SECOND ORDER STATISTICS OF THE MIMO κ-μ KEYHOLE FADING CHANNELS

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Key words: Level crossing rate (LCR), Average fade duration (AFD), Keyhole multiple-input and multiple-output (MIMO) fading channels, κ-μ fading, Multiplicative fading.

In this paper novel analytical closed-form expressions for the level crossing rate (LCR) and the average fade duration (AFD) of the double κ-μ random process are presented. Capitalizing on them, second order statistics of multiple input multiple output keyhole fading channels with space-time block coding (STBC) are analyzed. Numerically obtained results are graphically presented to show the effects of various parameters such as the fading severity and number of transmit/receive antennas on the overall system's performances.

1. INTRODUCTION

MIMO (multiple-input and multiple-output) communication technique, is based on multiple antennas usage at both the transmitter and receiver for the purpose of improving communication performances. It has recently attracted attention in wireless communications, because it offers significant increases in data throughput and link range without additional bandwidth or transmit power by achieving higher spectral efficiency (more bits per second per hertz of bandwidth) and link reliability or diversity (reduced fading). Capitalizing on these properties, MIMO is an important part of modern wireless communication standards such as IEEE 802.11n (Wifi), 4G, 3GPP Long Term Evolution, WiMAX and is also planned to be used in such as recent High-Speed Packet Access plus (HSPA+) and Long Term Evolution (LTE) [1–3].

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MIMO communications systems potentials are not always reachable even for a fully uncorrelated transmit and receive channels. It has been shown that for some environments, the performances of wireless MIMO systems can become very low, even for uncorrelated signals. This effect has been termed "keyhole" or "pinhole", and is attributed to the rank deficiency of the MIMO channel [4]. The keyhole MIMO channels have been discussed through physical examples, considering uncorrelated channels with a single degree of freedom [5–6]. It is shown, that under the keyhole effect, the entries of the channel matrix H, follow statistics described as a product of two independent single-path gains [5].

Multiplicative fading models have been given special attention for some time, because they have been used for keyhole channel modelling of MIMO systems. The first order statistical properties of the double Nakagami-m fading model has been considered in [7], where the fading between each pair of transmit and receive antennas in presence of the "keyhole" is characterized as Nakagami-m fading. In [6] analytical solutions for level crossing rate (LCR) and average fade duration (AFD) for the double Nakagami-m model have been determined, and results are applied to study the second order statistics of the keyhole channels applicable to MIMO systems with space-time block coding (STBC), operating in rich-scattering environments. It is assumed that the channels are independent.

Nakagami fading (m-distribution) describes multi-path scattering with relatively large delay-time spreads, with different clusters of reflected waves [8]. It provides good fits to collected data in indoor and outdoor mobile-radio environments and is used in many wireless communications applications. κ-μ fading model was recently proposed [9]. This distribution is more realistic than other special distributions, since its derivation is completely based on a non-homogeneous scattering environment, and is a general physical fading model which includes Rayleigh, Rician and Nakagami-m fading models as special cases [10]. The model is written in terms of two physical parameters, κ and μ with μ related to multi-path clustering, while the parameter κ denoting the ratio between the total power of dominant components and the total power of the scattered waves.

In this paper analytical results for level crossing rate (LCR) and average fade duration (AFD) for this model have been determined, and results are applied to study the second order statistics of the keyhole channels applicable to MIMO systems with space-time block coding (STBC), operating in rich-scattering environments. It is assumed that the channels are independent. Moreover to the best of author's knowledge, no analytical study of switch and stay combining involving assumed correlated κ-μ fading model has been reported in the literature.

