FINE SPATIAL DISTRIBUTION ANALYSIS OF THE MOBILE TERMINAL POSITIONING ERROR USING A 3D MODEL

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Many applications have been developed or are under development in the areas of environmental monitoring, detecting construction malfunctions of smart houses, and tracking targets. These applications require a good orientation of the nodes relative to a global reference system in order to provide geographical information that is very important. In this paper, we review several localization principles, discuss issues in localization algorithm design and present a numerical simulation with a focus on the evaluation of the error spatial distribution of the mobile terminal positioning in cellular networks. We prove that the position error depends on the placement of network nodes in relation with the terminal location. Also, we demonstrate that the optimal solution of the algorithm is achieved when the reference nodes positions are almost equidistant to the mobile terminal position. Future sensor networks will involve movable sensor nodes, so new localization algorithms will need to be developed to handle these moving nodes and the proposed positioning algorithm is able to solve this issue in a more accurate manner.

1. INTRODUCTION

Locating mobile radio equipment using positioning systems (PS) was necessary since the interwar period and, more acutely, in the Second World War [1, 2], when the localization of soldiers in difficult situations was critical. Twenty years later, during the Vietnam War, the U.S. Department of Defense (USDOD) launched a series of satellites, components of the global positioning system (GPS) to ensure precise location of operations on the battlefield. Currently, GPS includes 30 satellites and provides facilities widely spread in civil telecommunications market for personal navigation applications [3].

Despite the undisputed success of GPS technology, location accuracy is significantly reduced in urban and indoor environment, where the signals used are strongly affected, especially because of attenuation and multipath propagation of electromagnetic waves.

The commercial applications of supervision and medical assistance at home require localization algorithms to track people with special needs (*e.g.* elderly people or

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those with disabilities) and children who are not always under direct supervision, or to guide travelers and tourists during travels [4,5]. Some of the many commercial location-based services are tracking vehicles on the roads, monitoring employees in the offices and finding the location of on-demand equipment in the industrial area, supervising patients in the hospitals, and tracking animals in the wildlife conservation areas and farms [6].

In recent years, it has been recognized that location-based services are not the only applications, where the location information can be used, but also it can be utilized to solve some other issues in the wireless networks. The applications based on the utilization of location information are folded under four categories: location-based services, network optimization, transceiver algorithm development and optimization, and environment characterization.

The error distribution of localization algorithms is a problem which is discussed for years now. Several research approaches have focused on the statistical distribution of the position error [5, 7, 8] and others discussed the relation between the number of anchors and the position error [9], but few approaches discuss the relationship between anchor placement and terminal position.

This paper analyses the accuracy of the localization algorithms in a spatial positioning error distribution point of view and is organized as follows. Section 2 contains a description of three localization algorithms: circular, hyperbolic and spherical. Functioning of the proposed spherical algorithm is illustrated in a numerical simulation of the real problems of positioning in Section 3. In Section 4, conclusions are drawn and aspects regarding future work are presented.

2. LOCALIZATION ALGORITHMS

In mobile communications, location could be defined simply as the estimate of the space position at a time of a mobile terminal. More rigorously, localization involves a set of logical (based on the principles of localization) and physical (location-based methods) that contribute to determining the position and terminal positioning error. Rigorous understanding of location mathematical principles is strictly necessary both for efficient design of computing algorithms and for optimizing the technological processes involved. Localization principles can be classified by the dimensionality imposed by concrete practical needs. Two-dimensional localization, although satisfactory in many cases and easier to understand, still has important limitations that justify the generalization to three–dimensional models.

2.1. TWO-DIMENSIONAL LOCALIZATION ALGORITHMS

In case of two-dimensional localization the distances or differences in distance between the mobile terminal and at least three base stations (called anchors) are required. Both involve solving a nonlinear system of N equations for calculating the terminal position, where N refers to the number of anchors [7], [10]. For $N = 3$, lateration is known as the trilateration. If the location of the mobile terminal is based on distance measurements, the position is calculated using circular lateration, while knowing the differences in distance is the basis for hyperbolic lateration.

