SMOOTH FREQUENCY RESPONSE DIGITAL AUDIO EQUALIZERS WITH SMALL RINGING IMPULSE RESPONSE

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Key words: Audio equalizers, Grouped B-spline functions, Smooth frequency response, Small ringing.

This paper describes a design method for optimum digital audio equalizers with small ringing impulse responses, associated with smooth frequency responses. The procedure benefits from superior order derivatives of the B-spline functions to reduce the ripples in the time domain and to ensure a natural audition. The B-splines will be grouped to generate the desired frequency responses, with improved interband independence and suppressed ringing of the impulse responses.

1. INTRODUCTION

Spectral sound equalization is an important technique for processing audio signals. We will analyze the design of the linear-phase digital equalizers for the individual frequency sub-bands, with appropriate frequency-response functions. If the frequency sub-bands have different magnitudes, the summed frequency response is discontinuous and the impulse response will have significant ringing. It is important for the equalizer to have a smooth frequency response [1, 2] in order to optimize the transient performance. A tradeoff is possible between the independence and the flatness of the frequency bands [3]. The challenge is to shape the frequency response in order to minimize problems.

2. GROUPED B-SPLINE FILTERS

A smooth frequency response is associated with shorter finite impulse response (FIR) filters [4]. For audio equalizers the property translates into high quality (natural) output signals [1]. If the amplitude response can be differentiated k times with finite results, the impulse response coefficients are proportionate to $1/n^{k+1}$, where n is the discrete time index. B-spline functions will be used to

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approximate the frequency response for every sub-band. If f_i and B_i are the centre frequency and the frequency width (normalized values) associated with the i^{th} sub-band, we introduce the notation of the normalized variable

$$x = (f - f_i) / B_i, \tag{1}$$

where f is the frequency variable. The frequency response associated with a sub-band can be a symmetrical B-spline function $S_k(x)$ of order k. Such functions can be generated as [5]:

$$S_k(x) = \sum_{j=0}^{k+1} \frac{(-1)^j}{k!} C_{k+1}^j \left(x + \frac{k+1}{2} - j \right)^k u\left(x + \frac{k+1}{2} - j \right), \tag{2}$$

where C_m^n is the number of n combinations of m, and $u(x)$ is the unit step function:

$$u(x) = \begin{cases} 0, & x < 0, \\ 1, & x > 0. \end{cases} \tag{3}$$

The B-spline functions can be generated recursively as

$$S_k(x) = \left[\left(\frac{k+1}{2} + x \right) S_k\left(x + \frac{1}{2} \right) + \left(\frac{k+1}{2} - x \right) S_{k-1}\left(x - \frac{1}{2} \right) \right] \frac{1}{k}, \tag{4}$$

or computed by iteratively convolving with the same rectangular pulse:

$$S_k(x) = S_{k-1}(x) * S_0(x), \tag{5}$$

where:

$$S_0(x) = \begin{cases} 1, & x \in (-0.5,\ 0.5), \\ 0, & \text{otherwise.} \end{cases} \tag{6}$$

In order to demonstrate the properties of the designed FIR filters we compare performances with the eigenfilter design method. By minimizing a quadratic measure of the error in the frequency band, an eigenvector of an appropriate matrix is computed to create the filter coefficients. The algorithm is optimal in the least squares sense [6]. Figure 1 presents the approximated frequency response of a linear phase FIR filter of length $N = 101$, with a quarter-octave band centre at the normalized frequency $f_n = 0.25$. An ideal bandpass filter is also illustrated. Figure 2 shows the impulse response $h(n)$ of the filter. The abrupt variation in the frequency domain generates an undesired ringing effect in the time domain, increasing the length of the impulse response. In figure 2, the pre- and postringing reaches amplitudes of 0.2, which produces severe distorsions of the filter's output audio signal. The filters presented in the rest of the paper will have the length $N = 101$.

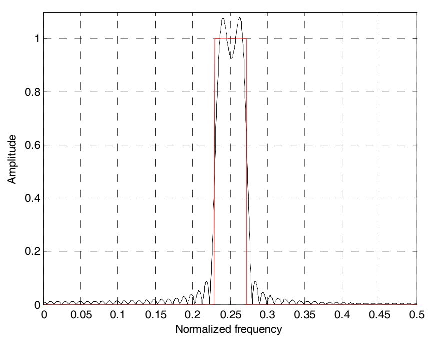

Fig.1 – Desired and approximated frequency response with linear phase FIR filter of length N=101 approximation, for an ideal bandpass filter.

