A PROBABILISTIC MODEL FOR POWER GENERATION ADEQUACY EVALUATION

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Key words: Power generating system, Interference model, Monte Carlo simulation.

The basic function of a modern electric power system is to supply the system load requirements as economically as possible and with a reasonable degree of reliability and quality. In power system generation planning, many models and techniques have been developed to evaluate the reliability performance. The objective of this paper is to develop a probabilistic model for power generating system reliability performance evaluation, based on the convolution technique.

1. **INTRODUCTION**

The quality and availability required of electrical energy is directly related to the power system reliability concept. The reliability associated with a power system, in a general sense, is a measure of the overall ability of the system to generate and supply electrical energy. Due to the complexity of the electric power system, it is divided into functional areas namely generation, transmission and distribution. Reliability of each functional area is usually analyzed separately for an easier evaluation and eventually combined to assess the system reliability.

The electricity has a characteristic due to the fact that it can't be stored in the system for long time and important quantity. As a result, electricity must be used when it is produced and generated there and then when there is demand. Therefore, generating systems are designed and operated in order to meet the demand load of the system with a certain reliability level. Generating system reliability is used to evaluate the capacity of the generating power system to satisfy the total system load. The reliability assessment of a power generating system can be divided into two main aspects: system adequacy, which is related to the existence of sufficient facilities to satisfy system load demand, and system security, which is related to the ability of the system to respond to dynamic or transient disturbances.

Load demand can exceed the generating capacity for two main reasons. First, if there is a very high load peak demand that exceeds the installed capacity of the

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system, the system cannot supply the load peak. Second, if some generating capacity units are out of service because of failures or periodic maintenance, a high load peak demand that does not exceed the installed capacity of the system can exceed the available capacity at that moment.

In this paper, the authors focus on an evaluating model for the generating system adequacy, having in view the behaviour of generating units. The developed model is based on the combination of random variables that describe the generating capacity and system load demand and, finally compared with the results of the other models or simulation techniques.

2. GENERATING SYSTEM ADEQUACY INDICES

The adequacy associated to a generating power system, in a general sense, is a measure of the ability of system generating capacity to satisfy the total system load. Generally, the generating power system evaluation process does not consider the transmission and distribution systems, only concentrates on the balance between generating capacity and load demand. The approach to generating adequacy evaluation is to develop a capacity model for all the capacity from the system and to join this with an established load model.

The most popular indices used in generating power system are the Loss of Load Expectation and the Loss of Energy Expectation, which have to base the Loss of Load Probability (LOLP) [1, 2].

The Loss of Load Expectation (LOLE) [hours/year] indicates the average number of hours in a given period (usually one year) in which the load is expected to exceed the available generating capacity. It is obtained by calculating the loss of load duration in hours for that daily peak demand exceeding the available capacity for each day and adding these times for all the days for a number of sample years.

The Loss of Energy Expectation (LOEE) [MWh/year], sometime known as the Expected Energy Not Supplied (EENS), specifies the expected energy that will not be supplied by the generation system due to those occasions when the load demanded exceeds the available generating capacity.

Other indices can be defined as generating system reliability performance measures, such as: Expected Loss of Load Frequency [occurrences/year], Expected Duration of Loss of Load [hours/occurrence], Load Not Supplied per Interruption [MWh/occurrence], Expected Energy Not Supplied per Interruption [MWh/occurrence], Energy Index of Reliability [%], and others. Frequency and duration of loss of load are a basic extension of the LOLE index, in that they identify the average frequency and the average duration of the occurrence, for a certain period (usually one year).

3. A PROBABILISTIC MODEL FOR GENERATING ADEQUACY

The basic approach to evaluate the adequacy of an electrical power generating system consists of two parts: capacity model and load model. The capacity and load models are joined using a probabilistic model which evaluates the reliability performance of the generating system in terms of adequacy indices.

This problem can be analyzed like the problem of interference of the stress-strength random variable model, described in many areas of reliability [3, 4]. Mathematically, the interference model conducts to a joined variable of the two random variables of generating capacity and load demand, as is showed in Fig. 1.

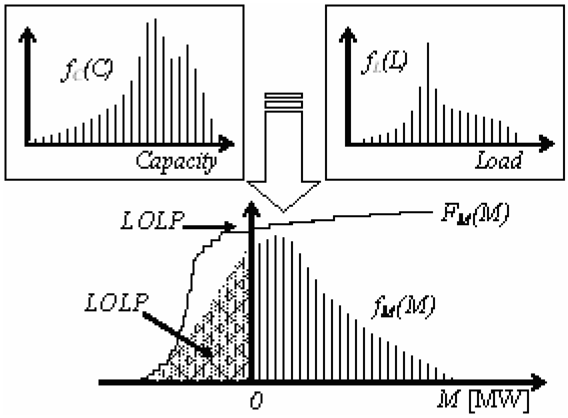

Fig. 1 – The interference model of the generating capacity and load demand.