Regardless of the localization principles used, distance measurements on which they are based are always subject to error (intrinsic imprecision, in a statistic point of view). This justifies the name pseudo-distances that will be adopted in the following to refer to distances resulting from a physical process of measurement, a priori affected by errors.

2.1.1. CIRCULAR LATERATION ALGORITHM

To understand circular lateration, is assumed that R_i distances, $i = 1, 2, \ldots, N$, between the mobile terminal T and, respectively, base stations S_i, $i = 1, 2, \ldots, N$, are known accurately. Coordinates are calculated using the Pythagorean Theorem. If (x_i, y_i) are the Cartesian coordinates of the reference points in predefined coordinate system, and (x_T, y_T) are the unknown coordinates of the mobile terminal which must be located, the exact distance R_i between the terminal and base stations are determined by the following equation:

$$\sqrt{(x_i - x_T)^2 + (y_i - y_T)^2} = R_i,\ i = 1, 2, 3. \tag{1}$$

The (x_i, y_i) coordinates of the reference points are assumed to be precisely known and the distances between the mobile terminal which must be located and these, will be measured by different methods: the received signal strength, time of arrival etc. With notations $\sqrt{x_i^2 + y_i^2} = \rho_i$, $i = 1, 2, 3$, where ρ_i represent the distances from base stations to predefined coordinate system origin, and using T for transpose operation of a matrix, the initial system of nonlinear equations can be rewritten in a compact form:

$$2 \cdot \mathbf{A} \cdot \mathbf{X} = \mathbf{B}, \tag{2}$$

where $\mathbf{X} = (x_T, y_T)^{\mathrm{T}}$, $\mathbf{A} = \begin{pmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{pmatrix}$ and

$$\mathbf{B} = \begin{pmatrix} (R_1^2 - R_2^2) - [(x_1^2 - x_2^2) + (y_1^2 - y_2^2)] \\ (R_1^2 - R_3^2) - [(x_1^2 - x_3^2) + (y_1^2 - y_3^2)] \end{pmatrix} = \begin{pmatrix} (R_1^2 - R_2^2) - (\rho_1^2 - \rho_2^2) \\ (R_1^2 - R_3^2) - (\rho_1^2 - \rho_3^2) \end{pmatrix}.$$

If the matrix $\mathbf{A}$ is nonsingular, the system has the unique solution

$$\mathbf{X} = \frac{1}{2} \cdot \mathbf{A}^{-1} \cdot \mathbf{B} . \tag{3}$$

If the matrix A is singular, then

$$\begin{vmatrix} x_2 - x_1 & y_2 - y_1 \\ x_3 - x_1 & y_3 - y_1 \end{vmatrix} = 0 \Leftrightarrow \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0 , \tag{4}$$

situation which corresponds to the colinearity of the base stations positions and with a undetermined system, with two distinct solutions.

In all practical situations, the quantities involved in the system are obtained by estimates and observations inherently imprecise, the system of equations having no precise mathematical solution. However, in order to obtain the estimated results $\tilde{\mathbf{X}} = \left(\tilde{x}_T , \tilde{y}_T \right)^{\mathrm{T}}$, we applied the method of least squares and obtained the following general solution

$$\tilde{\mathbf{X}} = \frac{1}{2} \cdot \left(\mathbf{A}^{\mathrm{T}} \cdot \mathbf{A} \right)^{-1} \cdot \mathbf{A}^{\mathrm{T}} \cdot \mathbf{B} . \tag{5}$$

If, in particular, the matrix **A** is a square matrix and has an inverse, then

$$\left(\mathbf{A}^{\mathrm{T}} \cdot \mathbf{A} \right)^{-1} \cdot \mathbf{A}^{\mathrm{T}} = \mathbf{A}^{-1} \cdot \left(\mathbf{A}^{\mathrm{T}} \right)^{-1} \cdot \mathbf{A}^{\mathrm{T}} = \mathbf{A}^{-1} \cdot \mathbf{I} = \mathbf{A}^{-1} .$$

The measured distances (pseudo-distances) P_i , $i = 1, 2, 3$, differ from precise distances, with a particular relative error ε caused by the inaccuracy of the measurement [11] (including the effects of clocks synchronization mismatch, refraction and multi-path propagation etc.)

