Rev. Roum. Sci. Techn.– Électrotechn. et Énerg.
Vol. **62**, *2*, pp. 179–184, Bucarest, 2017

TRANSMISSION OVER KAPPA-MU FADING CHANNELS WITH GAMMA DISTRIBUTED RANDOM LINE-OF-SIGHT COMPONENTS

DJOKO BANDJUR[1], BRANIMIR JAKSIC[1], STEFAN PANIC[2], MILOS BANDJUR[1], ANA MATOVIC[3], EDIS MEKIC[4]

Key words: Average bit error probability (ABER), Cumulative distribution function (CDF), Kappa-mu fading, Outage probability (OP), Probability density function (PDF).

In this paper we will observe wireless propagation in non-homogenous environment under non-deterministic line-of-sight (LOS) conditions, when the random nature of dominant/scattering components ratio has been considered and modeled as Gamma distribution. First closed-form expressions have been derived for the probability density function (PDF) and cumulative distribution function (CDF) of the observed fading process. Further performance analysis of wireless transmission over given conditions has been carried out through the standard criterions, such as outage probability (OP) and average bit error probability (ABER). Numerically obtained results have been graphically presented and discussed in the function of transmission parameters.

1. INTRODUCTION

Kappa-mu fading model describes the multipath propagation in non-homogeneous scattering environment when line-of-sight (LOS) signal components are present [1]. Also this model, which provides a very good fit to experimental measurements, has general properties, since includes in itself Rayleigh, Rician, and Nakagami-m fading models as its special cases.

The fading parameter, which defines the ratio between the total power of dominant components and the total power of scattered components, and so defines the severity of observed fading process, often is assumed to be deterministic and is used as constant value in propagation scenarios modeling. Knowledge and estimation of this parameter is crucial in link budget determination, design of receiver structure and adaptive modulation and coding technique selection [2]. However, due to the non-stationary nature of vehicular communication channels this assumption should not be taken for granted. Urban scenario measurements, presented in [3], taken at 1.9 GHz with a chip rate of 1.2288 MHz, show time-varying nature of observed fading parameter. In a comprehensive study [4], group of authors, based on measuring 16 individual single input single-output channels, with a bandwidth of 240 MHz, for a duration of 20 s, have shown that this fading parameter should not be assumed to have constant value in time, and that its values non stationarity depend on various factors. A need for deriving new analytic model that takes the randomness of this fading parameter into account has been pointed out in [3] and [5]. The ratio of powers dominant and scatter components has been already treated as random process in [6]. It has been found that ratio of components is lognormal, with the median being a simple function of antenna height, antenna beam width, and distance and with a corresponding standard deviation. Randomness of the ratio has also been observed in papers [7, 8], when high speed railway (HSR) viaduct and terrain cutting scenarios have been analyzed.

Various researches have proven the fact that lognormal and Gamma distributions can closely approximate each other [9, 10].

The aim of this paper was to obtain novel characterization of propagation in LOS conditions, by observing general kappa-mu fading model and considering its ratio of powers dominant and scatter components as Gamma distributed random process.

Standard first order statistical characterization for this model will be determined, *i.e.* probability density function (PDF) and cumulative distribution function (CDF) of random envelope process will be obtained in closed form, in the function of Laguerre polynomials. Based on derived PDF and CDF expressions, performance analysis of transmission in given environment will be carried out, through evaluating and observing some standard performance criterions, such as outage probability (OP) and average bit error rate (ABER). Numerically obtained results will be discussed through function of the system parameters.

2. SYSTEM MODEL

PDF of the kappa-mu distributed random envelope process is given with [2]:

$$f(y)=\frac{\mu(1+\kappa)^{\frac{\mu+1}{2}}y^{\frac{\mu-1}{2}}}{\kappa^{\frac{\mu-1}{2}}\exp(\mu\kappa)\Omega^{\frac{\mu+1}{2}}}\cdot \exp\left(-\frac{\mu(1+\kappa)y}{\Omega}\right)\cdot I_{\mu-1}\left(2\mu\sqrt{\frac{\kappa(1+\kappa)y}{\Omega}}\right). \quad (1)$$

Here $I_{\mu-1}(x)$ denotes the modified Bessel function of the first kind and order $\mu-1$ [11, Eq. 8.445]. Parameter μ is related to the number of multipath clusters, through which signal propagates, and so can describe severity of fading process, since as μ decreases fading severity increases [2]. Parameter κ represents the ratio between the total power of dominant components and the total power of scattered components (*i.e.* $\kappa = d^2/2\mu\sigma^2$, d^2 representing the mean power of the dominant components and total power for each cluster scattered components being $2\sigma^2$). Ricean model