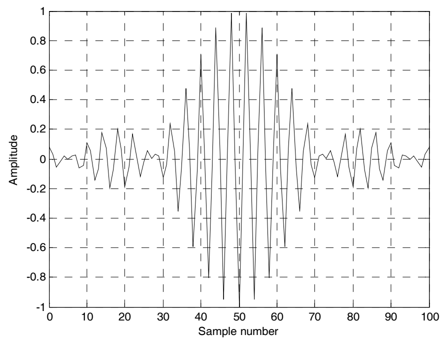

Fig. 2 – Impulse response of the approximated frequency response presented in Fig. 1.

The grouping of B-spline functions can reduce the region where the bandpass frequency response is nonzero. The frequency overlap with neighbouring sub-band filters can be decreased with three grouped B-spline functions of order k:

$$SS_k(x) = \beta S_k(\alpha x - \gamma) + S_k(\alpha x) + \beta S_k(\alpha x + \gamma), \tag{7}$$

where α, γ and β are real valued coefficients. The parameters α and γ control the support of function $SS_k(x)$. Thus, $SS_k(x) \neq 0$ for the interval $x \in \left(-\frac{\gamma}{\alpha} - \frac{k+1}{2\alpha}, \frac{\gamma}{\alpha} + \frac{k+1}{2\alpha}\right)$. By imposing $SS_k(0) = 1$, parameter β becomes:

$$\beta = \frac{1 - S_k(0)}{S_k(-\gamma) + S_k(\gamma)}. \tag{8}$$

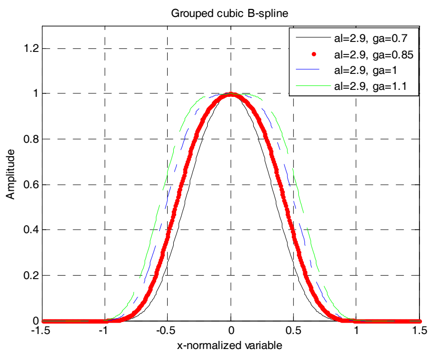

Fig. 3a – Grouped cubic B-spline for different values of α (α = al); γ = 0.85 (γ = ga).

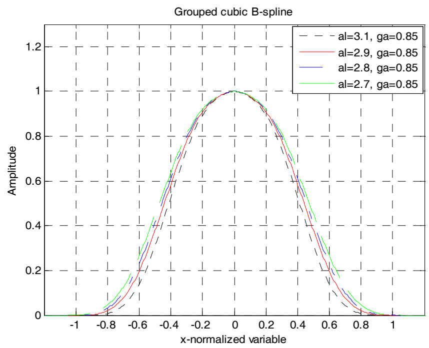

Fig. 3b – Grouped cubic B-spline for different values of γ (γ = ga); α = 2.9 (α = al).

Figure 3a and 3b illustrate the dependence of the grouped cubic B-splines on the parameters α and γ. Figure 3a shows the grouped cubic B-spline for different values of γ. The parameter α was fixed at $\alpha = 2.9$. Figure 3b shows the dependence of the grouped cubic B-spline on the parameter α. The parameter γ was fixed at $\gamma = 0.85$. We can see that the parameters α and γ give a fine control for an optimum smooth approximation of the ideal filter.

3. OPTIMUM DIGITAL AUDIO EQUALIZERS

The impulse response for an ideal digital lowpass filter, with the normalized cutoff angular frequency ω_t, is given by

$$h(n) = \mathrm{IDTFT}\left\{H\left(\mathrm{e}^{\mathrm{j}\omega}\right)\right\} = \frac{1}{2\pi}\int_{-\pi}^{\pi} H\left(\mathrm{e}^{\mathrm{j}\omega}\right)\mathrm{e}^{\mathrm{j}\omega n}\mathrm{d}\omega = \frac{\omega_t}{\pi}\mathrm{sinc}\left(\omega_t n\right), \tag{9}$$

where

$$H\left(\mathrm{e}^{\mathrm{j}\omega}\right) = \begin{cases} 1, & \omega \in \left(\omega_t,\ -\omega_t\right) \\ 0, & \omega \in \left[-\pi,\ -\omega_t\right) \cup \left(\omega_t,\ \pi\right], \end{cases} \tag{10}$$