$$R_i = P_i \left(1 + \varepsilon \right), \ i = 1, 2, 3 . \tag{6}$$

All distances are measured with errors that are satisfactorily described by a normal statistical distribution and the probability density that the true distance over which was applied a method of measurement is given by the Gaussian distribution function

$$p_i = \frac{1}{\sqrt{2 \cdot \pi \cdot \sigma_i^{\,2}}} \exp\left[- \frac{\left(R_i - \overline{R}_i \right)^2}{2 \cdot \sigma_i^{\,2}} \right], \quad i = \overline{1,4} \tag{7}$$

where $\overline{R}_i$ is most likely distance value (average value for the normal distribution), and σ_i is the standard deviation of the mean. Probability densities p_i satisfy the normalization of probability relations

$$\int_{-\infty}^{+\infty} p_i \mathrm{d}R_i \approx \int_{0}^{+\infty} p_i \mathrm{d}R_i = 1 , \tag{8}$$

where the above approximation is valid because in practice $\overline{R}_i >> \sigma_i > 0$.

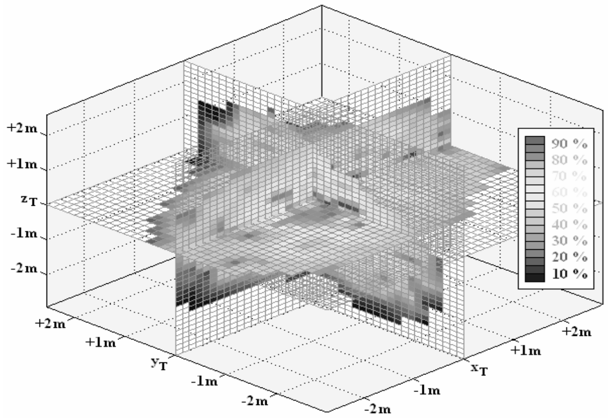

Fig. 1 – The position dependence of the relative probability density in case of mobile terminal localization using spherical algorithm.

Figure 1 illustrates the position dependence of the relative probability density of the mobile terminal localization, calculated using Matlab. The position is computed relative to the location of the terminal, estimated to be the most probable. The gradual scheme is related to the ratio of the probability density and its maximum value, reached in the terminal position.

Therefore, the three circles do not intersect in a geometric point, but will define a quasi-triangular location area which size depends on the distance measuring error values.

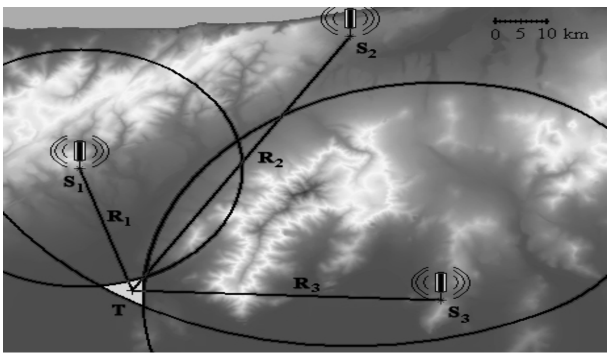

Fig. 2 – The location area of a mobile terminal determined by knowledge of pseudo distances using circular lateration.

As a result, the above-stated system of equations is in most cases inconsistent and has no unique solution. However, in order for a result to be obtained, different mathematical approaches may be used. One of the well developed and widely used methods is based on an iterative computation and is intended to minimize the mean square localization error.

2.1.2. HYPERBOLIC LATERATION ALGORITHM

Hyperbolic lateration appeared to solve the difficulties involved in circular lateration regarding time of arrival (TOA) measurement errors. To reduce these errors, a precise clock synchronization was needed at the mobile terminal, raising sometimes unacceptable costs. By using the time difference of arrival (TDOA) method, the error introduced by the terminal clock is eliminated. In fact, the propagation time difference expresses the distance difference $R_j - R_k$, $j \neq k$, of the terminal at the two separate reference points. In hyperbolic lateration terminal position can be determined by solving the system of nonlinear equations

$$\sqrt{\left(x_j - x_T\right)^2 + \left(y_j - y_T\right)^2} - \sqrt{\left(x_k - x_T\right)^2 + \left(y_k - y_T\right)^2} = R_j - R_k = \Delta R_{j,k}, \quad j \neq k \qquad (9)$$

where, by $\Delta R_{j,k}$ we noted the difference of distances from the mobile terminal to the reference points j and k.