The interference model depends on the nature of random variables involved in model, continuous or discrete. The stress-strength interference model and its practical evaluations were developed, examined and presented by authors in many others papers [5, 6]. In this paper, both variables have multiple power [MW] levels, with each level having a probability of occurrence, so that these variables can be modelled as discrete random variables.

The generation system adequacy indices can be obtained by observing the joined variable called the available capacity margin, and defined like the difference between available generating capacity C and load demand L variables [7]. A positive margin denotes that the system generation is sufficient to meet the system load, while a negative margin implies that the system load is not served.

The LOLP is a widely used indicator, as a criterion for generation system reliability, because it indicates the probability of the load to exceed the generating capacity of a proposed generation power system, during a given time interval.

$$\text{LOLP} = \Pr\left(C < L\right) = \sum_{i=1}^{N_L} \sum_{j=1}^{N_C} \Pr\left(L_i\right) \cdot \Pr\left(C_j\right) \cdot I_{ij} \,, \tag{1}$$

where C is a discrete random variable representing the available generating capacity and L is a discrete random variable of the load demand. In equation (1), $\Pr(L_i)$ is the probability of the i^{th} load level (L_i) and N_L the number of the load levels in the load probability function. $\Pr(C_j)$ is the probability of the j^{th} generation capacity level (C_j) and N_C the number of the generation capacity levels in the generation capacity probability function. I_{ij} is an indicator function defined as $I_{ij} = \left\{ 0, \text{ if } \; L_i \le C_j ; \, 1, \text{ if } \; L_i > C_j \right\}$.

The LOLP index is assessed only for the area in which load exceeds capacity, such the previous relationship can be rewritten as:

$$\text{LOLP} = \Pr\left(M < 0\right) = \sum_{k=1}^{N} \Pr\left(M_k\right) = F_M\left(0\right), \tag{2}$$

where M=C–L is a discrete random variable representing the available capacity margin. The $\Pr(M_k)$ is the probability of the k^{th} available capacity margin level (M_k) and N is the number of the available capacity margin levels from negative margin area. The $F_M(M)$ is the cumulative distribution function of M variable. A relationship between the probability function, cumulative distribution function of the available capacity margin variable, respectively the LOLP index is presented in Fig. 1.

In power generating system, sometimes others adequacy indices are necessary, beside the LOLP. It is important to know how many times the load exceeds the available generating capacity, or how much energy has been lost due to interruption. The LOLE and LOEE indices can be expressed using the LOLP and expected values of available capacity margin in negative margin area, for a given period using the following relationships:

$$\text{LOLE} = \text{LOPT} \cdot T \;\text{ and }\; \text{LOEE} = \sum_{k=1}^{N} M_k \cdot \Pr\left(M_k\right) \cdot T \,, \tag{3}$$

where T is the total number of hours from analysed period, M_i and $\Pr(M_i)$ have been defined for equation (2).

In the following, is presented a procedure for determining the available capacity probability function for a various number of capacity units, respectively to establish a load probability function, that combined to estimate the adequacy indices. The quantitative adequacy evaluation invariably leads to a data required supporting such studies, so, for our study we will use the IEEE Reliability Test Systems (IEEE-RTS), developed by the Subcommittee on the Application of Probability Methods in the IEEE Power Society, to provide a common test system which could be used for comparing the results obtained from different methods, available to [8].

The IEEE-RTS generation system is composed by a combination of 32 generation units ranging from 12 MW to 400 MW. Generation system data contains the type of generation units, rated capacity [MW] and number of each units, respectively the reliability data as mean time to failure (MTTF) and mean time to repair (MTTR), in hours.

The capacities of each generation units can be modelled like a discrete random variable. Let C_k be a discrete random variable representing the capacity that can be supplied by the k^{th} generation unit, with $k = 1, \ldots, 32$. Is assumed that each unit has two possible working states, down and up. The random variable can be described by its associated probability function that contains all the capacity states, in an ascending order, and its probabilities:

$$\{0 \times q_k, CU_k \times p_k\}, \tag{4}$$

where:

- CU_k is the rated capacity of k^{th} generation Unit (it is used CU_k, not to be confused with C_k, the generation capacity level of discrete random variable C);
- $q_k = \Pr(0) = \dfrac{\mathsf{MTTR}_k}{\mathsf{MTTF}_k + \mathsf{MTTR}_k}$ is probability that available capacity of k^{th} generation unit to be zero;
- $p_k = \Pr(CU_k) = \dfrac{\mathsf{MTTR}_k}{\mathsf{MTTF}_k + \mathsf{MTTR}_k}$ is probability that available capacity of k^{th} generation unit to be rated capacity, CU_k [MW].

