. 1c) were under scrutiny [7]: magnetization angle error, especially for the lifter magnet (angle $\theta$), diamagnetic resulting torque exerted by the diamagnetic plate(s) and verticality error affecting the lifter magnet positioning onto the mounting rod (angle $\delta$) (or the same error type for the mounting rod itself).

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**Fig. 1** – Diamagnetically stabilized vertical permanent magnet levitation ($m_l$ and $m_f$ are the magnetic moments of the lifter magnet and floating magnet, respectively); a) classical levitation setup with collinear magnetization similarly oriented; b) ideal levitation setup with antiparallel magnetization directions; c) error affected variant of the ideal structure of point “b” (for illustration purposes angles $\theta$ and $\delta$ are drawn oversized; the Cartesian frame is attached to the lifter magnet, with the origin at its center of mass; $g$ is the gravitational acceleration vector).
Note that the $z$-axis diamagnetic force components were excluded from the very beginning since the investigated phenomenon not only exists in single diamagnetic plate configurations, but also when both stabilizers are simultaneously present. (In this last case, the two diamagnetic forces are supposed to balance each other, since the floater is considered having equal separations from the two plates.)

First, although inferred as a potential source of the phenomenon, preliminary calculations showed the magnetization angle error ($\theta$) being less correlated to the floater’s $z$-axis shift. Secondly, the diamagnetic torque has magnitudes in the order of $10^{-12}$–$10^{-11}$ N·m, and cannot affect significantly the computed equilibrium point coordinates, as we shall further prove. Therefore, this second potential cause was also ruled out. The third remaining possibility – verticality error in positioning the lifter magnet – gave the most conclusive results, showing a strong correlation of angle $\delta$ with the floater $z$-axis displacement.

The global investigation (first experimental and then theoretical) [7] was particularly difficult because of the extremely small value of angle $\delta$, impossible to indentify by simple visual inspection (less than 1°). Consequently, a model of the levitator focused on this issue is proposed and developed in Section 2. In Section 3, the obtained analytical results are used to derive the theoretical dependence of $z$-axis displacement vs. angle $\delta$ and to also assess the sensibility of a possible tilt meter based on the investigated levitation device. Final conclusions are highlighted in Section 4.

![Fig. 2 – Quasi-vertical permanent magnet levitation; a) middle plate case; b) bottom plate case.](image-url)
2. ANALYTICAL MODEL OF THE LEVITATOR

2.1. STATEMENT OF THE MAGNETOSTATICS PROBLEM

As stated at the end of Section 1, the proposed model uses two NeFeB parallelepiped magnets (referred further on as the lifter and the floater) with no magnetization angle errors ($\theta = 0$ – ideal magnets) and a single (middle or bottom) stabilization pyrolytic graphite slab. The only significant considered error is the verticality angle error, as shown in Fig. 2.

Furthermore, magnetizations of both lifter and floater are supposed uniform (with no edge effect) and rigid throughout their volume. Parameter $d$ is the distance from the floater’s geometric center either to the middle or the bottom plate. As a result of the symmetry with respect to $xz$-plane ($y = 0$) presented by the two structures shown in Fig. 2, the equilibrium point will belong to the same plane. The goal of the present section is to solve the above stated magnetostatics problem, namely to determine the coordinates of the equilibrium point $x_0$ and $z_0$, but also the corresponding tilt angle $\alpha_0$ made by the floater with the $x$-axis.

2.2. STATIC EQUILIBRIUM CONDITIONS

In order to achieve the levitation state, the resulting force acting on the suspended magnet, as it is reflected for the two positions of the diamagnetic plate shown in Fig. 3, must be zero:

![Fig. 3 - Balance of forces and torques acting on the suspended magnet; a) middle plate case; b) bottom plate case.](image-url)
\[ F = F_m + F_d + W = 0, \]  
\[ \tau = \tau_m + \tau_d = 0, \]  
where \( F_m \) is the attraction force exerted by the lifter, \( F_d \) the stabilizing force applied by the diamagnetic plate and \( W \) the levitated magnet’s weight. Due to the aforementioned symmetry property, all the forces can be considered as being contained within the \( xz \)-plane.