[1] University of Pristina, Faculty of Technical Science, Knjaza Milosa 7, 38220 Kosovska Mitrovica, Serbia, E-mail: djoko.bandjur@pr.ac.rs, branimir.jaksic@pr.ac.rs, djoko.bandjur@pr.ac.rs
[2] University of Pristina, Faculty of Natural Science and Mathematics, Lole Ribara 29, 38220 Kosovska Mitrovica, Serbia, E-mail: stefanpnc@yahoo.com
[3] University of Nis, Faculty of Electrical Engineering, Aleksandra Medvedeva 14, 18000 Nis, Serbia, E-mail: ana.matovic4@gmail.com
[4] State University of Novi Pazar, Vuka Karadzica, 36300 Novi Pazar, Serbia, E-mail: emekic@np.ac.rs

could be easily obtained from above by setting value to parameter $\mu = 1$, with parameter κ defining the Ricean K factor. Furthermore, if $\kappa = 0$, the κ-μ distribution is equivalent to the Rayleigh distribution. From (1) one can see how the κ-μ distribution reduces to the Nakagami-*m* distribution as $\kappa \to 0$.

Let us take into consideration the randomness of κ parameter behavior. As mentioned, well-known fact is that lognormal and Gamma distributions can closely approximate each other. Also the $2\mu\sigma^2$ part from κ factor has already been observed as Gamma random process in various composite processes such are K and K_G distributions [10, 12]. Parameter d^2 was already treated as Gamma-type (Nakagami-m) distributed random process in land mobile satellite (LMS) channel model from [13]. Here we will obtain novel characterization of propagation in LOS conditions by observing entire κ factor as Gamma distributed random process modeled by:

$$p_\kappa(\kappa) = \frac{1}{\Gamma(c)} \frac{\kappa^{c-1}}{\kappa_0^c} \exp\left(-\frac{\kappa}{\kappa_0}\right), \qquad (2)$$

with $\kappa_0 = E(\kappa^2)$ and c describing the random κ factor severity change parameter, where E is expectation operator.

Now, novel random proccess envelope PDF can now be obtained by averaging over Gamma distributed proccess of κ factor change as:

$$p(y) = \int_0^\infty f(y/\kappa) p_\kappa(\kappa) \mathrm{d}\kappa \,. \qquad (3)$$

Now after substituting (2) into (3), by performing some mathematical transformations with respect to [11, Eq. 3.384.5], we obtain closed form expression (Appendix):

$$p(y) = \frac{\mu \cdot y^{\frac{\mu-1}{2}} \cdot \exp\left(-\frac{\mu y}{\Omega}\right)}{\Omega^{\frac{\mu+1}{2}} \cdot \Gamma(c) \cdot \kappa_0^c} \sum_{p=0}^{\infty} \frac{\mu^{\mu-1+2p} \cdot \left(\frac{y}{\Omega}\right)^{\frac{\mu-1+2p}{2}}}{\Gamma(p+\mu)\cdot p!} \cdot$$
$$\cdot \frac{\pi^2}{\left(\mu + \frac{\mu y}{\Omega} + \frac{1}{\kappa_0}\right)^{p+c} \cdot \Gamma(-p-\mu) \cdot \sin(\pi(2p+c+\mu))} \cdot$$
$$\cdot \left[\frac{\left(\mu + \frac{\mu y}{\Omega} + \frac{1}{\kappa_0}\right)^{-p-\mu} \cdot L_{p+\mu}^{-2p-c-\mu}\left(\mu + \frac{\mu y}{\Omega} + \frac{1}{\kappa_0}\right)}{\sin(\pi(-p-\mu)) \cdot \Gamma(1-p-c)} - \right. \qquad (4)$$
$$\left. - \frac{\left(\mu + \frac{\mu y}{\Omega} + \frac{1}{\kappa_0}\right)^{p+c} \cdot L_{-p-c}^{2p+c+\mu}\left(\mu + \frac{\mu y}{\Omega} + \frac{1}{\kappa_0}\right)}{\sin(\pi(p+c)) \cdot \Gamma(1+p+\mu)} \right],$$

with $L_n^k(x)$ denoting Laguerre polynomial defined by [14, Eq. 1.3.1]. Infinite-series from above rapidly converge with only few terms needed to achieve accuracy at 5th significant digit.

In Fig. 1, PDF of observed proccess is shown for some combination of system parameter values. Parameter κ_0 is defined in (2).