2. SYSTEM MODEL

Let us consider MIMO "keyhole" channel model. Complex path gain of baseband equivalent signal transmitted over the channel between the i-th transmit and j-th receive antenna at arbitrary moment t is expressed as [11]:

$$h_{i,j}(t)=\alpha_i(t)\,\beta_j(t)\,\mathrm{e}^{\mathrm{j}(\phi_i(t)+\psi_j(t))}, \qquad 1\le i\le M,\ 1\le j\le N, \tag{1}$$

where $\left\{\alpha_i(t)\mathrm{e}^{j\phi_i(t)}\right\}_{i=1}^{M}$ denote the complex path gains introduced by the rich-scattered channel from the i-th transmitting antenna to the “keyhole”, and $\left\{\beta_i(t)\mathrm{e}^{j\psi_j(t)}\right\}_{j=1}^{N}$ denote the complex path gains introduced by the rich-scattered channel from the “keyhole” to the j-th receiving antenna.

Phases $\{\phi_i(t)\}_{i=1}^{M}$ and $\{\psi_i(t)\}_{j=1}^{N}$ are independent and uniformly distributed over [0,2π). The amplitudes $\{\alpha_i(t)\}_{i=1}^{M}$ and $\{\beta_i(t)\}_{j=1}^{N}$ are modelled by κ-μ random variables.

The fading parameters of $\alpha_i(t)$ will be denoted with κ_1, μ_1 and Ω_T, where Ω_T = E [α_T^2] for all i. In the similar manner, the fading severity parameters of $\beta_j(t)$ will be denoted with κ_1, μ_1 and Ω_R, whereas Ω_R = E [β_R^2] for all j. With assumed mobility of both the transmitter and receiver with respect to the “keyhole”, all channel gains are time-correlated random processes with maximum Doppler shifts $f_{\alpha i}=f_\alpha$ and $f_{\beta j}=f_\beta$, respectively. Then, the time derivatives $\dot{\alpha}_i$ and $\dot{\beta}_j$, independent from α_i and β_j, will both follow zero-mean Gaussian probability density functions (PDFs) with variances $\sigma_{\dot{x}}^2=2\pi^2 f_d^2\Omega_x$, where $1\le i\le M$ and $\sigma_{\dot{y}}^2=2\pi^2 f_d^2\Omega_y$ where $1\le j\le N$.

The orthogonal space-time block coding (STBC) signal combining transform a MIMO fading channel into an equivalent single input single output (SISO) fading channel at arbitrary moment t with a path gain of the squared Frobenius norm of the MIMO channel matrix $H(t)=[h_{ij}(t)]_{MxN}$ given with:

$$\|H(t)\|_F^2=\sum_{i=1}^{M}\sum_{j=1}^{N}\left|h_{ij}(t)\right|^2=\left(\sum_{i=1}^{M}\alpha_i^2(t)\right)\left(\sum_{j=1}^{N}\beta_j^2(t)\right). \tag{2}$$

The instantaneous output value of signal-to-noise (SNR) per symbol, after the process of space-time block decoding, is given by:

$$\gamma(t)=\frac{\overline{\gamma}}{MR}\|H(t)\|_F^2, \tag{3}$$

where $\overline{\gamma}=E_S/N_0$ denotes the average SNR value per receive branch, and R is the rate of the STBC.

Let us define auxiliary random process $Z(t)$ by:

$$Z(t) = \sqrt{\|H(t)\|_F^2} = X(t)\,Y(t), \tag{4}$$

with $X(t) = \sqrt{\sum_{i=1}^{M} \alpha_i^2(t)}$ and $Y(t) = \sqrt{\sum_{j=1}^{N} \beta_j^2(t)}$ denoting the κ-μ distributed random processes with probability distribution functions (PDFs) given by [12]:

$$p(x) = \frac{2L_1\mu_1(1+k_1)^{\frac{L_1\mu_1+1}{2}}}{k_1^{\frac{L_1\mu_1-1}{2}} \mathrm{e}^{L_1\mu_1 k_1} \Omega_x^{\frac{L_1\mu_1+1}{2}}} x^{L_1\mu_1} \mathrm{e}^{-\frac{\mu_1(1+k_1)x^2}{\Omega_x}} I_{L_1\mu_1-1}\left[2\mu_1\sqrt{\frac{L_1 k_1(1+k_1)x^2}{\Omega_x}}\right] \tag{5}$$

