DIAMAGNETIC LEVITATION SETTING
WITH ENLARGEMENT OF THE STABILITY AREA

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Pieces of diamagnetic materials are used for stabilizing the free levitation of permanent magnets in various micro-systems that require high sensitivity to the gravitational field alternation. The stability area of the levitation floater in such configurations is relatively narrow and limits the performance of these devices. For a vertical symmetric levitation configuration, this paper proposes a method of extending the stability interval by using a magnetic field source with a particular optimized geometry, which provides a larger magnetic field curvature at the levitation point.

1. INTRODUCTION

The free stable levitation of permanent magnets in stationary fields can only occur in the presence of a diamagnetic body. The diamagnetic material, due to its magnetic features, acts as a very weak servo-system and keeps the suspended magnet in a very small stability area [1, 2]. Usually, these tiny intervals of stability are increased adopting a configuration that contains active materials with very good physical properties – levitated magnets with large values for remanent flux density and diamagnetic material with high absolute values for magnetic susceptibility [3]. This paper suggests an enlargement of the stability area by means of special geometry for the magnetic field source that is able to create larger values of the magnetic field curvature at the levitation points. There is also analyzed the possibility of getting an extended stability zone along with minimum energy consumption.

2. EQUILIBRIUM AND STABILITY RESTRICTIONS

In order to achieve the free stable levitation of a permanent magnet in a stationary field, the minimum of its total energy, along with the satisfied


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equilibrium condition, at the levitation point must be fulfilled [4]. The potential energy \( U \) of a dipole magnet of moment \( M \) imbedded in a field of magnetic flux density \( B \) is:

\[
U = -MB + mgz = -MB + mg, \tag{1}
\]

where \( mgz \) is the gravitational energy. The magnet will align itself with the local field direction because of the torque and, therefore, the energy is only dependent on the magnitude \( B \) of the flux density.

Assuming a circular symmetry for the magnetic field \( B(r, z) \), the equilibrium points will be on the \( z \)-axis of symmetry. Then, the condition that \((z_0, 0)\) to be an equilibrium point is:

\[
F = -\nabla U \bigg|_{z=0} = 0 \Rightarrow \nabla B \bigg|_{z=0} = -\frac{mg}{M}. \tag{2}
\]

For stabilizing the equilibrium a piece of diamagnetic material is to be placed beneath the suspended magnet. Thus, in the relation of potential energy (1) a new term \( Cz^2 \) must be added, representing the diamagnetic material influence [4].

Expressing the magnitude of the magnetic flux density \( B(r, z) \) in terms of its \( z \) component \( B_z(r, z) \) only, and taking into account that \( \nabla B = 0 \) and \( \nabla \times B = 0 \), the following extension for total energy yields [5]:

\[
U = -M \left[ B_0 + \left( B_0 - \frac{mg}{M} \right)(z - z_0) + \frac{1}{2} B_0'(z - z_0)^2 + \right.
\]

\[+ \left. \frac{1}{4} \left( \frac{B_0'}{2B_0} - B_0'' \right) r^2 + \cdots \right] + C(z - z_0)^2. \tag{3}
\]

Here, \( B_0 = B_z(0, z) \bigg|_{z=z_0} \); \( B_0' = \frac{\partial B_z(0, z)}{\partial z} \bigg|_{r=r_0} \); \( B_0'' = \frac{\partial^2 B_z(0, z)}{\partial z^2} \bigg|_{r=r_0} \).

Stability conditions in \((z_0, 0)\) ask for a minimum value of energy, which means positive curvature function in every direction. Considering the equilibrium condition (2) and the expression for potential energy (3), the stability relations for both vertical and radial direction can be written as:

\[
\left\{
\begin{array}{l}
\frac{\partial^2 U}{\partial z^2} \bigg|_{z=z_0} \bigg|_{r=0} > 0 \Rightarrow D_v = C - \frac{MB_0''}{2} > 0; \\
\frac{\partial^2 U}{\partial r^2} \bigg|_{z=z_0} \bigg|_{r=0} > 0 \Rightarrow D_h = \frac{M}{4} \left( \frac{B_0' - B_0''^2}{2B_0} \right) > 0.
\end{array}
\right. \tag{4}
\]
For achieving a stable static levitation of the floating magnet, the above set of conditions must be simultaneously satisfied. The left-hand side quantities of the inequalities, denoted by $D_v$ and $D_h$, are called stability discriminants, corresponding to the vertical and radial directions respectively.