Now, starting from CDF infinte-series expression for the kappa-mu random proccess [2]:

$$F_r(r) = \sum_{p=0}^{\infty} \frac{\kappa^p \cdot \mu^p \cdot \gamma\left(\mu + p, \frac{\mu(\kappa+1)}{\Omega} r^2\right)}{\exp(\kappa\mu) \cdot \Gamma(\mu+p) \cdot p!}, \qquad (5)$$

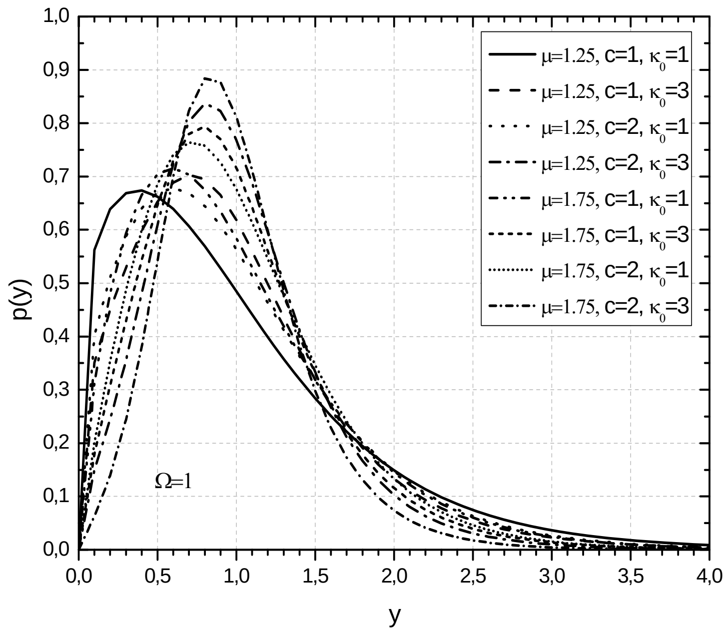

Fig. 1 – PDF for various values of system parameters.

where $\gamma(a, x)$ stands for the incomplete Gamma function [11, Eq. 8.443], by taking into account Gamma modeled randomness of its dominant/scatering component ratio, novel CDF could be obtained after averaging (5) as:

$$F_R(r) = \int_0^\infty F_{R/\kappa}(r/\kappa) p_\kappa(\kappa) \mathrm{d}\kappa \, . \tag{6}$$

After substituting (2) and (5) into (6), with respect to [11, Eq. 3.384.5] and properties of Laguerre polynomials we obtain CDF expression in a closed-form as (Appendix):

$$\begin{aligned} F_R(r) = &\sum_{p=0}^{\infty}\sum_{s=0}^{\infty} \frac{\mu^{2p+\mu+s} \cdot r^{2p+2\mu+2s} \cdot \exp\left(-\frac{\mu \cdot r^2}{\Omega}\right)}{\Gamma(\mu+p) \cdot p! \cdot \kappa_0^c} \cdot \\ &\cdot \frac{\Gamma(1+\mu+p)}{\Gamma(c) \cdot (\mu+p) \cdot \Omega^{\mu+p+s} \cdot \Gamma(1+\mu+p+s)} \cdot \\ &\cdot \frac{1}{\left(\frac{1}{\kappa_0}+\mu+\frac{\mu \cdot r^2}{\Omega}\right) \cdot (p+c) \cdot \Gamma(-\mu-p-s)} \cdot \\ &\cdot \frac{\pi^2}{\sin\left[\pi(2p+c+\mu+s)\right]} \left[\frac{\left(\frac{1}{\kappa_0}+\mu+\frac{\mu \cdot r^2}{\Omega}\right)^{-\mu-p-s}}{\sin\left[\pi(-\mu-p-s)\right] \cdot \Gamma(1-p-c)} \cdot \right. \\ &\cdot \sum_{n=0}^{\mu+p+s} (-1)^n \left(\frac{1}{\kappa_0}+\mu+\frac{\mu \cdot r^2}{\Omega}\right)^n \cdot \\ &\cdot \frac{(-p-c)!}{(\mu+p+s-n)! \cdot (-\mu-2p-s-c+n)! \cdot n!} - \\ &- \frac{\left(\frac{1}{\kappa_0}+\mu+\frac{\mu \cdot r^2}{\Omega}\right)^{p+c}}{\sin\left[\pi(p+c)\right] \cdot \Gamma(1+\mu+p+s)} \cdot \\ &\cdot \sum_{m=0}^{-p-c} (-1)^m \left(\frac{1}{\kappa_0}+\mu+\frac{\mu \cdot r^2}{\Omega}\right)^m \cdot \\ &\left. \cdot \frac{(p+\mu+s)!}{(-p-c-m)! \cdot (2p+c+\mu+s+m)! \cdot m!} \right] \, . \end{aligned} \tag{7}$$

Rapidly converging sums from above expression need only few terms before truncation to provide accuracy at 5th significant digit.