One of the methods discussed in the literature for the linearization of the nonlinear system of equations obtained for hyperbolic lateration assumes that the location of the terminal is determined with four reference points S_1, S_2, S_3, and SR, and the last of which is considered a basic reference (or primary) to estimate the differences in distance to the other three reference points (Fig. 3). Considering that the distances from the mobile terminal T to the reference points $S_{1,2,3}$ and SR are R_i with $i = \overline{1,4}$, the system of equations corresponding to this configuration will contain three independent linear equations

$$\begin{cases} \sqrt{\left(x_1 - x_T\right)^2 + \left(y_1 - y_T\right)^2} - \sqrt{\left(x_4 - x_T\right)^2 + \left(y_4 - y_T\right)^2} = R_1 - R_4 = \Delta R_{1,4} \\ \sqrt{\left(x_2 - x_T\right)^2 + \left(y_2 - y_T\right)^2} - \sqrt{\left(x_4 - x_T\right)^2 + \left(y_4 - y_T\right)^2} = R_2 - R_4 = \Delta R_{2,4} \\ \sqrt{\left(x_3 - x_T\right)^2 + \left(y_3 - y_T\right)^2} - \sqrt{\left(x_4 - x_T\right)^2 + \left(y_4 - y_T\right)^2} = R_3 - R_4 = \Delta R_{3,4} \end{cases}.$$

Following the same steps as for the circular lateration, we get the same solution: $\mathbf{X} = \frac{1}{2} \cdot \mathbf{A}^{-1} \cdot \mathbf{B}$, where

$$\mathbf{A}=\begin{pmatrix}\left[(x_1-x_4)\cdot\Delta R_{3,4}-(x_3-x_4)\cdot\Delta R_{1,4}\right] & \left[(y_1-y_4)\cdot\Delta R_{3,4}-(y_3-y_4)\cdot\Delta R_{1,4}\right]\\ \left[(x_2-x_4)\cdot\Delta R_{3,4}-(x_3-x_4)\cdot\Delta R_{2,4}\right] & \left[(y_2-y_4)\cdot\Delta R_{3,4}-(y_3-y_4)\cdot\Delta R_{2,4}\right]\end{pmatrix},$$

$$\mathrm{X}=(x_T,y_T)^{\mathrm{T}}$$

$$\mathbf{B}=\begin{pmatrix}\left[(\rho_1^2-\rho_4^2)-\Delta R_{1,4}^2\right]\cdot\Delta R_{3,4}-\left[(\rho_3^2-\rho_4^2)-\Delta R_{3,4}^2\right]\cdot\Delta R_{1,4}\\ \left[(\rho_2^2-\rho_4^2)-\Delta R_{2,4}^2\right]\cdot\Delta R_{3,4}-\left[(\rho_3^2-\rho_4^2)-\Delta R_{3,4}^2\right]\cdot\Delta R_{2,4}\end{pmatrix}.$$

Differences in measured distances (distances pseudo-differences) $\delta P_{i,j}$, $i \neq j = 1, 2, 3$, differs from actual distance values differences $\Delta R_{i,j}$, $i \neq j = 1, 2, 3$, with some relative error ε determined by propagation time measuring accuracy: $\Delta R_{i,j} = \delta P_{i,j}(1+\varepsilon)$, $i \neq j = 1, 2, 3$. Accordingly, the hyperboles do not intersect in a geometric point but defines an area which depends on the value of distance difference measuring error [10].

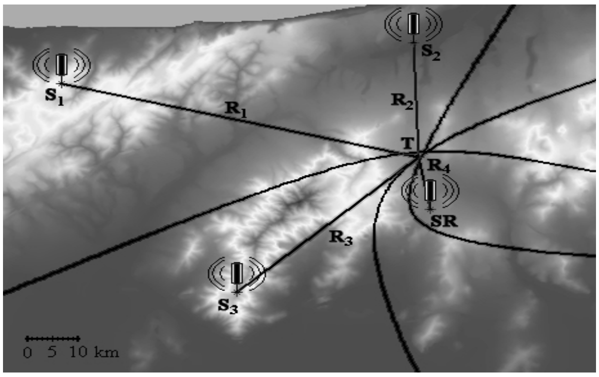

Fig. 3 – Hyperbolic lateration with four reference points. The mobile terminal position is determined by the intersection of three hyperboles with the reference points as outbreaks.