The second static equilibrium condition, expressed along the \( y \)-axis only, requires a zero resulting torque (Fig. 3):

\[ \tau_m \tau_d = 0, \]

where \( \tau_m \) and \( \tau_d \) are the moments of the force couples imparted by the lifter magnet and the diamagnetic plate, respectively. Note that \( \tau_d \) is similarly oriented in both cases depicted in Fig. 3. When two plates are present \( \tau_d \) is still opposite to the \( y \)-axis direction, but doubles its magnitude. In this particular case \( F_d = 0 \). The significant size contrast presented by size of the two magnets allows us to consider the suspending magnet as a dipole of magnetic moment magnitude:

\[ m_i = \frac{B_{f,\text{OP}}}{\mu_0}, \]

where \( B_{f,\text{OP}} \) is the open-loop operating point flux density of the floater, \( V \) its volume and \( \mu_0 \) the free space permeability. The lifting magnetic force becomes [8]:

\[ F_m = (m_i \cdot \nabla)B_i = \left( m_{i,x} \frac{\partial B_{i,x}}{\partial x} + m_{i,z} \frac{\partial B_{i,z}}{\partial z} \right) i + \left( m_{i,x} \frac{\partial B_{i,z}}{\partial x} + m_{i,z} \frac{\partial B_{i,x}}{\partial z} \right) k, \]

where \( B_i(x,0,z) = B_{i,x}(x,0,z) i + B_{i,z}(x,0,z) k \), is the magnetic flux density associated to the lifter magnet, \( m_f = (-m_i \sin \alpha) i + (-m_i \cos \alpha) k \) and \( i \) and \( k \) are the \( x \)- and \( z \)-axis unit vectors, respectively. The magnetic torque produced by the lifter magnet is [8]:

\[ \tau_m = m_f \times B_i = \left( m_{i,z}B_{i,x} - m_{i,x}B_{i,z} \right) j, \]

where \( j \) is the \( y \)-axis unit vector. For a given distance \( d \), the diamagnetic force and torque depend only on variable \( \alpha \), the tilt angle of the floater with respect to the \( x \)-axis (Fig. 2). We can thus write:

\[ F_d(\alpha) = F_{d,x}(\alpha) i + F_{d,z}(\alpha) k \text{ and } \tau_d = \tau_{d,y}(\alpha) j. \]

On components, for a given value of \( \delta \), the equilibrium conditions (1) and (2), along with (3)–(6), yield the following nonlinear partial derivative system of equations in the unknowns \( x, z \) and \( \alpha \):
In order to solve (7) one has to determine the $x$- and $z$- components of the vector field $B_ℓ$ and the diamagnetic force $F_d$ and the torque $τ_d$. For determining these last two quantities, we opt for a more accurate approach than the classical one based on the method of images. Although accurate enough for modeling the single diamagnetic vertical force component appearing in Fig. 1a type structures, when the floater is tilted (Fig. 1c) one must take into account the anisotropy of the stratified structure of the pyrolytic graphite plate. That arises the necessity of establishing the flux density formula associated to the tilted floater magnet ($B_f$).

### 2.3. FLUX DENSITIES OF LIFTING AND FLOATING MAGNETS

Let us denote by $w$, $t$ and $l$ the lifting magnet’s width, thickness, and length, respectively, as shown in Fig. 4a. Similarly, in Fig. 4b $w_f$, $t_f$ and $l_f$ are the corresponding dimensions of the suspended magnet. Both magnets are magnetized through their length with remanence values $B_r$, $ℓ$ and $B_{r,f}$, respectively. In general, all these parameters are available from the magnet data sheets. Although relatively close to the remanence value, a better accuracy of the following computation is
ensured by the actual open-loop operating point flux densities values $B_{l,\text{OP}}$ and $B_{f,\text{OP}}$. The procedure described in detail in [5–7, 9] implies first determining the permeance coefficient $P_c$, and then intersecting the $B-H$ demagnetization branch of the magnets with the operating line of slope $P_c + 1$. The so obtained values $B_{l,\text{OP}}$ and $B_{f,\text{OP}}$ divided by $\mu_0$ are going to represent the uniform and rigid magnetization magnitudes of the two magnets. This procedure accounts for both the edge effect and the slightly nonuniform magnetization distribution over the magnets’ volume.