3. NUMERICAL RESULTS

Outage probability (OP, P_{out}) is a measure of the system performance, used for helping the designers of wireless communications systems in order to meet required quality-of-service (QoS) and grade of service (GoS) demands. OP is defined as the probability that received signal falls below a given outage threshold R_{th} also known as a protection ratio, and can be determined according to relation [2]:

$$p_{out} = P(R < R_{th}) = F_R(R_{th}) \, . \tag{8}$$

Capitalizing on closed-form CDF expression from (7), OP values can be efficiently evaluated for various values of system parameters.

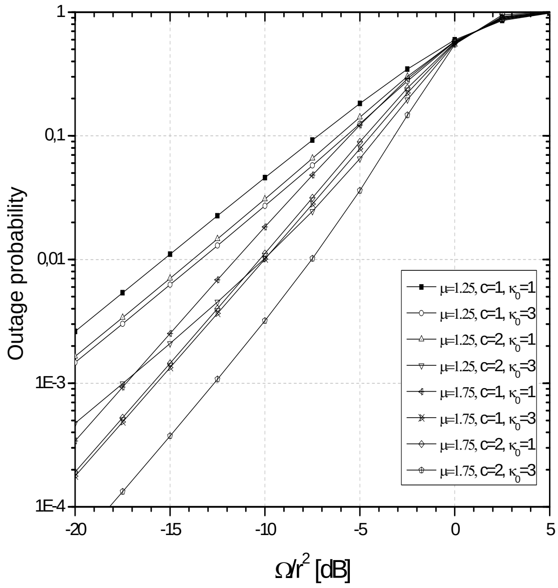

Fig. 2 – OP of transmission in observed propagation scenario, for various values of derived model parameters.

It can be seen from Fig. 2 how the change of parameters c and κ_0, which define the randomness of κ factor, affects the OP. Change of parameters c and κ_0 provides more significant effect at lower values of μ parameter, since fading is more severe, and its severity generally decreases with μ parameter growth. Also is interesting to observe how c parameter change influences more OP at higher values of κ_0. This could be explained through the fact that dynamics of parameter κ change (defined through parameter c) is more extensively at higher mean expected values of κ, since wider range of conditions is covered. Another standard performance criterion could be efficiently evaluated based on (4). Namely ABER in observed environment, when transmission is carried out over corresponding modulation format can be determined according to:

$$P_e = \int_0^\infty P_{e|y}(e \mid y) p(y) \mathrm{d}y \, , \tag{9}$$

with conditional ABER, $P_{e|y}(e \mid y)$, depending on modulation form being used.

Namely, for non-coherent modulation formats, *i.e.* for noncoherent frequency shift keying (NCFSK) and binary differential phase shift keying (BDPSK) $P_{e|y}(e \mid y) = 1/2 \exp(g \cdot y)$, with $g = 1/2$ and $g = 1$, respectively, while for coherent modulation formats, *i.e.* coherent frequency shift keying (CFSK) and coherent phase shift keying (cPSK) $P_{e|y}(e \mid y) = 1/2 \operatorname{erfc}\left(\sqrt{g \cdot y}\right)$, with erfc($x$) denoting complementary error function [11, Eq. 8.250.4], and $g = 1/2$ and $g = 1$, respectively [15]. We denote by g the modulation parameter.

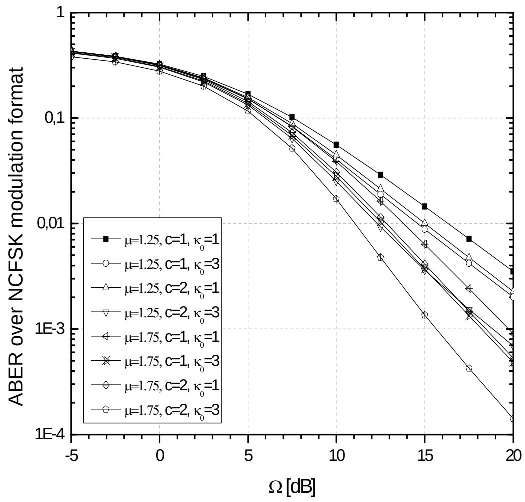

Fig. 3 – ABER over NCFSK modulaction format for various values of system parameters.