$\mathrm{sinc}(x) = \sin(x)/x$ and IDTFT is the inverse discrete time Fourier transform. The grouped B-spline functions exhibit desirable characteristics as bandpass amplitude functions (smooth frequency responses), with reduced impulse response ringing and a good attenuation in the stopband, by choosing the parameters α and γ. By convolving $S_0(x)$ a number of k times in the frequency domain, the result is reduced impulse response ringing

$$h_k(n) = \mathrm{IDTFT}\left\{H_k\left(\mathrm{e}^{\mathrm{j}\omega}\right)\right\} = \frac{\omega_t^{k+1}}{\pi^{k+1}}\mathrm{sinc}^{k+1}\left(\omega_t n\right), \tag{11}$$

where $H_k\left(\mathrm{e}^{\mathrm{j}\omega}\right) = H\left(\mathrm{e}^{\mathrm{j}\omega}\right) * H\left(\mathrm{e}^{\mathrm{j}\omega}\right) * \ldots * H\left(\mathrm{e}^{\mathrm{j}\omega}\right)$ is a k times convolution of $H\left(\mathrm{e}^{\mathrm{j}\omega}\right)$.

Furthermore, we introduce the mean square error (MSE) between the frequency response of the *ideal* equalizer, with M amplitude samples, A_k, and the frequency response of the *spline functions approximation*, with M amplitude samples, denoted as $A_k^{'}$

$$\mathrm{MSE} = \frac{1}{M}\sum_{k=0}^{M-1}\left(A_k - A_k^{'}\right)^2. \tag{12}$$

Figure 4 presents an example of *optimal* smooth frequency response of the ideal bandpass filter with a quarter-octave band centered at 0.25, using grouped B-spline functions with the order $k = 7$. We considered $SS_7(x) \neq 0$ with the support $x \in (-1,1)$, $\alpha = 5$, and $\gamma = 1$. By decreasing α, in the first simulations, and γ in the next stages, the attenuation in the stopband was increased and the ringing from the impulse response was minimized. The optimum results are obtained for $\alpha = 3.69$ and $\gamma = 0.53$.

The corresponding minimum attenuation in the stopband is $a_m = 93.37$ dB and also MSE = 0.016. The ringing amplitude is reduced to approximately 0.02 (Fig. 5), lower than the value associated with Fig. 2 (a ringing of 0.2). Grouped B-spline functions appear particularly attractive in this regard as bandpass frequency response functions [7–9]. There are no abrupt frequency response changes or irregularities to determine unnatural sounds.

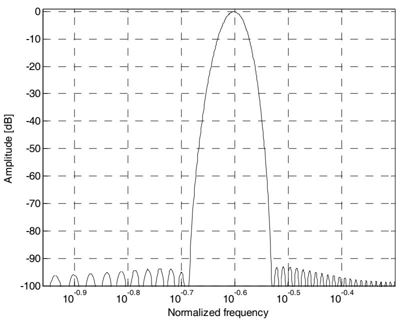

Fig. 4 – Frequency response, in dBs; grouped B-spline functions with k = 7; approximation performed for the ideal equalizer in Fig. 1.

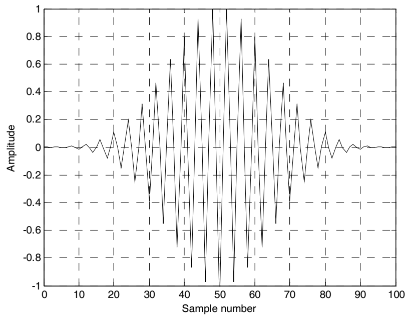

Fig. 5 – Impulse response of the filter presented in Fig. 4.