$$p(y) = \frac{2L_2\mu_2(1+k_2)^{\frac{L_2\mu_2+1}{2}}}{k_2^{\frac{L_2\mu_2-1}{2}} \mathrm{e}^{L_2\mu_2 k_2} \Omega_y^{\frac{L_2\mu_2+1}{2}}} y^{L_2\mu_2} \mathrm{e}^{-\frac{\mu_2(1+k_2)y^2}{\Omega_y}} I_{L_2\mu_2-1}\left[2\mu_2\sqrt{\frac{L_2 k_2(1+k_2)y^2}{\Omega_y}}\right], \tag{6}$$

with $I_k(\cdot)$ denoting the modified Bessel function of the first kind of k-th order [13] with M denoted with L_1 and N denoted with L_2. The time derivatives $\dot{X}$ and $\dot{Y}$ are independent from X and Y, respectively, and both follow the zero-mean Gaussian PDF with variances given above. Therefore, the random process $Z(t)$, defined by (4), is a double κ-μ process for which an exact and approximate LCR and AFD is obtained. The LCR and the AFD of instantaneous output SNR, given by (4) are respectively given by:

$$N_\gamma(\gamma) = N_z\left(\sqrt{\gamma MRT/\overline{\gamma}}\right), \tag{7}$$

$$T_\gamma(\gamma) = T_z\left(\sqrt{\gamma MR/\overline{\gamma}}\right), \tag{8}$$

with Z and γ defined as in (3) and (4).

3. ON THE SECOND ORDER STATISTICS OF THE OUTPUT SNR

The LCR of Z at threshold z is defined as rate at which the random process crosses level z in the negative direction. To extract LCR, we need to determine the joint PDF of Z and $\dot{Z}$, $f_{Z\dot{Z}}(z,\dot{z})$, and apply the Rice's formula:

$$N_Z(z) = \int_0^\infty \dot{z} f_{Z\dot{Z}}(z,\dot{z})\,\mathrm{d}\dot{z}. \tag{9}$$

The above expression can be rewritten as:

$$N_Z(z)=\int_0^{\infty}\left(\int_0^{\infty}\dot{z}f_{\dot{Z}|ZX}(\dot{z}\mid z,x)\,\mathrm{d}\dot{z}\right)f_{Z|X}(z\mid x)f_X(x)\mathrm{d}x, \tag{10}$$

where $f_{\dot{Z}|ZX}(\cdot,\cdot,\cdot)$ is the conditional PDF of $\dot{Z}$ conditioned on Z and X. The conditional PDF can be determined by finding the time derivative of both sides of (9),

$$\dot{Z}=Y\dot{X}+X\dot{Y}=\frac{Z}{X}\dot{X}+X\dot{Y}. \tag{11}$$

For fixed values of $Z=z$ and $X=x$, the time derivative $\dot{Z}$ is a zero-mean Gaussian random variable with variance $\sigma^2_{\dot{Z}|ZX}=\dfrac{z^2\sigma^2_{\dot{X}}}{x^2}+x^2\sigma^2_{\dot{Y}}$. Therefore, the integral inside the brackets in the expression (10) can be evaluated as:

$$\int_0^{\infty}\dot{z}f_{\dot{Z}|ZX}(\dot{z}\mid z,x)\,\mathrm{d}\dot{z}=\frac{\sigma_{\dot{Z}|ZX}}{\sqrt{2\pi}}. \tag{12}$$