From the probability theory, it is known that the sum of two random variables ($C_1 + C_2$) is a new random variable, with probability that both unit 1 and unit 2 to be simultaneously in operation during a specified period of time. New random variable can be obtained by convolving the both random variables:

$$C = \text{conv}\,(C_1, C_2), \tag{5}$$

described by its associated probability function:

$$\{0 \times q_1 \cdot q_2,\quad CU_1 \times p_1 \cdot q_2,\quad CU_2 \times q_1 \cdot p_2,\quad (CU_1 + CU_2) \times p_1 \cdot p_2\}. \tag{6}$$

So, all the capacity generation units can be modelled as discrete random variables and the capacity generation system for the whole system is a new random variable described by:

$$C = \text{conv}\,(C_1, C_2, \ldots, C_{32}). \tag{7}$$

The capacity probability function may be computed using the gradual convolution for all units of system. After convolving all 32 discrete random variables of the generation units, it is obtained the following random variable for the IEEE-RTS generating system (Fig. 2).

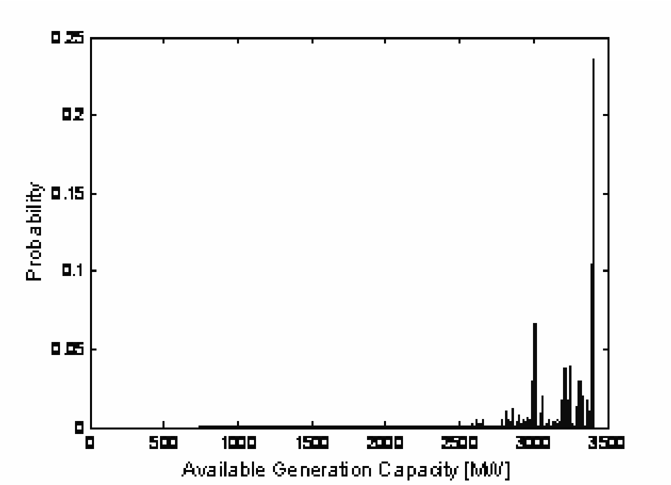

Fig. 2 – IEEE-RTS available capacity probability function.

The load demand in power systems is variable in time. There is no one unique profile or mathematical equation that can be adopted to represent the load characteristic curve. In this paper, the load characteristic curve has been modelled using the load model of IEEE-RTS. The annual load curve, included in IEEE-RTS, considers the seasonal, weekly, daily and hourly peak of load. The hourly load is percentage of daily peak, the daily peak load is percentage of weekly peak and weekly peak load is percentage of annual peak for three seasons, so the hourly load dependents by three indices (*i,j,k*). The index *i* represents a certain hour of the day, the index *j* represents a certain day of the week, and the index *k* represents a certain week of the year, respectively. The hourly load level HL*(i,j,k)* can be obtained from multiplication the three percentages of the peak of load, as:

$$\mathrm{HL}(i,j,k)=\left[\mathrm{pr\_hour}(i)\times\mathrm{pr\_day}(j)\times\mathrm{pr\_week}(k)\right]\times\mathrm{annual\_peak\_of\_load}\,, \quad (8)$$

where:

pr_hour*(i)*(%) is the percentage of i^{th} hour from the daily peak of load, $i = 1,...,24$;

pr_day*(j)*(%) is the percent of j^{th} day from the weekly peak of load, $j = 1,..,7$;

pr_week*(k)*(%) is the percent of k^{th} week from the yearly peak of load, $k = 1,...,52$.

The final hourly load for one year HL, can be described like a discrete random variable, with 8,736 hourly load levels (24 hours/day × 7 days/week × 52 weeks/year), having the following probability function:

$$\left\{\mathsf{HL}(i,j,k)\times\left(\Pr\left(\text{hour }\ i\ \text{ of day}\right)\cdot\Pr\left(\text{day }\ j\ \text{ of week}\right)\cdot\Pr\left(\text{week }\ k\ \text{ of }\ \text{year}\right)\right)\right\} \quad (9)$$

Taking into account that some hourly load levels may have the same values, the probability of any value of HL variable, can be evaluated using the count number of the discrete random variable levels with the same values. For that, the authors have been developed a Matlab-Simulink function that allowed determination of the probability function associated of the discrete random variable of IEEE-RTS hourly load, as is shown in Fig. 3.

