EQUIVALENT CIRCUIT PARAMETERS AND OPERATING PERFORMANCES OF THE THREE-PHASE ASYNCHRONOUS MOTOR

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Starting from usual rated data ($\eta_n$, $\alpha_{\eta,m}$, $\cos \phi_{1,n}$, $s_n$, $m_M$), a simple and efficient procedure is proposed for identifying T-equivalent circuit parameters of three-phase asynchronous motor. Subsequently, by numerical solving of equivalent circuit, the operating performances of a three-phase asynchronous motor supplied from voltage or current sources, at constant or variable frequency, can be obtained. A numerical example (75kW, 1485 rpm asynchronous motor) confirms the validity of proposed method and the relatively good agreement between numerical and analytical characteristics.

1. INTRODUCTION

Mathematical model identification (i.e. T equivalent circuit parameters) of three-phase asynchronous motors, most frequently used motor in electrical drives, can be an important objective with practical benefits, in both analysing and synthesis problems of asynchronous motors with imposed technical ($\eta_n$, $\alpha_{\eta,m}$, $\cos \phi_{1,n}$, $s_n$, $m_M$), and economical performances [1–3].

Among abovementioned performances, rated power factor ($\cos \phi_{1,n}$), rated efficiency ($\eta_n$) and p.u. optimal charge ($\alpha_{\eta,m}$) – which corresponds to maximum efficiency – are important requirements of the motor, and constitute the main data for the identification algorithm.

2. EQUIVALENT CIRCUIT PARAMETERS

The induction type machines (transformers, asynchronous machines) are represented by the well known T–equivalent circuit mathematical model, with $r_i$ and $x_{\sigma,1}$ the parameters of primary, $r_2$ and $x_{\sigma,2}$ of the secondary and $r_w$ and $x_m$ the parameters of the magnetization circuit, all expressed in p.u.

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2.1. RATED AND CATALOG DATA

Asynchronous motor rated no-load and short-circuit data is not standardized as in transformers. Therefore, mathematical model identification procedure must consider only the usual (normalized) data, as follows: rated power factor (\(\cos \varphi_{1,n}\)), rated efficiency (\(\eta_n\)) and optimal p.u. charge (\(a_{\eta,m}\)), rated slip (\(s_n\)) and p.u. maximum torque of the motor, voltage supplied \(m_{m,U} = m_{m,U} / \eta_n \cdot \cos \varphi_{1,n}\).

From the simplified vector diagram and angular characteristic of asynchronous machine [2], \(p_{e,n} = m_{m,U} \cdot \sin 2\varphi_{2,n} \equiv \cos \varphi_{1,n}\), it results the following rated values:

- rotor (induced) current: \(i_{2,n} = \cos \varphi_{1,n} / \cos \varphi_{2,n}\), \(\varphi_{2,n} = 0.5 \cdot \arcsin(p_{e,n} / m_{m,U})\);
- magnetization current: \(i_{m,n} = \sin \varphi_{1,n} - i_{2,n} \cdot \sin \varphi_{2,n}\);
- magnetizing reactance: \(x_m \equiv 1 / i_{m,n}\);
- total leakage reactance: \(x_{\sigma} \equiv 1 / 2m_{m,U}\);
- stator current corresponding to maximum torque: \(i_{1,M} \equiv i_{1,A} / \sqrt{2} \equiv \sqrt{2} \cdot m_{m,U}\).

![Fig. 1 – Universal characteristics.](image)

In case of an asynchronous motor with given power factor (\(\cos \varphi_{1,n}\)), the above values depend only on maximum torque (\(m_{m,U} \equiv 1.5...3\) p.u.), so, they are universal characteristics of the motor, voltage supplied, as in Fig. 1.
2.2. ELECTROMECHANICAL CHARACTERISTICS

The electromechanical characteristics $i_1(\alpha)$, $\cos\varphi_1(\alpha)$ and $\eta(\alpha)$ at rated voltage and frequency, for different slip values ($s$) or internal angle ($\theta$), may be determined against dimensionless parameter $\alpha = P_2/P_{2,\eta} \cong s/s_n = 0 \div 1$; simple analytical expressions for these characteristics have been proposed in [4]:

- stator current characteristic: $i_1(\alpha) \cong \sqrt{i_o^2 + (1-i_o^2) \cdot \alpha^2}$, where $i_o \cong i_{m,0}$; (2)

- power factor characteristic: $\cos\varphi_1(\alpha) \cong \alpha / \sqrt{\alpha^2 + (i_o + x_\alpha \cdot \alpha^2)^2}$, with a peak at $\alpha_{\varphi,0} = \sqrt{i_o/x_\alpha} \cong \sqrt{2 \cdot m_{n,/} \cdot i_o} = f(m_{n,/})$, an universal characteristic, Fig. 1;