2.2. THREE-DIMENSIONAL LOCALIZATION ALGORITHM

If the vertical location of the mobile terminal is necessary or if the reference points are situated at an altitude of which differences cannot be neglected in relation to the horizontal distances between the base stations and the terminal, we need to use three-dimensional localization.

2.2.1. SPHERICAL LATERATION ALGORITHM

For implementation of the spherical lateration algorithm is necessary to determine the distances from the mobile terminal to at least four reference points (base stations). The terminal will be located at the intersection of four spheres of which rays are equal to the distances between the terminal and the four reference points [11, 12].

Thus, equation (1) is generalized to

$$\sqrt{\left(x_i - x_T\right)^2 + \left(y_i - y_T\right)^2 + \left(z_i - z_T\right)^2} = R_i, \quad i = 1, 2, 3, 4, \tag{10}$$

where $\mathbf{X} = \left(x_T, y_T, z_T\right)^{\mathrm{T}}$ is the Cartesian coordinates triplet of the mobile terminal, and $\left(x_i, y_i, z_i\right)$ are the coordinates of the reference point i spaced from the terminal with R_i. Generalizing with a dimensional extension the steps exposed in Section 2.1.1 in the particular case of two-dimensional circular lateration we can write directly following solution

$$\mathbf{X} = \frac{1}{2} \cdot \left(\mathbf{A}^{\mathrm{T}} \cdot \mathbf{A}\right)^{-1} \cdot \mathbf{A}^{\mathrm{T}} \cdot \mathbf{B}. \tag{11}$$

where matrix notation has been used

$$\mathbf{A} = \begin{pmatrix} x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \\ x_4 - x_1 & y_4 - y_1 & z_4 - z_1 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} \left(R_1^2 - R_2^2\right) - \left[\left(x_1^2 - x_2^2\right) + \left(y_1^2 - y_2^2\right)\right] \\ \left(R_1^2 - R_3^2\right) - \left[\left(x_1^2 - x_3^2\right) + \left(y_1^2 - y_3^2\right)\right] \\ \left(R_1^2 - R_4^2\right) - \left[\left(x_1^2 - x_4^2\right) + \left(y_1^2 - y_4^2\right)\right] \end{pmatrix}.$$

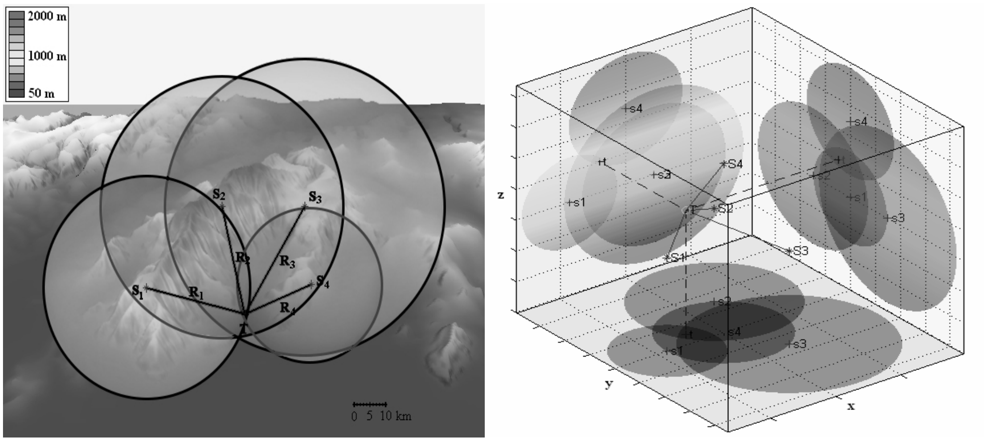

Fig. 4 – Spherical lateration with minimum number of reference points. The position of the mobile terminal *T* is uniquely determined by the intersection of the four spheres, with centres of the reference points (base stations).