The conditional PDF of Z for some fixed value of $X=x$, $f_{Z|X}(z\mid x)$, is determined by the transformation of random variables $f_{Z|X}(z\mid x)=\dfrac{f_Y(z\mid x)}{x}$. After some algebraic transformations, it is obtained:

$$\begin{aligned}&N_z(z)=4\sqrt{\pi\Omega_y}f_d\sum_{p=0}^{+\infty}\sum_{q=0}^{+\infty}C_1\int_0^{+\infty}\sqrt{1+\frac{z^2}{x^4}\left(\frac{\Omega_x}{\Omega_y}\right)}\,x^{2p-2q+2\mu_1L_1-2\mu_2L_2}\mathrm{e}^{-\frac{\mu_1(1+k_1)x^2}{\Omega_x}}\mathrm{e}^{-\frac{\mu_2(1+k_2)z^2}{\Omega_y x^2}}\mathrm{d}x\\&C_1=\frac{k_1^p k_2^q(1+k_1)^{p+\mu_1L_1}(1+k_2)^{q+\mu_2L_2}L_1^pL_2^q\mu_1^{2p+\mu_1L_1}\mu_2^{2q+\mu_2L_2}}{\mathrm{e}^{\mu_1L_1k_1+\mu_2L_2k_2}\Omega_x^{p+\mu_1L_1}\Omega_y^{q+\mu_2L_2}\Gamma(p+\mu_1L_1)\Gamma(q+\mu_2L_2)p!\,q!}\end{aligned} \tag{13}$$

with $\Gamma(x)$ denoting the Gamma function [13]. Using some software the above integral can be evaluated numerically with desired accuracy. On the other side, applying Laplace approximation allows obtaining a highly accurate closed-form solution of the expression (13), and that is presented in the following subsection.

Average fade duration (AFD) of the random variable Z is defined as the average time that the double κ-μ process remains below level z after crossing that level in negative direction:

$$T_z(z)=\frac{F_z(z)}{N_z(z)}, \tag{14}$$

where $F_z(Z)$ denotes the cumulative distribution function of Z. For the double κ-μ random process, it attains the form:

$$F_z(z)=\sum_{p=0}^{+\infty}\sum_{q=0}^{+\infty}C_2\; z^{p+q+\mu_1 L_1+\mu_2 L_2}$$

$$\times G_{1,3}^{2,1}\left(\frac{\mu_1\mu_2(1+k_1)(1+k_2)}{\Omega_x\Omega_y}z\left|\begin{matrix} 1-\frac{p+q+\mu_1 L_1+\mu_2 L_2}{2} \\ \frac{p-q+\mu_1 L_1-\mu_2 L_2}{2},\frac{q+\mu_2 L_2-p-\mu_2 L_2}{2},\frac{p+q+\mu_1 L_1+\mu_2 L_2}{2}\end{matrix}\right.\right), \quad (15)$$

$$C_2=C_1\mu_1^{\frac{p+q+\mu_1 L_1+\mu_2 L_2}{2}}\mu_2^{\frac{p+q+\mu_1 L_1+\mu_2 L_2}{2}}(1+k_1)^{\frac{q+\mu_2 L_2-p-\mu_2 L_2}{2}}(1+k_2)^{\frac{p-q+\mu_1 L_1-\mu_2 L_2}{2}}$$

where G [ ] denotes and Meijer's functions [13]. Using [14], Laplace type integral can be approximated as:

$$\int_0^{+\infty} g(x)\,\mathrm{e}^{-\lambda f(x)}\mathrm{d}x\approx\sqrt{\frac{2\pi}{\lambda}}\frac{g(x_0)}{\sqrt{f''(x_0)}}\mathrm{e}^{-\lambda f(x_0)}, \quad (16)$$

when the real-valued parameter λ is large. In (12) *f(x)* and *g(x)* are real-valued functions of x, and x_0 is the point at which *f(x)* has an absolute minimum (interior critical point of *f(x)*). *f″(x)* is the second derivative of *f(x)* with respect to x. Since the above expression provides very accurate results even for small values of λ [13], *f(x)* and *g(x)* are set as:

$$f(x)=\frac{\mu_1(1+k_1)x^2}{\Omega_x}+\frac{\mu_2(1+k_2)z^2}{\Omega_y x^2}-\ln\left(x^{x^{2p-2q+2\mu_1 L_1-2\mu_2 L_2}}\right), \quad (17)$$

$$g(x)=\sqrt{1+\frac{z^2}{x^4}\left(\frac{\sigma_{\dot{x}}}{\sigma_{\dot{y}}}\right)^2}, \quad (18)$$

where

$$f''(x_0)=(2\mu_1(1+k_1)/\Omega_x)+\left(6\mu_2(1+k_2)z^2/\Omega_y x^4\right)+\left(2(p+\mu_1 L_1-q-\mu_2 L_2)\right)/x^2$$

and $\lambda=1$. The critical point of *f(x)* is determined as a value of x, for which $\partial f/\partial x=0$,

$$x_0 = \left[\frac{\left(p + \mu_1 L_1 - q - \mu_2 L_2\right) + \sqrt{\left(p + \mu_1 L_1 - q - \mu_2 L_2\right)^2 + \dfrac{4\mu_1\left(1 + k_1\right)\mu_2\left(1 + k_2\right)z^2}{\Omega_x \Omega_y}}}{2\dfrac{\mu_1\left(1 + k_1\right)}{\Omega_x \Omega_y}} \right]^{\frac{1}{2}} . \quad (19)$$

Using expressions for $f(x)$, $g(x)$, and x_0, the approximate closed-form solutions for LCR and AFD for double κ-μ process is obtained:

$$N_z(z) = 4\pi\sqrt{2\Omega_y} \sum_{p=0}^{+\infty} \sum_{q=0}^{+\infty} C_1 z^{2q+2\mu_2 l_2 - 1} \times \frac{x_0^{\,2p-2q+2\mu_1 L_1 - 2\mu_2 L_2} \sqrt{1 + \dfrac{z^2}{x^4}\left(\dfrac{\Omega_x}{\Omega_y}\right)}}{\sqrt{\dfrac{2\mu_1\left(1 + k_1\right)}{\Omega_x} + \dfrac{6\mu_2\left(1 + k_2\right)z^2}{\Omega_y x_0^{\,4}} + \dfrac{2p - 2q + 2\mu_1 L_1 - 2L_2\mu_2}{x_0^2}}} \mathrm{e}^{-\frac{\mu_1\left(1+k_1\right)x_0^{\,2}}{\Omega_x}} \mathrm{e}^{-\frac{\mu_2\left(1+k_2\right)z^2}{\Omega_y x_0^{\,2}}} . \quad (20)$$

4. **NUMERICAL RESULTS**

Numerical results obtained for the LCR of the STBC MIMO communications system operating over a keyhole fading channel are presented at Figs. 1 and 2. It is assumed that the mobile transmitter and mobile receiver introduce same maximum Doppler shifts due to same relative speeds with respect to the "keyhole", yielding $f_\alpha = f_\beta = f_d$. Normalized LCR (N_z / f_d) of the instantaneous output SNR is depicted vs. normalized SNR threshold, calculated as $10\log\left[\gamma MR / \left(\overline{\gamma}\left(\Omega_x / \mu_T\right)\left(\Omega_y / \mu_R\right)\right)\right]$.

At Fig. 1, the results are presented for four different pairs of numbers of transmit and receive antennas (M, N), including the SISO (single input single output) case.

As it was expected better performances (smaller LCR values) are obtained when higher number of transmit/receive antennas are used. From Fig. 2 is obvious that for higher values of κ-μ fading severity parameter μ and for higher values of dominant/scattered components power ratio κ LCR values decrease, since for smaller κ and μ values, dynamic in channel is larger.