Without diamagnetic materials, setting $C = 0$, we see that if the curvature of flux density of the field is positive and large enough to create horizontal stability:

$$B'' > \left(\frac{mg}{2M^2B_0}\right)^2 \Rightarrow D_h > 0$$

the magnet will become unstable vertically: $D_v < 0$.

3. CONFIGURATION WITH ENLARGED STABILITY AREA

If the magnetic field curvature $B''$ is positive and large enough to determine a horizontal stability ($D_h > 0$), a vertical stabilized configuration can be easily obtained by placing a diamagnetic material nearby the floater. A simple vertical configuration that uses a common cylindrical coil for the magnetic field source, a tiny cylindrical shaped floater of Nd$_2$Fe$_{14}$B and pyrolytic graphite as the stabilizing diamagnetic material, was detailed analysed in [6], where the thickness of the diamagnetic piece is taken into account.

According to (2) and (4) the inflection point of the magnetic field, where levitation is possible to occur, is fixed by the geometry of the lifter magnet, not by its strength. Instability is related to the curvature of the lifter field and force balance depends on its gradient. That makes it feasible to engineer the location of the stable zones by adjusting the geometry of the field source and to control the gradient by adjusting its strength. The vertical stability condition can be rewritten as follows:

$$\frac{2C}{M} > B''_0 > \frac{B_0^2}{2B_0} = \left(\frac{mg}{2M^2B_0}\right)^2. \quad (5)$$

The $C$ term is proportional to diamagnetic susceptibility $\chi$ and the magnetic moment $M$ of the floater and gets smaller if the gap $d$ between levitated magnet and the diamagnetic body increases. An analytical expression for $C$ is obtained in [4] by using the dipole approximation of the floater and the method of magnetic images in the diamagnetic piece, considering an infinite thickness of the diamagnetic plate:

$$C = \frac{3\chi\mu_0M^2}{\pi(2d+L)^5}, \quad (6)$$

where $\mu_0$ represents the vacuum permeability and $L$ is the height of the levitated magnet.

Taking into account (5) and (6) the maximum value for the gap distance between the floater and the diamagnetic material $d_{\text{max}}$ (the stability area) can be analytically predicted:
A large curvature for the magnetic flux density at levitation point can be reached by choosing for the magnetic field source a double winding coil with a particular design as shown Fig. 1. This special arrangement of the winding layers, in which flows a DC current, is able to provide in certain areas high magnetic field curvature along with the preservation of equilibrium requirement and leads to an enlargement of the initially narrow stability zone.

For a certain magnetic field source geometry, the offset \( \delta \), shown in Fig.1, should be selected after an optimization process, which also considers a uniform electric and thermal stress for both magnetic field source windings.

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\[
d_{\text{max}} \leq \left( \frac{3|x|\mu_0 B_0 M^3}{8\pi(mg)^2} \right)^{1/5} \frac{L}{2},
\]

\[
A_{\text{large}} = \frac{1}{2} \left[ \pi \mu \chi \right]^{1/2}.
\]
4. NUMERICAL RESULTS

Aiming to perform a quantitative analyse over the above studied configuration, we set up a very simple model with specific geometrical data and material properties.