3. SIMULATION RESULTS AND PERFORMANCE ANALYSIS

The two-dimensional and three-dimensional localization algorithms presented above assume knowledge of measured distances using positioning methods affected by errors (*e.g.* TOA, TDOA). This assumption implies inaccurate geometric relationships between the positions of the base stations and mobile terminal, difficulty that is overcomed by implementing iterative calculation. The programs had as input information the exact positions of the base stations and the inaccurate distances to the mobile terminal. A step in the refinement of the iterative algorithm requires additional knowledge of the errors with which the distances are virtually measured. This allows, in addition to finding the terminal's position, the estimation of its error. In other words, the iterative method provides the estimated mobile terminal location as the most probable, directly dependent on the most probable values of the measured distances. But these distances should be understood as having an average (for a symmetric statistical distributions, the mean is the most probable) and a dispersion depending on the localization method accuracy. This imprecision is reflected in the existence of a dispersion of possible terminal positions, known with different probabilities.

We illustrate the refinement of the localization algorithms with the numerical development we designed for the three-dimensional algorithm of spherical iteration.

To analyse the performance of the previous exposed localization algorithms and test the possibilities of practical applications, we developed a numerical simulation of localization in a known urban perimeter.

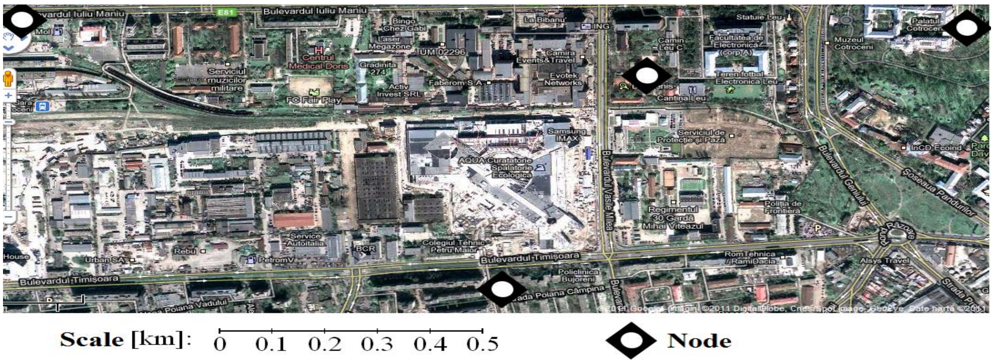

Fig. 5 – Known sites of GSM antennas in urban perimeter of Bucharest.

In this area, we chose four locations for base stations where there are real GSM antennas, which we assume are properly equipped to perform localization. Their coordinates are specified by longitude, latitude (in degrees, minutes and seconds) and quota relative to the average altitude of the area (in meters): $\mathbf{S}_1(26°2'26.73''; \; 44°26'3.19''; \; 37)$, $\mathbf{S}_2(26°3'7.18''; \; 44°25'39.46''; \; 36)$, $\mathbf{S}_3(26°3'19.49''; \; 44°25'58.06''; \; 32)$ and $\mathbf{S}_4(26°3'47.23''; \; 44°26'1.89''; \; 30.3)$. Their positions, relative to the map of the area, were established with good approximation using Google Maps and represented in the Fig. 5 with a circular symbol. After delimitation area, we associate it an arbitrary reference point against which we performed the localization computations, using a set of Matlab programs and functions.

The algorithm implemented was that of spherical lateration as base stations and mobile terminal were considered to be at different elevations relative to an average altitude urban plan. For a clear presentation of the results, we developed a chart of the tracking error according to the position of the mobile terminal in the perimeter described in relation to the known locations of base stations. It should be noted that the results are exclusively based on the four defined base stations. The wireless networks contain several antennas in the same area, which in reality would make the localization process more accurate (for spherical lateration the minimum number of base stations required is 4, but the principle of localization can work with any number greater than 4 antennas).