For uniformly magnetized block magnets, a very convenient approach capable of providing analytical formulas for the flux density components is offered by the magnetic charge equivalence [5–13]. The introduction of uniformly distributed surface fictitious magnetic charges over the magnets’ poles can substitute the entire magnetized material volume. The equivalent surface magnetic charges for the two magnets are $\sigma_{m,l} = B_{l,\text{OP}}$ and $\sigma_{m,f} = B_{f,\text{OP}}$. Consequently, the magnetic field is the analytical result of the Coulombian surface integrals performed over the pole surfaces [5–13]. Starting from the formulas presented in [13], the needed $B$-field components for the two magnets are as follows.

a. For the lifter, let us first consider the vectors: $a = (-t/2, t/2), b = (-w/2, w/2)$ and $c = (-l/2, l/2)$. Using the elements of those vectors, we define also the quantities: $x_m = x - a_m, y_n = y - b_n$ and $z_k = z - c_k$ with $m, n, k = 1, 2$, their respective indexes. The x- and z- components become [13]:

$$B_{l,x}(x, y, z) = \frac{B_{l,\text{OP}}}{4\pi} \sum_{m=1}^{2} \sum_{k=1}^{2} (-1)^{m+k} \ln \left( \frac{y_1 + \sqrt{x_m^2 + y_n^2 + z_k^2}}{y_2 + \sqrt{x_m^2 + y_n^2 + z_k^2}} \right);$$

$$B_{l,z}(x, y, z) = \frac{B_{l,\text{OP}}}{4\pi} \sum_{m=1}^{2} \sum_{n=1}^{2} \sum_{k=1}^{2} (-1)^{m+n+k} \arctan \left( \frac{x_m y_n}{y_2 + \sqrt{x_m^2 + y_n^2 + z_k^2}} \right).$$

b. For the floater, a similar procedure is applied with respect to the local Cartesian frame $(X, Y, Z)$, depicted in Fig. 4b: $a' = (-t_f/2, t_f/2), b' = (-w_f/2, w_f/2)$ and $c' = (-l_f/2, l_f/2)$; $X_m = X - a'_m, Y_n = Y - b'_n$ and $Z_k = Z - c'_k$ with $m, n, k = 1, 2$. We get [13]:

$$B_{f,x}(X, Y, Z) = -\frac{B_{f,\text{OP}}}{4\pi} \sum_{m=1}^{2} \sum_{k=1}^{2} (-1)^{m+k} \ln \left( \frac{Y_1 + \sqrt{X_m^2 + Y_n^2 + Z_k^2}}{Y_2 + \sqrt{X_m^2 + Y_n^2 + Z_k^2}} \right);$$

$$B_{f,z}(X, Y, Z) = -\frac{B_{f,\text{OP}}}{4\pi} \sum_{m=1}^{2} \sum_{n=1}^{2} \sum_{k=1}^{2} (-1)^{m+n+k} \arctan \left( \frac{X_m Y_n}{Y_2 + \sqrt{X_m^2 + Y_n^2 + Z_k^2}} \right).$$

2.4. EVALUATION OF THE DIAMAGNETIC FORCE AND TORQUE

In a first approximation the diamagnetic stabilizing force can be computed utilizing the method of images [2]. The floater and its image contained within the diamagnetic material are reduced to small dipoles separated by a distance of $2d$. To deal with the close proximity of the tilted floater bottom end and the pyrolytic graphite plate, a more accurate diamagnetic force calculation is needed. Moreover, in this case the anisotropic structure of the diamagnetic material implies a full field calculation. A similar conclusion also holds for the upper plate case.

As suggested by the system of equation (7), our approach dissociates the diamagnetic effects from the main magnetostatics problem referenced to the $(x, y, z)$ Cartesian frame attached to the lifter. Let us denote by $s$ the separation between the lowest extremity of the floater and the diamagnetic plate. In practice, separation $s$ is slightly inferior to 1 mm, for all possible values of variable $\alpha$. Therefore, a constant $s$ value was used for all performed calculations resulting a distance $d$ function of only variable $\alpha$: $d(\alpha) = 0.5(t_c \cos \alpha + l_t \sin \alpha) + s$, with $0 \leq \alpha \leq \pi/2$. Finally, we get the diamagnetic force $F_d$ and the torque $\tau_d$ as functions of only the tilt angle $\alpha$.