For the lifter magnet we used a cylindrical symmetric coil made having \( l_1 = 20 \text{ cm}, l_2 = 10 \text{ cm} a_1 = 2 \text{ cm}, a_2 = 3 \text{ cm} \) and \( N_1I = N_2I = 500 \text{ Asp} \). A tiny Nd\(_2\)Fe\(_3\)B cylinder (diameter \( 2R = 4 \text{ mm} \) and height \( L = 4 \text{ mm} \) – Fig. 1) was chosen as the floater. Its magnetic moment and mass are \( M = 0.48 \text{ Am}^2 \) and \( m = 0.39 \text{ g} \), respectively. A piece of pyrolytic graphite (\( \chi = 450 \cdot 10^{-6} \)) placed under the levitated magnet stabilises the equilibrium. The magnetic flux density variation along the symmetry \( z \)-axis versus the offset distance \( \delta \) is presented in Fig. 2.

The same quantity variation for \( \delta = 3.77 \text{ cm} \), denoted \( B_{\text{source}} \), is shown in Fig. 3. A corresponding single-winding coil having \( N_1 = N_2 = 1000 \text{ Asp} \) produces \( B_{\text{coil}} \), whose variation is also plotted in Fig. 3. Geometry of this cylindrical coil is defined by the same height \( l_1 = 20 \text{ cm} \) and by the two radii \( a_1 = 2 \text{ cm} \) and \( a_2 = 4 \text{ cm} \).

![Fig. 2 – Variation of magnetic flux density along z-axis.](image1)

![Fig. 3 – Comparison between the two magnetic flux density field sources.](image2)

The levitation configuration that uses the one winding coil as the magnetic field source gives an equilibrium point located at \( z_0 = 12.70 \text{ mm} \). The maximum value of the gap distance (stability area) is \( d_{\text{max}} = 1.46 \text{ mm} \). That corresponds to \( C = 0.078 \) which still ensures a positive value for the vertical discriminant of stability \( D_v \) in the levitation point. Thus, the stability area is restricted by \( d \in (0 - 1.46) \text{ mm} \). Fig. 4 shows, for this case, the stability discriminants variation and the resulting equilibrium point.

Adopting the proposed magnetic field supplier, we first had to choose an optimal value for the offset distance \( \delta \), in order to achieve the maximal gap distance of the considered levitation setting. The goal was fulfilled by using an
optimization procedure based on the criteria imposed by (2), (5) and (8). Hence, a value of \( \delta = 3.77 \) cm was obtained. This leads to an equilibrium point \( z_0 = 43.76 \) mm, where the maximum value of the gap distance reaches \( d_{\text{max}} = 2.195 \) mm, as presented in Fig. 5.

![Fig. 4 – Discriminant’s variation in the case of a single winding magnetic field source.](image)

![Fig. 5 – Discriminant’s variation in the case of a double winding magnetic field source.](image)

The new extended stability area of the floater in the suggested configuration is 50 \% larger compared to the first one. Furthermore, it is important to notice that the equilibrium point is relocated far for its initial position. This drawback can be overcome by properly adjusting the intensity of electrical current through the windings.
5. CONCLUSIONS

Stabilization problem of permanent magnet levitation in stationary fields by diamagnetic materials is treated aiming to gain a larger stability area in a particular case of a vertical symmetric magnetic field source.

Equilibrium and stability conditions for determining the location of equilibrium point and its stability area are analyzed. A numerical simulation presenting the quantitative aspect for these developed devices was also done. From this study we achieve a significant increase of the stability area due to the special geometry of the field source which uses two-separated layers for its windings.

This simple and inexpensive method of the stability interval enlargement can also be used along with diamagnetic pieces with higher absolute value for magnetic susceptibility, stronger magnetic field sources and floater magnets having the highest magnetization – mass ratio. This could lead to a significant improvement of all micro devices based on the diamagnetic phenomenon.

Practical applications of the permanent magnet levitation can be found in very high-sensitive gravity sensors or in frictionless suspension design, whose parameters (such rigidity) may be controlled by adjusting the magnetic field profile. Because of the simplicity and flexibility of such configurations they can be incorporated in optical detection schemes, being an attractive alternative to devices based on other complex levitation techniques.

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REFERENCES