From Fig. 3, beside expected conclusions that better system performances (lower ABER values) are reachable in the area of higher values of parameters μ, c and κ_0, one can see that statements given about the nature and influence of parameters c and κ_0 change (which define the randomness of κ- factor) on system performance values change, which are valid for OP criterion, also stand in this case.

As addition in Fig.4 ABER values for observed propagation scenario are shown for some of commonly used modulation techniques.

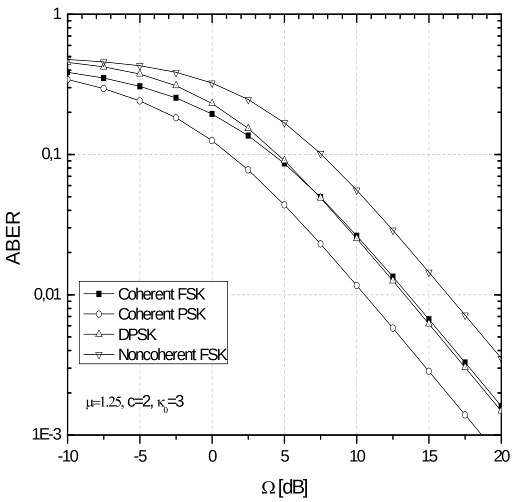

Fig. 4 – ABER for different modulation formats.

4. **CONCLUSIONS**

Random change of the ratio of powers dominant/scattering components in non-homogenious propagation under LOS conditions has been considered in this paper, instead of considering this ratio as a constant value deterministic variable. Novel closed-form expressions have been derived for the PDF and CDF of observed general fading model, when the random nature of kappa-mu dominant/scattering components ratio parameter has been modeled with Gamma distribution. Further, based on these expressions OP and ABER have been efficiently evaluated and analyzed in the function of system parameters.

ACKNOWLEDGEMENTS

This work has been partially performed within the framework of the project: Performance optimization of energy-efficient computer and communication systems, TR32023, funded by Ministry of Science and Technological Development of the Republic of Serbia.

APPENDIX

DERIVATION PDF FORMULAS

Substituting (1) and (2) into (3), we obtain:

$$p(y)=\int_0^\infty \frac{\mu(1+\kappa)^{\frac{\mu+1}{2}} y^{\frac{\mu-1}{2}}}{\kappa^{\frac{\mu-1}{2}}\exp(\mu\kappa)\Omega^{\frac{\mu+1}{2}}}\exp\left(-\frac{\mu(1+\kappa)y}{\Omega}\right)\cdot I_{\mu-1}\left(2\mu\sqrt{\frac{\kappa(1+\kappa)y}{\Omega}}\right)\frac{1}{\Gamma(c)}\frac{\kappa^{c-1}}{\kappa_0^c}\exp\left(-\frac{\kappa}{\kappa_0}\right)\mathrm{d}\kappa. \qquad (A1)$$

Expressing the Bessel functions $I_{\mu\text{-}1}(x)$ as a series [11, Eq. 8.445], (A1) can be written as:

$$p(y)=\int_0^\infty \frac{\mu(1+\kappa)^{\frac{\mu+1}{2}} y^{\frac{\mu-1}{2}}}{\kappa^{\frac{\mu-1}{2}}\exp(\mu\kappa)\Omega^{\frac{\mu+1}{2}}}\cdot \exp\left(-\frac{\mu(1+\kappa)y}{\Omega}\right)\cdot\sum_{p=0}^{\infty}\frac{\left(\frac{2\mu\sqrt{\kappa(1+\kappa)y}}{\Omega}\right)^{\mu-1+2p}}{2^{2p+\mu-1}\Gamma(p+\mu)\cdot p!}\cdot \frac{1}{\Gamma(c)}\frac{\kappa^{c-1}}{\kappa_0^c}\exp\left(-\frac{\kappa}{\kappa_0}\right)\mathrm{d}\kappa. \qquad (A2)$$

After some straightforward mathematical manipulations, (A2) can be written as:

$$p(y)=\frac{\mu\cdot y^{\frac{\mu-1}{2}}\cdot\exp\left(-\frac{\mu y}{\Omega}\right)}{\Omega^{\frac{\mu+1}{2}}\cdot\Gamma(c)\cdot\kappa_0^c}\sum_{p=0}^{\infty}\frac{\mu^{\mu-1+2p}\cdot\left(\frac{y}{\Omega}\right)^{\frac{\mu-1+2p}{2}}}{\Gamma(p+\mu)\cdot p!}\int_0^\infty \kappa^{p+c-1}\cdot(1+\kappa)^{\mu+p}\cdot\exp\left(-\left(\mu+\frac{\mu y}{\Omega}+\frac{1}{\kappa_0}\right)\kappa\right)\mathrm{d}\kappa. \qquad (A3)$$