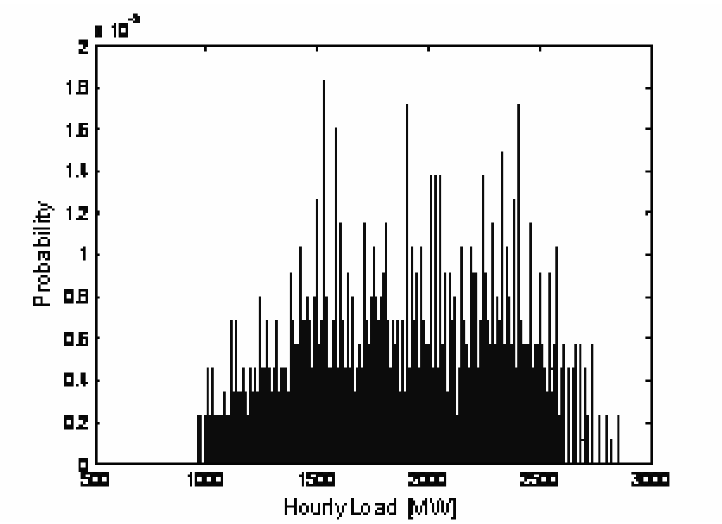

Fig. 3 – IEEE-RTS hourly load probability function.

Capacity margin is the amount by which generating capacity exceeds system load demand, expressed as a discrete random variable. The available capacity margin variable results from difference of generating capacity and load demand variables, and its probability function arises from the load-capacity probability functions interference. The probability function of capacity margin variable is presented in the Fig. 4.

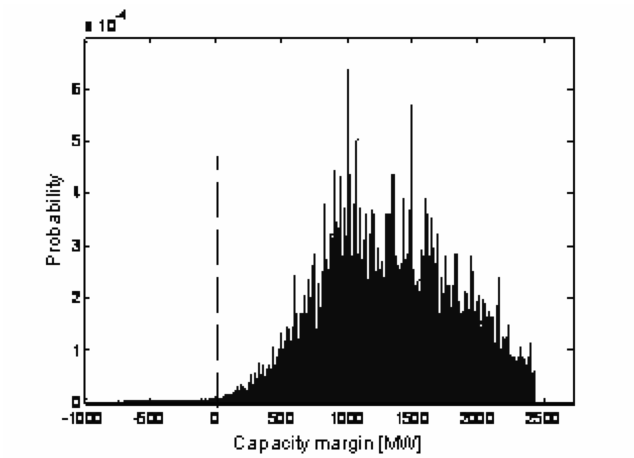

Fig. 4 – The available capacity margin probability function.

4. STUDY RESULTS

The Matlab program based on available capacity margin variable has been developed to compute the LOLP, LOLE and LOEE indices using the relations (2–3). The program results of previous model have been compared with results from other model. Sequential Monte Carlo simulation has been used to provide information relates to the average values of the adequacy indices. This technique generates a chronological operating cycles for each generation unit. The simulation is done by using the MTTF and MTTR parameters to produce two sequence of up and down times, necessary to estimate the chronological operating unit's cycles. The generating system operating model can then be obtained by combining the operating cycles of all units. The required adequacy indices may be observed from the overlapping generation operating model to a chronological load curve, over a long time period. The simulation can be stopped when a specified degree of confidence has been achieved. Figure 5 shows an example of LOLE and LOEE indices evaluation, using the sequential Monte Carlo simulation, for 10,000 sample years, necessary to ensure the convergence process.

The IEEE-RTS generating system adequacy indices provided by probabilistic model and sequential Monte Carlo simulation are shown in Table 1. For a better comparison between models, three levels of annual peak of load were considered in the adequacy indices evaluation. It can be seen that the results obtained from both methods are very close. The probabilistic method provides comparative results with Monte Carlo simulation, but the computing time is considerably less than the simulation technique, requiring fewer CPU resources.