The area containing base stations (1855 m length and 795 m width) is covered by a two-dimensional network of nodes with a required discretization (25 m × 25 m). The factors that are taken into consideration in the determination of the mesh have been the resolution of the localization error chart on the one hand and the computation time on the other hand. The real position of the mobile terminal was chosen by one in each plane network nodes covering map perimeter. A Matlab function was designed to simulate the TOA measurement method. We took into consideration some pertinent estimates of characteristic quantities of the simulated measurement physical process: the refractive index of the air is set to 1.0008, the absolute error of base stations time measurement equipment is fixed to 1ns, the absolute error of time measurement by mobile device is fixed to 5 ns, while the time determining error associated with frequency bandwidth is 1 ns.

The results provided to the main program by the function that simulates measuring distances from base stations to the current position of the mobile terminal (considered "real" in the sense of the simulation) should be understood as the numerical equivalent of the results produced by a real experiment, when the test-position of the terminal is known and determine the estimated position based on localization algorithm. In order to give to the virtual measurements the inherently imprecise character of the real ones, distance values obtained from the determination of the time errors are spread around the mean value by a Gaussian distribution with standard deviation relative to the total temporal error.

Then, the main program estimated iteratively terminal position, following the algorithm described in Section 2. Estimated position is then compared, for each node of the two-dimensional network, with the "real" mobile terminal position. Difference between the two positions (real and estimated) is interpreted as absolute localization error, and average over 50 similar simulations of the absolute difference is depicted in Fig. 6.

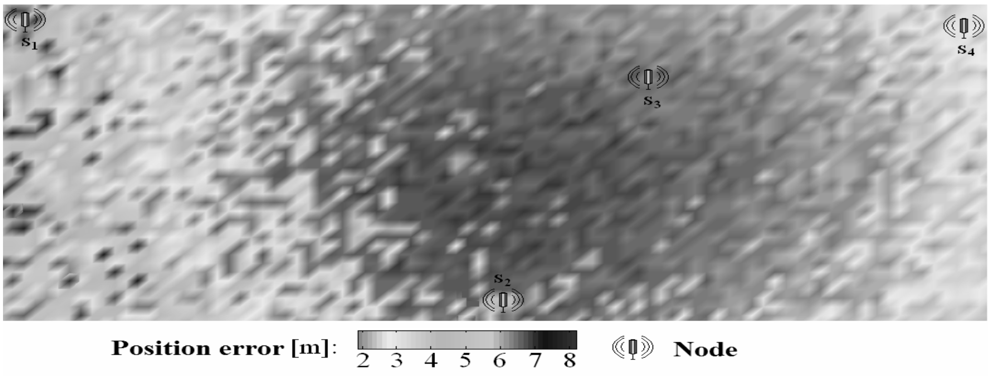

Fig. 6 – Spatial distribution of position error in the urban perimeter shown in Fig. 5.

We therefore sought to mediate over a number of virtual experiments to simulate the repetitive nature of the actual measurements necessary to exclude purely statistical fluctuations of results.

It is noted that in the center of the considered urban perimeter, at comparable distances to the S1 and S4, and S2 and S3 respectively, the mobile terminal location is more accurate, with errors of meters. In this area, the localization error reaches the minimum value, approximately 1–2 meters, depending on the channel properties. At the periphery of the delimited area, when two or three of the distances from antennas to the terminal can be much larger than the others, the location algorithm is less accurate, with errors which could increase with almost an order of magnitude. Here, the absolute errors reach the maximum values, approximately 7–8 meters.

This undesirable fact is compensated by using of multiple network base stations to perform the localization process, their election being determined by proximity relations with mobile terminal.

4. **CONCLUSION**

We developed a numerical simulation with a focus on the evaluation of the spatial position error distribution. We proved that the position error depends on the placement of network nodes in relation with the terminal location. Thus, the aim is to ensure position errors uniformity in a whole field of interest or automatic selection of network sensors participating at a certain moment in a mobile terminal localization process, so that position error to be minimal.

This simulation is a challenge for the further development of a location information management system in order to provide different applications such as location-based services (positioning and tracking), network optimization, transceiver algorithm optimization and environment characterization [13].

We admit that a location information management system is an important part of wireless networks [14]. Therefore, we think that the exploitation of location information in cognitive and cooperative networks is extremely useful. At the same time, it must take into account the compromise between the use of such information and user privacy.

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