Solution for the integral in (A3) is well-known and given in [11, Eq. 3.384.5]:

$$\int_0^\infty x^{q-1}\cdot(1+ax)^{-v}\cdot\exp(-ax)\mathrm{d}x = \\ = \frac{\pi^2}{pq\cdot\Gamma(v)\cdot\sin(\pi(q-v))}\cdot \left[\frac{\left(\frac{t}{a}\right)^{v}\cdot L_{-v}^{v-q}\left(\frac{t}{a}\right)}{\sin(\pi(-t-\mu))\cdot\Gamma(1-t-c)} - \frac{\left(\frac{t}{a}\right)^{q}\cdot L_{-q}^{q-v}\left(\frac{t}{a}\right)}{\sin(\pi q)\cdot\Gamma(1-v)}\right] \tag{A4}$$

For this case applies

$$x=\kappa \\ t=\mu+\frac{\mu y}{\Omega}+\frac{1}{\kappa_0} \\ q=p+c \\ a=1 \\ v=-p-\mu \tag{A5}$$

Capitalizing on (A4) and (A5), (A3) can be expressed as:

$$\int_0^\infty \kappa^{p+c-1}\cdot(1+\kappa)^{\mu+p}\cdot\exp\left(-\left(\mu+\frac{\mu y}{\Omega}+\frac{1}{\kappa_0}\right)\right)\mathrm{d}\kappa = \\ = \frac{\pi^2}{\left(\mu+\frac{\mu y}{\Omega}+\frac{1}{\kappa_0}\right)^{p+c}\cdot\Gamma(-p-\mu)\cdot\sin(\pi(2p+c+\mu))}\cdot \\ \cdot\left[\frac{\left(\mu+\frac{\mu y}{\Omega}+\frac{1}{\kappa_0}\right)^{-p-\mu}\cdot L_{p+\mu}^{-2p-c-\mu}\left(\mu+\frac{\mu y}{\Omega}+\frac{1}{\kappa_0}\right)}{\sin(\pi(-p-\mu))\cdot\Gamma(1-p-c)} - \frac{\left(\mu+\frac{\mu y}{\Omega}+\frac{1}{\kappa_0}\right)^{p+c}\cdot L_{-p-c}^{2p+c+\mu}\left(\mu+\frac{\mu y}{\Omega}+\frac{1}{\kappa_0}\right)}{\sin(\pi(p+c))\cdot\Gamma(1+p+\mu)}\right] \tag{A6}$$

Eventually, by substituting (A6) in (A3), we obtain the final expression for the PDF given by (4).

DERIVATION CDF FORMULAS

Using the expression for the incomplete Gamma function $\gamma(a,x)$ [11, Eq. 8.443]:

$$\gamma(a,x)=\frac{x^a}{a}\exp(-x)\,{}_1F_1(1;1+a;x)= \\ =\frac{x^a}{a}\exp(-x)\sum_{s=0}^{\infty}\frac{(a)_s\cdot x^s}{(1+a)_s\cdot s!} \tag{A7}$$

the incomplete Gamma function in (5) can be expressed as:

$$\gamma\left(\mu+p,\frac{\mu(\kappa+1)}{\Omega}r^2\right)=\frac{\left(\frac{\mu(\kappa+1)}{\Omega}r^2\right)^{\mu+p}}{\mu+p}\cdot \\ \cdot\exp\left(-\frac{\mu(\kappa+1)}{\Omega}r^2\right)\cdot\sum_{s=0}^{\infty}\frac{(1)_s\cdot\left(\frac{\mu(\kappa+1)}{\Omega}r^2\right)^s}{(1+\mu+p)_s\cdot s!} \tag{A8}$$

where $(a)_{(n)}$ denotes the Pocchammer symbol [11].