For the sake of simplicity we present only the bottom plate case, in which we translate the initial $(x, y, z)$ frame to a new point of origin, as shown in Fig. 5. The performed translation does not affect the main computation since the force and torque components remain the same. In Fig. 5, $W$, $T$ and $L$ are the pyrolytic graphite plate width, thickness and length, respectively.

![Fig. 5 – Floating magnet in close proximity of a stabilizing pyrolytic graphite plate (dimensions and systems of coordinates).](image)

Our approach consists in first computing the force $F_p$ and torque $\tau_d$ exerted by the floater on the pyrolytic graphite plate. We have [8]:

\begin{align*}
F_p &= \\
\tau_d &= 
\end{align*}
\[
F_p = \int_{V_{\text{plate}}} \left( M_p \cdot \nabla \right) B_f \, dV \quad \text{and} \quad \tau_p = \int_{V_{\text{plate}}} \left( M_p \times B_f \right) \, dV ,
\]

where \( M_p \) is the magnetization inside the diamagnetic plate and \( V_{\text{plate}} \) its volume. For a convenient evaluation of these volume integrals, (9.1) and (9.2) must be expressed relative to the \((x, y, z)\) system of coordinates shown in Fig. 5a. The system \((X, Y, Z)\) represents the combined translation and rotation of the system \((x, y, z)\). Specifically, we find the following dependencies:

\[
\begin{align*}
\mathbf{e}_X &= i \cos \alpha - k \sin \alpha \\
\mathbf{e}_Y &= j \\
\mathbf{e}_Z &= i \sin \alpha + k \cos \alpha 
\end{align*}
\]

where \( \mathbf{e}_X, \mathbf{e}_Y \) and \( \mathbf{e}_Z \) are the unit vector of the system \((X, Y, Z)\). With (11), after substituting for \( X, Y \) and \( Z \) in (9.1) and (9.2), the \( B_f \)-field components become:

\[
B_{t,x} = B_{t,x}(x + d) \cos \alpha - z \sin \alpha; \quad B_{t,y} = B_{t,y}; \quad B_{t,z} = B_{t,z} \cos \alpha - B_{t,z} \sin \alpha .
\]

Magnetization \( M_p \) takes into account the anisotropic characteristic of pyrolytic graphite:

\[
M_p = \frac{1}{\mu_0} \begin{pmatrix} \chi_\perp & 0 & 0 \\ 0 & \chi_\parallel & 0 \\ 0 & 0 & \chi_\perp \end{pmatrix} \begin{pmatrix} B_{t,x} \\ B_{t,y} \\ B_{t,z} \end{pmatrix} ,
\]

where \( \chi_\perp \) and \( \chi_\parallel \) are the perpendicular and parallel magnetic susceptibilities.

Substituting (9), (12) and (13) in (10), and taking advantage of the symmetry about the \(xz\)-plane presented by the structure depicted in Fig. 5, we get:

\[
F_{p,x} = \frac{1}{\mu_0} \int_0^{L} \int_{-W/2}^{W/2} \int_{-L/2}^{L/2} \chi_\perp B_{t,x} \left( \frac{\partial B_{t,x}}{\partial x} + \chi_\parallel \left( \frac{\partial B_{t,y}}{\partial y} + \frac{\partial B_{t,z}}{\partial z} \right) \right) \, dx \, dy \, dz ;
\]

\[
F_{p,z} = \frac{1}{\mu_0} \int_0^{L} \int_{-W/2}^{W/2} \int_{-L/2}^{L/2} \chi_\parallel B_{t,z} \left( \frac{\partial B_{t,z}}{\partial z} + \chi_\parallel \left( \frac{\partial B_{t,y}}{\partial y} + \frac{\partial B_{t,z}}{\partial z} \right) \right) \, dx \, dy \, dz ;
\]

\[
\tau_{p,y} = -\frac{1}{\mu_0} \int_0^{L} \int_{-W/2}^{W/2} \int_{-L/2}^{L/2} B_{t,y} B_{t,z} (\chi_\parallel - \chi_\perp) \, dx \, dy \, dz .
\]