- efficiency characteristic: $\eta(\alpha) \cong \alpha / (\alpha + p_{o,e}^* + \alpha^2 \cdot p_{k,e}^*)$, with a maximum at $\alpha_{\eta,0} = \sqrt{p_{o,e}/p_{k,e}}$, $p_{o,e}^* = p_o + \epsilon_o$, $p_{k,e}^* = p_k^* - \epsilon_o$, with $\epsilon_o = i_o^2 \cdot p_{j,1,n}$, $p_o^* = P_{o,n}/P_{2,n}$, $P_{o,n} = P_{j,e,n} + P_{m,n}$, and $p_{j,e}^* = P_{j,1,n}/P_{2,n}$, $P_{j,n} = P_{j,1,n} + P_{j,2,n}$.

2.3. LOSSES DISTRIBUTION

Starting from the analytical expression of efficiency, losses can be separated:

- Joule losses in rotor winding: $p_{j,2,n} = s_n \cdot p_{e,n} = s_n \cdot \eta_n \cdot \cos\varphi_{1,n} / (1-s_n)$;

- Joule losses in stator winding: $p_{j,1,n} = \sum p_n \quad (1 + \alpha_{\eta,n}^2) \cdot (1-i_o^2) = p_{j,2,n}$; (3)

- constant losses: $p_{o,n} = \frac{\alpha_{\eta,n}^2 \cdot \sum p_n}{(1 + \alpha_{\eta,n}^2)} - p_{j,1,n} \cdot i_o^2$,

where $\sum p_n = (1 - \eta_n) \cdot \cos\varphi_{1,n}$.  

Implementing the condition of positive losses, $p > 0$, means:

a) from $p_{j,1,n} > 0$, the maximum rated slip value ($s_{n,\max}$) and the maximum value of constant losses ($p_{o,\max}$) result, both depending on $\eta_n$ and $\alpha_{\eta,n}$:

$s_n < s_{n,\max} = (1 - \eta_n) / (1 + \eta_n \cdot \alpha_{\eta,n}^2)$, 

$p_{o,n} < p_{o,\max} = \alpha_{\eta,n}^2 \cdot \sum p_n / (1 + \alpha_{\eta,n}^2)$. (4)
b) from \( p_{o,n} > 0 \) results the maximum value of Joule losses in stator winding:

\[
P_{j1,n} < P_{j1,max} = \frac{\alpha_{n,m}^2 \sum_i p_n}{(1 + \alpha_{n,m}) \cdot i_o^2} = \frac{\alpha_{n,m}^2 \cdot (1 - \eta_n) \cdot \cos \varphi_{1,n}}{(1 + \alpha_{n,m}) \cdot i_o^2},
\]

and the minimum value:

\[
\alpha_{n,m} > (\alpha_{n,m})_{\min} = i_o \sqrt{\sum p_n - P_{j2,n}} - i_o^2 \cdot \left( \sum p_n - P_{j2,n} \right).
\]

Therefore, for an asynchronous motor with imposed data, the relations (3) allows the separation of losses in the machine (Fig. 2), and eventually a balanced choice, which could also consider the reference losses \( p_{ref} \approx \sum p_n/4 \).

2.4. EQUIVALENT CIRCUIT PARAMETERS

Subsequently, the T-equivalent circuit parameters can be obtained:

- resistance of stator winding: \( r_1 = p_{j1,n} \) [p.u.];
- resistance of rotor winding: \( r_2 = p_{j2,n}/i_{2,n}^2 = k_{r,s} \cdot s_n \) (\( k_{r,s} \approx 1 \));
- total leakage reactance: \( x_{n,1} \equiv x_{n,2} = x_n/2 \approx 1/(4 \cdot m_{m,U}) \), or \( k_{x,1,2} = x_{n,2}/x_{n,1} \);
- magnetizing reactance: \( x_m \equiv x_{m,n}/i_{m,n} \), \( u_{m,n} = \text{abs}(u_{m,n}) \approx 1 - x_{n,1} \cdot \sin \varphi_{1,n} \);
- equivalent constant losses (iron and mechanical losses): \( r_e \equiv u_{m,n}/p_{0,n} \) [p.u.].

