STUDY OF THE VOLTAGE FREQUENCY DOUBLER WITH NONLINEAR IRON CORE MAGNETIC CHARACTERISTIC

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Key words: Frequency doubler, Transformer, Magnetic core, B-H characteristic modelling.

The simple and reliable method for doubling a given frequency by the use of a transformer-like device is investigated both qualitatively and quantitatively. The principle of generating an output voltage having a double frequency dominant harmonic component consists in associating two identical core transformers, each equipped with a supplementary dc winding. Their role is to pre-magnetize in opposite directions the two identical nonlinear iron cores. The secondary windings are connected in series-opposition to provide the double frequency dominant output voltage, which can be then separated if necessary by the use of a simple low-pass filter. The present study proposes an alternative connection of the primary windings, beneficial also for the inrush current mitigation, accompanied by a calculation procedure conducted analytically up to a point beyond which a numerical computation is performed. A numerical example is presented for illustrating and evaluating the proposed solution.

1. INTRODUCTION

The rather simple method for doubling a certain voltage frequency, as a result of the opposite dc pre-magnetization of the nonlinear iron cores of two transformers, without the use of semiconductor circuit elements, has been proposed a couple of decades ago for radiofrequency applications [1, 2]. A similar solution based on the same phenomenon was adopted for frequency triplers [1]. In spite of its simplicity and robustness the use of this specially designed transformer devices are quite neglected, the rather common necessity in practice for doubling or multiplying a certain voltage frequency relies almost entirely on semiconductor circuits. A downside to be mentioned for the differential transformer may consist in its massiveness, becoming important especially when a successive multiplying (by a factor of two, for example) of the base frequency requires cascading several devices of the sort. Relying entirely on the nonlinearity of B-H characteristic, the
working principle of the frequency doubler device heavily depends on the selection of the ferromagnetic material to be used as iron core. Traditionally, a rather empiric trial-and-error procedure was the primary choice for designing such devices, including here determining the number of turns for each winding, the primary excitation current, core geometry, etc. Moreover, to construct the multiplied frequency output voltage waveform, a graphical point-by-point method was used. To conclude, these approaches are often cumbersome and sometimes inconclusive.

To the author’s best knowledge, no quantitative assessment for this device has been presented for the last few decades, hence scientific literature is scarce on this particular subject [1, 2]. To fill this gap the paper proposes an analytical approach for a slightly modified configuration of the classical frequency doubler, presenting some advantages and a significantly reduced inrush current.

2. DEVICE STRUCTURE AND WORKING PRINCIPLE

Shortly after electrical transformers were invented and introduced in service at the end of the 19th century, the possible practical use of the $B-H$ iron core characteristic for frequency multiplying purposes by associating two such devices became evident. The classical structure of the frequency doubler is shown in Fig. 1,
as it has been presented by Zenneck in [1] at the beginning of the 1920s. Some less important elements relative to the working principle of the device were not included in Fig. 1. In brief, two identical core transformers (denoted by (α) and (β)) are energized from an ac generator G, supposed to provide a sine wave voltage $u_1(t)$ (whose frequency $f_1$ is to be doubled), through the adjustable inductor and capacitor $L_1$ and $C_1$ connected in series with the two primary windings. Thus, the same non-sinusoidal current $i_1(t)$ flows in both windings of $w_1$ turns. The secondary windings (with $w_2$ turns) are also connected in series in which flows $i_2(t)$, but in a manner that the two induced voltages subtract one from the other (“in opposition”) to give the output voltage $u_2(t)$, supposed to contain a double frequency dominant harmonic. The phenomenon is possible if each core is pre-magnetized in opposite directions by an identical winding (with $w_0$ turns) carrying a dc current $I_0$. In the first case the pre-magnetization flux assists the fascicular flux $\phi_\alpha(t)$ and opposes $\phi_\beta(t)$ for the second transformer.

In his work Zenneck [1] realizes a comprehensive and original study of the device on both instances as frequency doubler and also as frequency tripler – the latter when the secondary windings voltages are summed up. The author uses a first approximation of the $B-H$ curve under the form of a third degree polynomial

$$B(H) = c_1 H - c_2 H^3,$$

where $c_1$ and $c_2$ are constants specific to the used iron core material. The main disadvantage of this formula is that it may be only satisfactory accurate for the initial below-“knee” part of the $B-H$ curve. After attaining a maximum value the function (1) becomes monotonic decreasing, thus less succeeding to properly model the saturation branch of the characteristic. Unfortunately, as previously mentioned, the very functioning of the device resides in the saturation phenomenon of the iron core. Consequently, a more accurate $B-H$ dependency model must be envisaged. Nonetheless, with (1) it is easily shown that the fascicular fluxes are also third degree polynomials [1]

$$\phi_j(t) = K_j \theta_j(t) - K_j \theta_j(t) \quad \text{with} \quad j \in \{\alpha', \beta'\},$$

where $K_1$, $K_2$ are constants specific to the two identical cores (material and geometry) and $\theta_\alpha(t)$, $\theta_\beta(t)$ represent the total amount of ampere-turns (the magnetomotive force) corresponding to the two core mean path loops depicted by dashed line in Fig. 1, in no-load conditions ($i_2(t) = 0$) [1]:

$$\theta_\alpha(t) = w_1 i_1(t) + w_0 I_0 \quad \text{and} \quad \theta_\beta(t) = w_1 i_1(t) - w_0 I_0.$$  