Further, after replacing (2), (5) and (A8) into (6) we obtain:

$$F_R(r)=\int_0^\infty\sum_{p=0}^{\infty}\frac{\kappa^p\cdot\mu^p\cdot\frac{\left(\frac{\mu(\kappa+1)}{\Omega}r^2\right)^{\mu+p}}{\mu+p}}{\exp(\kappa\mu)\cdot\Gamma(\mu+p)\cdot p!}\cdot \\ \cdot\exp\left(-\frac{\mu(\kappa+1)}{\Omega}r^2\right)\cdot\sum_{s=0}^{\infty}\frac{(1)_s\cdot\left(\frac{\mu(\kappa+1)}{\Omega}r^2\right)^s}{(1+\mu+p)_s\cdot s!}\cdot \\ \cdot\frac{1}{\Gamma(c)}\frac{\kappa^{c-1}}{\kappa_0^c}\exp\left(-\frac{\kappa}{\kappa_0}\right)\mathrm{d}\kappa \tag{A9}$$

After some straightforward mathematical manipulations, (A9) can be written as:

$$F_R(r)=\sum_{p=0}^{\infty}\sum_{s=0}^{\infty}\frac{\mu^{2p+\mu+s}\cdot r^{2p+2\mu+2s}\cdot\exp\left(-\frac{\mu\cdot r^2}{\Omega}\right)}{\Gamma(\mu+p)\cdot p!\cdot\kappa_0^c}\cdot \\ \cdot\frac{\Gamma(1+\mu+p)}{\Gamma(c)\cdot(\mu+p)\cdot\Omega^{\mu+p+s}\cdot\Gamma(1+\mu+p+s)}\cdot \\ \int_0^\infty(\kappa+1)^{\mu+p+s}\kappa^{p+c-1}\exp\left(-\kappa\left(\frac{1}{\kappa_0}+\mu+\frac{\mu r^2}{\Omega}\right)\right)\mathrm{d}\kappa \tag{A10}$$

Solution of the integral in (A10) is already known in the literature [11, Eq. 3.384.5] and given by (A4).

For this case applies

$$x=\kappa \\ t=\mu+\frac{\mu\cdot r^2}{\Omega}+\frac{1}{\kappa_0} \\ q=p+c \\ a=1 \\ v=-p-\mu-s. \tag{A11}$$

Capitalizing on (A4) and (A11), solution for the integral in (A10) can be expressed as:

$$\int_0^\infty(\kappa+1)^{\mu+p+s}\kappa^{p+c-1}\exp\left(-\kappa\left(\frac{1}{\kappa_0}+\mu+\frac{\mu r^2}{\Omega}\right)\right)\mathrm{d}\kappa = \\ =\frac{1}{\left(\frac{1}{\kappa_0}+\mu+\frac{\mu r^2}{\Omega}\right)^{-\mu-p-s}}\cdot \\ \cdot\frac{\pi^2}{(p+s)\cdot\Gamma(-\mu-p-s)\cdot\sin(\pi(2p+c+\mu+s))}\cdot \\ \cdot\left[\frac{\left(\frac{1}{\kappa_0}+\mu+\frac{\mu r^2}{\Omega}\right)^{-p-\mu-s}\cdot L_{p+\mu+s}^{-2p-c-\mu}\left(\frac{1}{\kappa_0}+\mu+\frac{\mu r^2}{\Omega}\right)}{\sin(\pi(-p-\mu-s))\cdot\Gamma(1-p-c)} - \frac{\left(\frac{1}{\kappa_0}+\mu+\frac{\mu r^2}{\Omega}\right)^{p+c}\cdot L_{-p-c}^{2p+c+\mu}\left(\frac{1}{\kappa_0}+\mu+\frac{\mu r^2}{\Omega}\right)}{\sin(\pi(p+c))\cdot\Gamma(1+p+\mu+s)}\right]. \tag{A12}$$

Further, by using the well-known expression for Laguerre polynomials [11]

$$L_a^b = \sum_{m=0}^{\infty} (-1)^m \frac{(a+b)!}{(a-m)! \cdot (b+m)! \cdot m!} x^m \quad \text{(A13)}$$

solution for the integral in (A10) can be expressed as:

$$\int_0^{\infty} (\kappa+1)^{\mu+p+s} \kappa^{p+c-1} \exp\left(-\kappa\left(\frac{1}{\kappa_0}+\mu+\frac{\mu r^2}{\Omega}\right)\right) d\kappa =$$

$$= \frac{1}{\left(\frac{1}{\kappa_0}+\mu+\frac{\mu r^2}{\Omega}\right)^{-\mu-p-s}} \cdot$$

$$\cdot \frac{\pi^2}{(p+s)\cdot\Gamma(-\mu-p-s)\cdot\sin\left(\pi(2p+c+\mu+s)\right)} \cdot$$

$$\cdot \left[\frac{\left(\frac{1}{\kappa_0}+\mu+\frac{\mu\cdot r^2}{\Omega}\right)^{-\mu-p-s}}{\sin\left[\pi(-\mu-p-s)\right]\cdot\Gamma(1-p-c)} \cdot \right.$$