Furthermore, we propose the possibility of approximation using the grouped B-spline functions in the transition bands. We express the combination between a smoothed frequency response using grouped B-spline functions of k order in the transition bands and an ideal bandpass filter with a quarter-octave band centered at $f_n = 0.25$:

$$\mathrm{MOD}(x) = \begin{cases} SS_k(x)/sc, \; x \le x_1 \\ 1, \; x \in [x_1, \, x_2] \\ SS_k(x)/sc, x \ge x_2 \end{cases}, \tag{13}$$

where x, x_1 and x_2 are expressed similar to (1), based on the *ideal* bandpass filter width B, respectively on the *margins* f_1 and f_2. Also, $sc = SS_k(x_1)$, and we choose $f_1 = 0.245$, $f_2 = 0.255$, with the quarter-octave bandwidth $B = 0.2726–0.2295$.

Figure 6 presents the optimal smooth frequency response of the ideal bandpass filter with a quarter-octave band centered at 0.25, using grouped B-spline functions of sixth order in the transition bands, for $\alpha = 3.65$, $\gamma = 1.1$. The constant amplitude is chosen between the normalized frequencies 0.245 and 0.255. Figure 7 shows an approximation of the smooth frequency response from Fig. 6. The obtained minimum attenuation in the stopband is a_m = 58.25 dB and also MSE = 0.0082, lower than the value 0.0124 associated with the optimal smooth frequency response of the ideal bandpass filter with a quarter-octave band centered at 0.25, using grouped B-spline functions of sixth order (see Table 1). As a consequence of the lower MSE, we observe in Fig. 8 a ringing amplitude of aproximately 0.05 for the impulse response associated with the filter in Fig. 7 (higher than 0.012, see Table 1 for comparison with the first method). Table 2 shows the parameters obtained for a filter approximation (of the ideal bandpass filter from Fig. 1) using grouped B-spline functions of third to seventh order, in the transition bands. The amplitude is set constant for the normalized frequency interval [0.245, 0.255].

Figures 9 and 10 illustrate the ideal frequency response and the corresponding impulse response for an equalizer with three frequency sub-bands, centered on the normalized frequencies: f_1 = 0.1768, f_2 = 0.2102 and f_3 = 0.25. Furthermore, the associated smooth equalizer, using grouped B-spline functions of sixth order, is displayed in Fig. 9. Figures 11 and 12 illustrate the frequency response and the impulse response for the filter approximation asociated with the smooth equalizer in Fig. 9. The ringing values are up to 0.016 (lower than the ringing amplitude in Fig. 10). We considerred $SS_6(x) \neq 0$ for $x \in (-1,1)$, with $\alpha = 4.5$ and $\gamma = 1$. After decreasing α and γ in several simulations, the attenuation in the stopband is increased and ringing from the impulse response is minimized. The optimum results are obtained for $\alpha = 3.06$ and $\gamma = 0.96$. Moreover, Table 3 presents the parameters obtained for the approximation with grouped B-spline functions of third, fourth, fifth, sixth and seventh order, respectively, for the ideal equalizer in Fig. 9.

Figure 13 presents an ideal equalizer and the optimal smooth frequency response using grouped B-spline functions of seventh order in the transition bands, for $\alpha = 3.45$, $\gamma = 1.21$. The three frequency bands are centered on the normalized frequencies f_1 = 0.1768, f_2 = 0.2102 and f_3 = 0.25. The constant amplitudes are chosen between the normalized frequencies $f_i - 0.005$ and $f_i + 0.005$, for i = 1, 2, 3. Figure 14 shows a eigenfilter approximation with 101 coefficients for the smooth frequency response in Fig. 13. The attained minimum attenuation in the stopband is a_m = 47.63 dB and also MSE = 0.045 (lower than the value corresponding to the first method, for $k = 7$, see Table 3). In Fig. 15 we observe the associated ringing amplitude of 0.028, which is increased in comparison with the value associated with the first method (Table 3). Table 4 shows the parameters obtained by $N = 101$ approximation to grouped B-spline functions of third, fourth, fifth, sixth and seventh order, respectively, in the transition bands, for the ideal equalizer in Fig. 13. By comparing the two proposed methods, it can be noticed that a compromise is required between the accuracy of the frequency response and the ringing amplitude.