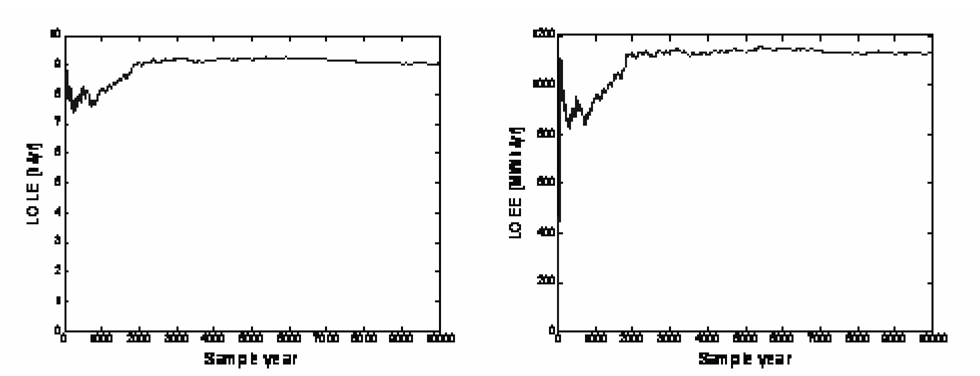

Fig. 5 – Results from sequential Monte Carlo simulation: a) LOLE [10^3 h/yr]; b) LOEE [MWh/yr].

The proposed model may be used in a generation source planning, to analyze different adding new generation unit scenarios and select the suitable sources as the generation system to meet a load growth, maintaining the same level of adequacy.

Table 1

IEEE-RTS Generating system adequacy indices

Index	LOLP [%]			LOLE [h/yr]			LOEE [MWh/yr]		
Annual peak load [MW]	2750	2850	2950	2750	2850	2950	2750	2850	2950
Probabilistic model	55.841 $\times 10^{-3}$	109.421 $\times 10^{-3}$	201.765 $\times 10^{-3}$	4.878	9.559	17.626	566.950	1171.50	2331.60
Monte Carlo Simulation	55.058 $\times 10^{-3}$	108.219 $\times 10^{-3}$	199.823 $\times 10^{-3}$	4.810	9.454	17.457	559.620	1158.80	2311.30

For IEEE-RTS case study, if the system with 2,850 MW annual peak of load is considered having an acceptable adequacy level, the developed model may be used to establish the best solution for generating system expansion, assuming an annual peak load growth. It is assumed that the annual peak load of the IEEE-RTS system is increased with 100 MW, from 2,850 MW to 2,950 MW. So, the LOLE and LOEE indices increase from 9.559 h/yr and 1171.5 MWh/yr for base case (with 2,850 MW peak load) to 17.626 h/yr, respectively 2,331.6 MWh/yr for the new annual peak of load (2,950 MW). The 100 MW growth of peak of load may be supplied from different adding new generation unit scenarios, until the LOLE and

LOEE indices decrease under the acceptable adequacy level, as is presented in Table 2. It shows the adequacy indices for all possible adding generation unit scenarios, to supply the increased peak of load, having in view various number and type of generation units available in IEEE-RTS. As shown Table 2, the scenario of adding a 1×50 MW, 1×20 MW and 2×12 MW generation units, leads to a minimum added capacity (94 MW).

Table 2

Reliability indices for various expansions of IEEE-RTS

Cases	System	Added capacity [MW]	LOLE [h/yr]	LOEE [MWh/yr]
1	IEEE-RTS with 8×12 MW	96	9.0120	1 123.3
2	IEEE-RTS with 5×20 MW	100	9.3306	1 166.2
3	IEEE-RTS with 2×50 MW	100	8.6911	1 080.8
4	IEEE-RTS with 1×100 MW	100	8.9763	1 120.8
5	IEEE-RTS with 1×20+7×12 MW	104	8.6396	1 069.3
6	IEEE-RTS with 2×20+6×12 MW	112	8.2240	1 017.6
7	IEEE-RTS with 3×20+3×12 MW	96	9.3615	1 170.7
8	IEEE-RTS with 4×20+2×12 MW	104	8.9712	1 114.6
9	IEEE-RTS with 1×50+1×20+2×12 MW	**94**	9.1828	1 152.8
10	IEEE-RTS with 1×50+2×20+1×12 MW	102	8.8513	1 097.6

5. **CONCLUSIONS**

In this paper, a probabilistic model for the power generating capacity adequacy evaluation is presented. The main aspect of the probabilistic model is to evaluate a random variable that describes the difference between capacity of generation units and load demand and it evaluating in the negative domain.

The model enables the evaluation of the most popular indices in generating capacity planning, namely the loss of load probability, loss of load expectation, and the loss of energy expectation, respectively. The results were validate using the sequential Monte Carlo simulation and found that provided results are very close. The model has the advantage that can be easily implemented in computer programs and require a computing time considerably less than in the case of simulation methods. The developed model can be used not only to evaluate the previous mentioned indices, but also may be used in a generation source planning, to select the suitable sources as the generation system to meet a load growth, maintaining the same level of adequacy level. The probabilistic model can provide more significant information for system planning, since they consider probabilistic aspects of generating units. The future work to be done will use the probabilistic model presented in this paper in a generation source planning with integrated wind energy sources.

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