$$\cdot \frac{\sum_{n=0}^{\mu+p+s} (-1)^n \left(\frac{1}{\kappa_0}+\mu+\frac{\mu\cdot r^2}{\Omega}\right)^n \cdot (-p-c)!}{(\mu+p+s-n)! \cdot (-\mu-2p-s-c+n)! \cdot n!} -$$

$$- \frac{\left(\frac{1}{\kappa_0}+\mu+\frac{\mu\cdot r^2}{\Omega}\right)^{p+c}}{\sin\left[\pi(p+c)\right]\cdot\Gamma(1+\mu+p+s)} \cdot$$

$$\left. \cdot \frac{\sum_{m=0}^{-p-c} (-1)^m \left(\frac{1}{\kappa_0}+\mu+\frac{\mu\cdot r^2}{\Omega}\right)^m \cdot (p+\mu+s)!}{(-p-c-m)! \cdot (2p+c+\mu+s+m)! \cdot m!} \right].$$

Eventually, by substituting (A14) in (A10), we obtain the final expression for the closed-form CDF given by (7).

Received on October 16, 2015

REFERENCES

1. M.D. Yacoub, *The κ-μ distribution and the η-μ distribution*, IEEE Antennas Propag. Mag., **49**, pp. 68–81 (2007).
2. S. Panic, M. Stefanovic, J. Anastasov, P. Spalevic, *Fading and Interference Mitigation in Wireless Communications*, CRC Press, New York, 2013.
3. G.G. Messier, J.A. Hartwell, *An Empirical Model for Nonstationary Ricean Fading*, IEEE Transactions on Vehicular Technology, **58**, *1*, pp. 14–20 (2009).
4. L. Bernado *et al.*, *Multi-Dimensional K-Factor Analysis for V2V Radio Channels in Open Sub-urban Street Crossings*, 21st IEEE International Symposium on Personal Indoor and Mobile Radio Communications, Istanbul, Turkey, 2010.
5. N. Sekulovic, M. Stefanović, *Performance Analysis of System with Micro- and Macrodiversity Reception in Correlated Gamma Shadowed Rician Fading Channels*, Wireless Personal Communications, **65**, *1*, pp. 143–156 (2012).
6. L.J. Greenstein, S. Ghassemzadeh, V. Erceg, D.G. Michelson, *Ricean K-factors in narrow-band fixed wireless channels: Theory, experiments, and statistical models*, IEEE Transactions on Vehicular Technology, **58**, *8*, pp. 4000–4012 (2009).
7. F. Luan, Y. Zhang, L. Xiao, C. Zhou, S. Zhou, *Fading Characteristics of Wireless Channel on High-Speed Railway in Hilly Terrain Scenario*, International Journal of Antennas and Propagation, Article ID 378407 (2013).
8. J. Lu, G. Zhu, B. Ai, *Fading Analysis for the High Speed Railway Viaduct and Terrain Cutting Scenarios*, International Journal of Antennas and Propagation, Article ID 862945 (2012).
9. V. Milenkovic, N. Sekulovic, M. Stefanovic, M. Petrovic, *Effect of Microdiversity and Macrodiversity on Average Bit Error Probability in Gamma Shadowed Rician Fading Channels*, ETRI Journal, **32**, *3*, pp. 464–467 (2010).
10. B. Nikolic, M. Stefanovic, S. Panic, J. Anastasov, B. Milosevic, *Closed-form expressions for selection combining system over correlated Generalized-K (KG) fading channels in the presence of co-channel interference*, ETRI Journal, **33**, *3*, pp. 320–325 (2011).
11. I.S. Gradshteyn, I.M. Ryzhik, *Table of Integrals, Series, and Products*, Seventh Edition. Academic Press, New York, 2007.
12. P.C. Sofotasios, S. Freear, *On the κ-μ/ gamma Composite Distribution: A Generalized Multipath/Shadowing Fading Model*, IEEE International Microwave and Optoelectronics Conference (IMOC '11), Natal, Brazil, 2011.
13. A. Abdi, W.C. Lau, M. S. Alouini, M. Kaveh, *A new simple model for land mobile satellite channels: first- and second-order statistics*, IEEE Trans. Wireless Commun., **2**, pp. 519–528 (2003).
14. R. Koekoek, *Generalizations of the Classical Laguerre Polynomials and some q-Analogues*, Escher Heirs/Cordon Art, Baarn, 1990.
15. M.K. Simon, M.S. Alouini, *Digital Communications over Fading Channels*, 2nd Ed, Wiley, New York, 2005.