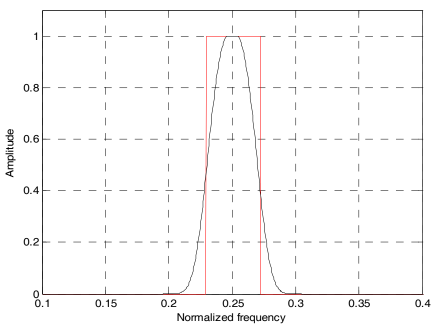

Fig. 6 – Smoothed frequency response of a bandpass filter with a quarter-octave band centered at 0.25, using grouped B-spline functions of sixth order in the transition band; the ideal bandpass filter is also illustrated.

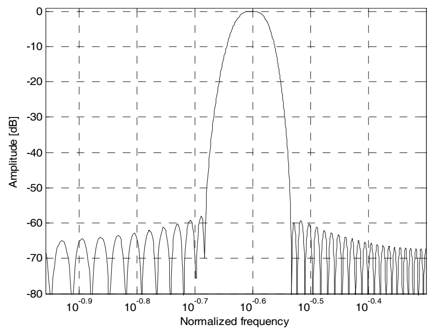

Fig. 7 – Frequency response, in dB, for $N = 101$ approximation with grouped B-spline functions of sixth order in the transition band, of the ideal equalizer from Fig. 6, using eigenfilter method.

Table 1

Parameters for grouped B-spline bandpass filters that approximate the ideal bandpass filter from Fig. 6.

k	MSE	a_m [dB]	Ring. amplit.	α	γ
3	0.076	73.83	0.0408	2.9	0.85
4	0.084	75.4	0.0349	3.21	0.9
5	0.096	71.31	0.0303	3.44	0.94
6	0.012	59.1	0.0203	3.9	0.9
7	0.016	93.37	0.0233	3.69	0.53

Table 2

Bandpass filters approximating the ideal filter in Fig. 6, using the grouped B-spline functions in transition bands.

k	MSE	a_m [dB]	Ring. amplit.	α	γ
3	0.085	53.75	0.051	2.9	0.85
4	0.074	75.79	0.068	3.4	1.06
5	0.095	52.57	0.040	3.44	0.94
6	0.082	58.25	0.049	3.65	1.1
7	0.007	47.37	0.068	3.85	1.1

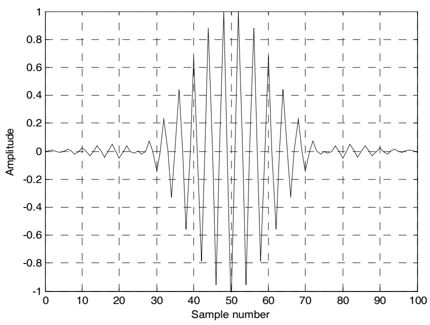

Fig. 8 – Impulse response of the filter presented in Fig. 7.

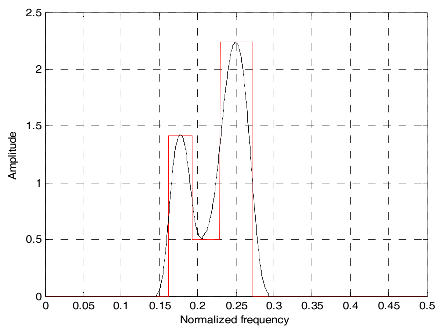

Fig. 9 – Frequency response for an ideal equalizer and smoothed frequency response using grouped B-spline functions of sixth order.

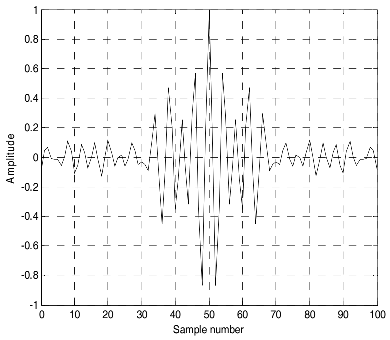

Fig. 10 – Impulse response for an approximation to the ideal equalizer showed in Fig. 9.