Note. In case of encapsulated induction motors (i.e. IP44), mechanical and ventilation losses can be determined from their frame size (H, mm.). Based on relation \( D_e \equiv k_{es} \cdot H \) [3], we obtain:
\[ p_{m,n} = \frac{P_{m,n}}{S_n} \approx \frac{1}{S_n} \cdot \left( \frac{n_1}{1000} \right)^2 \cdot \left( \frac{k_{1H} \cdot H}{100} \right)^4 \]  \hspace{1cm} (7)

and the equivalent parallel resistances of equivalent circuit:

\[ r_{Fe} \approx u_{m,n}^2 / p_{Fe}, \quad r_{m,v} \approx u_{m,n}^2 / p_{m,v}. \]

2.5. APPLICATION

A three-phase induction motor with the following rated data is considered:

\[ P_n = 75 \text{ kW}, \ IP = 44, \ H = 280 \text{ mm}, \ D_e = 425 \text{ mm}, \ p = 2, \ \eta_n = 0.94, \]  \hspace{1cm} (8)

\[ \cos \varphi_1, n = 0.88, \ F_{mr}, M_{m} = 2.4; \]

The distribution of losses versus \( \alpha \eta_m \) is shown in Fig. 2. Balanced losses (close to \( p_{ref} \)) are obtained for \( \alpha \eta_m = 0.9 \ldots 1.2. \)

The following losses in [p.u.] are computed for \( \alpha \eta_m = 1 \) (catalog data):

\[ p_{j1,n} = 0.0196, \quad p_{j2,n} = 0.0084, \quad p_{o,n} = 0.025, \quad p_{m,v} = 0.0081, \quad p_{ref} = 0.0132, \]

and corresponding parameters in [p.u.], respectively:

\[ r_1 = 0.0196; \quad r_2 = 0.0103; \quad x_{\alpha,1} = 0.085; \quad x_{\alpha,2} = 0.17 (k_{x,1,2} = 2); \quad x_m = 3.34; \]

\[ r_w = 35.7 (r_{Fe} = 52.8 \text{ and } r_{m,v} = 110.2), \quad u_{m,n} = 0.945, \quad i_{m,n} = 0.283, \quad i_{1,n} = 0.9. \]  \hspace{1cm} (9)

3. MOTOR PERFORMANCES AT RATED FREQUENCY

The electromechanical characteristics \( i_1(\alpha), \ \cos \varphi_1(\alpha), \ \eta(\alpha), \) and mechanical characteristic \( m = f(\alpha), \) are of major interest in anticipating the induction motor performance, when fed at constant voltage and frequency.

a) Electromechanical characteristics obtained by solving the T-equivalent circuit with constant parameters as shown in (9) are represented in Fig. 3, where \( \alpha = s/s_n, \ r_2/s = k_{r,s} \alpha, \) and \( k_{r,s} = r_2/s_n = 1.03. \)

The following rated values were obtained at \( i_1 = 1 \) [p.u.]:

\[ s_n = 0.00984, \ \eta_n = 0.941, \ \alpha_{n,m} = 1, \ \cos \varphi_{1,n} = 0.876, \]  \hspace{1cm} (10)

which are in a good agreement with imposed values shown in (8).

Fig. 3. shows also the analytical characteristics (2). A good correspondence is again observed, which confirms the validity of proposed relations.
b) Mechanical characteristics, or torque characteristics of the motor supplied by constant voltage and frequency, $m_U = f(\alpha)$, are represented in Fig. 4.

Relatively significant discrepancies (10–15%) are observed between numerical (obtained directly from equivalent circuit) and analytical (Kloss, $m_U,T$) results. For comparison, similar motor characteristics when fed at constant current
and frequency are represented in Fig. 4. In this case, a better agreement of data is observed.

c) Starting characteristics. At starting, the squirrel-cage motor is characterized by starting torque \( m_p^* \) and starting current \( i_{p} \), which are dependent on each other.

Indeed, torque can be written \( m_p = r_{2,p} \cdot i_{2,p}^2 \), with \( r_{2,p} = k_{R,2} \cdot r_2 \) rotor starting resistance and \( k_{R,2} > 1 \) the increasing factor that takes into account the skin effect.

Assuming the approximations \( r_2 \cong s_n \) and \( i_{2,p} \cong i_{i_p} = i_p \), the increasing factor \( (k_{R,2}) \) can be immediately evaluated [4]:

\[
k_{R,2} = \frac{m_p}{s_n i_p^2} \cong \frac{m_p^* s_n \cos \phi_{u_2}}{s_n i_p^2 (1 - s_n)} \geq 1.
\]  

A similar relation is obtained for the starting decreasing factor \( k_{x,p} \) of the leakage reactance \( (x_{\alpha,p} = k_{x,p} x_{\alpha} \), \( k_{x,p} < 1 \)), due to skin and saturation effects:

\[
k_{x,p} = \frac{x_{\alpha,p}}{x_\alpha} \cong \frac{2 \cdot m_{m,u}}{i_p^2} \sqrt{i_p^2 - (1 + r)^2 m_p^2} \leq 1,
\]

where

\[
r = r_1 / r_{2,p}.
\]

Application. For the 75 kW asynchronous motor (8), with \( i_p = 6.7 \) and \( m_p^* = 2.1 \) or \( m_p = m_p^* \cdot \eta_{u_2} \cos \phi_{u_2} = 1.74 \), it results: \( k_{R,2} = 3.9 \) and \( k_{x,p} = 0.5 \), very important results for motor design, especially for sizing the squirrel-cage rotor.