We also obtain \( F_{p,y} = 0 \) and \( \tau_{p,x} = \tau_{p,z} = 0 \).
For the floater and pyrolytic plate used for the computation performed in Section 3, $\alpha \in [0, \pi/2]$ and $s = 0.9$ mm, we numerically solved (14.1)–(14.3) using a general purpose math package. Values for $F_{p,x}$ were in the order of $10^{-4}$ N, but for $F_{p,z}$ in the order of $10^{-9}$ N. Similarly, the $y$-component of the torque $\tau_{p,y}$ has values in the order of $10^{-12}$–$10^{-11}$ N·m. Consequently, the last two quantities can be neglected for further calculations. Finally, in (7) $F_{d,x}(\alpha) = -F_{p,x}(\alpha)$ and $F_{d,z}(\alpha) = -F_{p,z}(\alpha) \approx 0$ [14], but also $\tau_{d,y}(\alpha) \approx 0$. The need of a convenient analytic function in (7), imposed the use of a polynomial interpolation of the discrete values of $F_{d,x}(\alpha)$, as shown by the graph plotted in Fig. 6. As expected, the bottom diamagnetic plate pushes the floater in an upward direction ($F_{d,x}(\alpha) < 0$).

For the middle plate case, a similar computation, or simply by symmetry reasons, we can conclude that $F_{d,x}(\alpha)_{\text{middle plate}} = -F_{d,x}(\alpha)_{\text{bottom plate}}$.

3. COMPUTATION RESULTS

The procedure described in Section 2, having as main objective solving system (7), was implemented in a professional math package. The computed results correspond to commercially available magnets, as follows: the N52 grade lifter $90 \times 40 \times 30$ [mm] ($t \times w \times l$), $B_{l,OP} = 1.4257$ T; the N52 grade floater $5 \times 5 \times 2$ [mm] ($t_{f} \times w_{f} \times l_{f}$), $B_{f,OP} = 1.446$ T. Stabilization was ensured by a 5 mm thick PG plate of perpendicular and parallel magnetic susceptibilities of $\chi_{\perp} = 45 \times 10^{-5}$ and $\chi_{\parallel} = 8.5 \times 10^{-5}$, respectively. The two magnets are magnetized through length (namely, $l$ and $l_{f}$). A value of $s = 0.9$ mm was considered for the separation
between the pyrolytic graphite plate and the floater’s most protruding end, for each tilt angle $\delta$.

In order to estimate the feasibility of the levitator as highly responsive tilt meter, we define its sensitivity as: $S = \frac{d|z_0(\alpha)|}{d\delta}$. Computation results are shown in Fig. 7 for tilt angles ranging up to 15°. Values of $|z_0(\alpha)|$ exhibit unexpectedly high values for small tilt angles, resulting a very high sensitivity around $\delta = 0$, when the latter exhibits an almost vertical variation graph. The smaller the tilt, the higher the sensitivity of the device becomes. Notice the insignificant difference between the sensitivity graphs in the two cases shown in Fig. 7, with slightly superior values for the bottom plate case (BP).

![Graph](image)

Fig. 7 – Absolute value of $z$-axis displacement and sensitivity vs. tilt angle (bottom plate – BP and middle plate – MP; circular marker – discrete computed values; continuous line – spline interpolation).

For all computed equilibrium points, stability was checked following the procedure presented in [2, 7]. The influence diamagnetic term along the $x$-axis was estimated and its value added to the corresponding stability function, which, as expected, changed to a positive value, proving that stability has been achieved.

4. CONCLUSIONS

The recently discovered and reported quasi-vertical counterintuitive diamagnetically stabilized levitation is further investigated in the present paper. A magnetic charge analytical model of the setup is proposed for computing the levitated magnet equilibrium point coordinates and rotation angle. The apparently abnormal deviation of the suspended magnet from the symmetry $x$-axis is
investigated and explained utilizing the proposed model. The main analytically proven cause of the phenomenon resides in the unintended and inherent very small verticality error affecting either the lifting magnet or its mounting rod. Their positioning error might be not identifiable by simple visual inspection, therefore recommending the use of this levitator for highly sensitive verticality detection and measurement devices in non-magnetic environments.

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