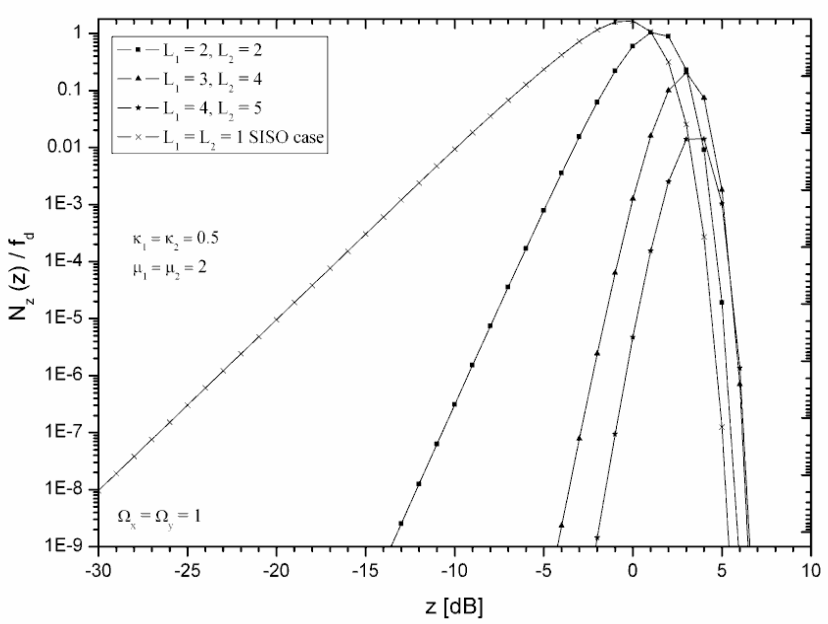

Fig. 1 – Normalized LCR for various values of transmit and receive antennas.

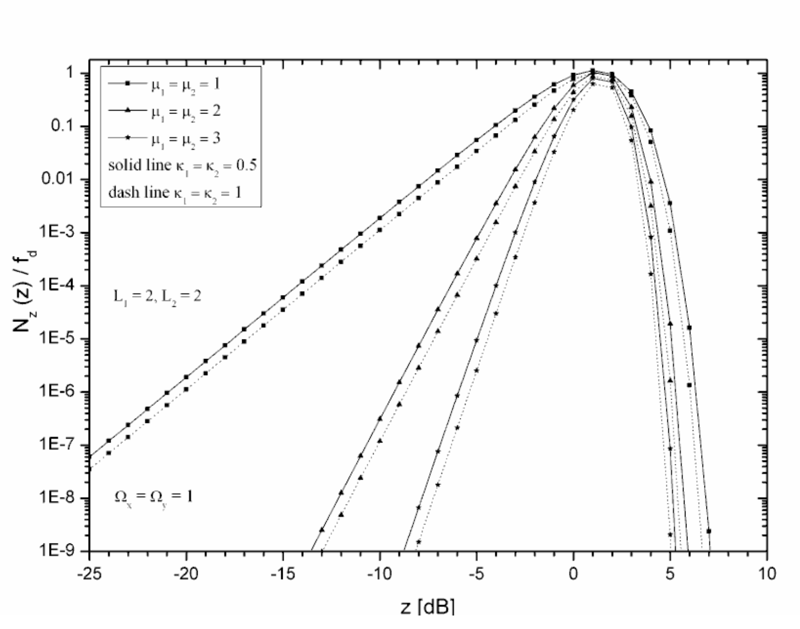

Fig. 2 – Normalized LCR for various values of fading severity parameters κ_i and μ_i.

5. **CONCLUSIONS**

An approach to the performance analysis of multiple input multiple output keyhole fading channels with space-time block coding is presented in this paper. Second order statistics of the instantaneous output SNR, i.e. the LCR and AFD, are given in the infinite series expressions form. Numerically obtained results were graphically presented and discussed in order to point out the effects of fading severity and number of transmit/receive antennas on the overall system's performances. Our analysis has high level of generality because it has been done for case of κ-μ distribution, which includes as special cases other important distributions, such as Rayleigh, Rician, and Nakagami-*m*.

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