4. ASYNCHRONOUS MOTOR PERFORMANCES AT VARIABLE FREQUENCY

The frequency characteristics are represented in Fig.5 for two slip values: \( s = s_n \), the rated slip, and \( s = s_m(f) = s_{m,df} \), which corresponds to maximum torque. The motor is supplied at: \( u_1 = f \) for \( f \leq 1 \), and \( u_1 = 1 \) for \( f \geq 1 \).

An important decrease of the maximum torque \( m(s) \) is noticed in the low frequency domain \( (f = f_1/f_{1,n} < 1) \), in contrast to usual estimations (Kloss) [5, 8]:

\[
m(s,f) \cong \frac{2 \cdot m_m(f)}{s \cdot s_m(f) + s_m(f)/s},
\]
with \( m_m(f) = m_{mU} \cdot u_1^2 / f^2 = m_{mU} = \text{const.} \)

\[
\begin{align*}
\text{Fig. 5 – Frequency characteristics. }
\end{align*}
\]

Starting frequency characteristics (\( s = 1 \)). For asynchronous motors supplied by variable frequency it is interesting to evaluate their starting performances (starting torque and starting current) at very low frequencies (\( f < 0.1 \ldots 0.2 \)), when skin effect is not present.

\[
\begin{align*}
\text{Fig. 6 – Starting frequency characteristics (} u_1 = f). 
\end{align*}
\]
In case of \( u_1 = f \) it results \( m_{P,m} \approx 0.8 \text{ [p.u.]} \) at \( f_P \approx 0.12 \) (Fig. 6), which are very different than the estimated values \( (m_{P,m} = m_{m,U}, \text{ and } f_P = s_{m,u}) \), computed with Kloss formula.

For comparison, in Fig. 7 there are also represented the starting \((s = 1)\) characteristics of the motor supplied by a current source \((i_{1,P} = 1 \text{ [p.u.]})\) and variable frequency. In the later, the results are in a good agreement with the estimates values: \( m_{P,m} = m_{m,l} \approx x_{m,u}/2 \approx 1.7 \text{ [p.u.]} \) at \( f_P = s_{m,a} = r_2/j_{m,u} \approx 0.003 \text{ [p.u.]} \).

Also, the starting performances are much better, which means a greater torque at a lower frequency, despite the inherent magnetic saturation at current supply.

5. CONCLUSIONS

The paper presents a simple and reliable procedure for determining the mathematical model (T – equivalent circuit parameters) of a three-phase asynchronous motor on the basis of rated and catalogue usual data \( \eta_n, \alpha_{n,m}, \cos \varphi_{1,n}, \) \( s_n, \) and \( m_{m,U}. \)

Considering maximum torque \((m_{m,l})\) as a variable parameter, the universal characteristics of three-phase asynchronous motor, \( i_{2,n}, i_{m,n}, x_{m,u}, x_{a}, \alpha_{a,m} = f(m_{m,l}) \), shown in Fig. 1, are outlined.

Based on proposed analytical relations for electromechanical characteristics (2), there were obtained a set of simple relations for determining losses \( p_{J1,n}, p_{J2,n}, p_{o,n} \), and consequently for resistances \( r_1, r_2, r_{us} \), with \( \eta_n, \) and \( \alpha_{n,m}, \) as a main parameters.
A numerical example of a three-phase asynchronous motor with 75 kW, 1485 rpm, confirms with a good accuracy the proposed analytical relations for electromechanical characteristics of the motor supplied by constant voltage and frequency (Fig. 3). There are, however, some discrepancies in torque evaluation (Fig. 4), if compared to usual simplified Kloss formula.

Interesting results are obtained for variable frequency when important discrepancies are again identified for maximum torque variation at low frequencies (Fig. 5). The same discrepancies are identified for starting torque (s = 1), at variable voltage and frequency (\(u_1 = f\)), Fig. 6. A better agreement between proposed method and Kloss formula are found at constant current.

The results obtained in this paper, especially the parameters identification method, can be a useful tool not only in asynchronous motors analysis, but also in synthesis (design) problems for asynchronous motors with imposed technical and economical performances (see the Poynting algorithm [2]).

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