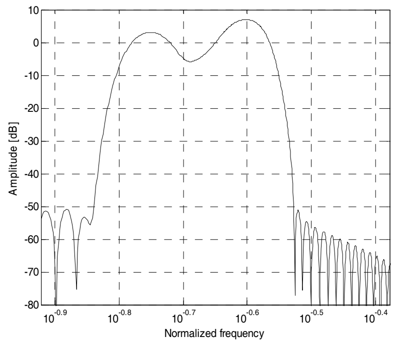

Fig. 11 – Frequency response, for the approximation using grouped B-spline functions with $k = 6$, related to the smoothed frequency response showed in Fig. 9.

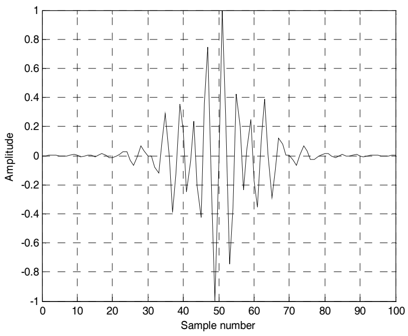

Fig. 12 – Impulse response of the filter presented in Fig. 11.

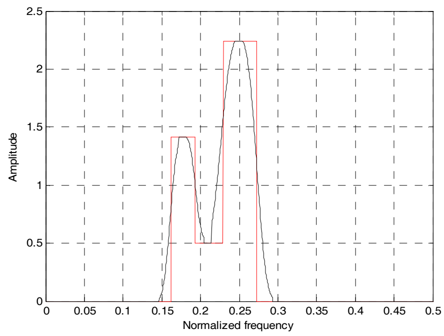

Fig. 13 – Frequency response for an ideal equalizer and smoothed frequency response using grouped B-spline functions of seventh order in the transition bands.

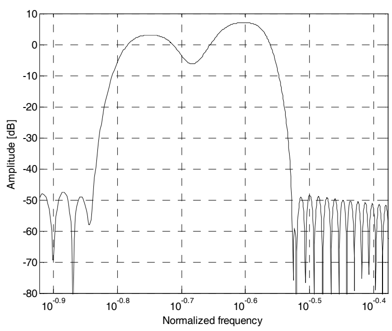

Fig. 14 – Frequency response approximation to grouped B-spline functions of seventh order in the transition bands, for the ideal equalizer illustrated in Fig. 13, using eigenfilter method.

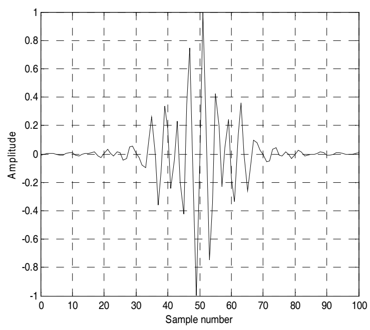

Fig. 15 – Impulse response of the filter presented in Fig. 14.

4. CONCLUSIONS

We observe that is desirable for the equalizer to have a smooth frequency response in order to minimize the ringing of the impulse response. It is impossible to have full independence between the frequency sub-bands and the smoothness of frequency responses. A flat response must have some overlap between the sub bands.

Table 3

Parameters for grouped B-spline three bandpass filters that approximate the ideal equalizer in Fig. 9

k	MSE	a_m [dB]	Ring. amplit.	α	γ
3	0.046	44.1	0.045	2.70	1
4	0.046	47.3	0.028	2.79	0.99
5	0.046	50.1	0.019	2.90	0.95
6	0.048	51.5	0.016	3.06	0.96
7	0.051	53	0.017	3.21	0.94

Table 4

Parameters for grouped spline three bandpass filters that approximate the ideal equalizer in Fig. 13, by using grouped B-spline functions in the transition bands

k	MSE	a_m [dB]	Ring. amplit.	α	γ
3	0.037	34.4	0.060	3.15	1.12
4	0.046	47.1	0.032	2.78	0.99
5	0.044	44.77	0.021	2.96	0.96
6	0.049	47.93	0.036	3.21	1.17
7	0.045	47.63	0.028	3.45	1.21

We can make the frequency response as smooth as desired, considerably reducing the ringing from the impulse response, and the preferred bandpass can be accurately approximated with FIR filters of resonable lengths. Grouped B-spline functions appear particularly attractive for the generation of transition bands or bandpass frequency responses. Decreasing the frequency bandwidth increases the length of the impulse response. Thus, a compromise must be made between the length of the filter and the quality of the approximation.

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