Another important hypothesis [1] of the proposed approach is that the primary current $i_1(t)$ is purely sinusoidal, in spite of the well-known non-sinusoidal periodic waveform variation of an iron core coil supplied at his terminals with a sinusoidal voltage. That assumption only holds when the adjustable elements $L_1$
and $C_1$ are tuned to the ac generator frequency $f_1$, namely to achieve $R–L–C$ series resonance in the primary windings circuit. Thus, the superior order harmonics magnitudes of $i_1(t)$ may be neglected with respect to the fundamental. Furthermore, this procedure allows the calculation of this current, which is then used in (3). Finally, with (2) and (3), the secondary windings being series-opposed, an analytical formula for the output voltage is obtained from Faraday’s induction law applied along the oriented closed paths ($\Gamma_\alpha$) and ($\Gamma_\beta$) [1]:

$$u_2(t) = u_{2\alpha}(t) - u_{2\beta}(t) = -w_2 \frac{d}{dt} [\varphi_\alpha(t) - \varphi_\beta(t)],$$

(4)

where $u_{2\alpha}(t), u_{2\beta}(t)$ are the no-load secondary voltages. From (4), performing all the necessary calculations analytically, a single $2f_1$ sinusoidal component is obtained for the output voltage.

More recently, a compact single-primary variant of the device was reported in [3], as shown in Fig. 2. Here, a detailed flux flow diagram produced by each winding is presented and used to graphically construct the output voltage waveform over a period of the input voltage period guided by the nonlinearity of the $B - H$ curve. A disadvantage of this constructive variant is that mutual parasitic influences are more susceptible to appear in contrast to the classical structure where the two constitutive magnetic circuits may be easily placed apart at a safe separating distance. Except for the already known notations from Fig. 1, a filtering capacitor $C$ is to be noticed in Fig. 2.

Fig. 2 – Compact constructive variant of frequency doubling device.
3. PROPOSED VARIANT AND OUTPUT VOLTAGE COMPUTATION

The present study proposes a new operation variant for the doubling frequency device in which the two primary windings are fed with the same sinusoidal voltage of the frequency to be doubled $f_1$ (e.g. in parallel from the mains), as shown in Fig. 3. Supplementarily, an identical additional resistor $R_a$ is connected in series with the two relatively small primary resistances $R_1$ to give a total resistance $R_{1t} = R_a + R_1$.

The preliminary stage of adjusting $L_1$ and $C_1$ for achieving resonance will be no longer necessary since producing the double frequency component does not rely any more on the existence of a sine wave primary current flow.

Further calculations are made in the classical flux tube assumption specific to magnetic circuit theory meaning that there is no flux leakage for the fascicular fluxes $\phi_a(t)$ and $\phi_b(t)$.

Let

$$u_1(t) = U \sqrt{2} \sin(2\pi f_1 t + \gamma)$$

(5)

Fig. 3 – Frequency doubling device with primary windings connected in parallel to the mains through an additional resistor $R_a$ (proposed variant).
be the sinusoidal applied sinusoidal voltage of root mean square value $U$ and initial phase $\gamma$. In the receptor convention, Faraday’s induction law for the two primary windings writes

$$R_i i_j(t) + w_1 \frac{d\varphi_j(t)}{dt} = u_i(t) \text{ with } j \in \{\alpha', \beta'\},$$  

where $i_\alpha(t)$ and $i_\beta(t)$ are the absorbed non-sinusoidal primary currents. Under no-load conditions, similarly to (3), Ampère’s theorem (applied along the median ferromagnetic core closed paths of length $l_i$) gives:

$$H_\alpha(t) l_f = w_1 i_\alpha(t) + w_0 I_0 \text{ and } H_\beta(t) l_f = w_1 i_\beta(t) - w_0 I_0,$$

where $H_\alpha(t)$ and $H_\beta(t)$ are the magnetic field strength time-dependant functions within the iron cores ($\alpha$) and ($\beta$).

As previously discussed in Chapter 2, for an accurate determination of the output voltage, a more realistic model for the magnetic core characteristic is to be employed.

Reference [4] presents a comprehensive study over the $H$-$B$ relationship first proposed by Brauer. One conclusion of this study underlines that above the Rayleigh region the Brauer formula is a reliable option, referred thus as “the Brauer region”. Using this model the two magnetic field strengths become [4, 5]

$$H_j(B_j) = B_j \left[ k_1 \exp(k_2 B_j^2) + k_3 \right] \text{ with } j \in \{\alpha', \beta'\},$$

where $B_\alpha$, $B_\beta$ are the corresponding flux densities of the two cores and $k_1$, $k_2$, $k_3$ are constants specific to the used ferromagnetic material. Also in [4], a fitting algorithm for determining these constants specific to a given material is presented.

With (5) – (8) combined, the following two compact first-order differential equations are obtained for the flux densities:

$$\frac{dB_\alpha}{dt} + \frac{R_\alpha l_f}{w_1 A_f} B_\alpha \left[ k_1 \exp(k_2 B_\alpha^2) + k_3 \right] = \frac{1}{w_1 A_f} \left[ U \sqrt{2} \sin(2\pi f_1 t + \gamma) + R_{11} I_0 \frac{w_0}{w_1} \right],$$

and

$$\frac{dB_\beta}{dt} + \frac{R_\beta l_f}{w_1 A_f} B_\beta \left[ k_1 \exp(k_2 B_\beta^2) + k_3 \right] = \frac{1}{w_1 A_f} \left[ U \sqrt{2} \sin(2\pi f_1 t + \gamma) - R_{11} I_0 \frac{w_0}{w_1} \right],$$

where $A_f$ is the cross-sectional constant area of the identical iron cores. As a result of their complexity there is no general analytical solution for the two equations (9) and (10) which have to be independently solved numerically under the initial conditions reflecting the pre-existing magnetization of the cores. More specifically, if $B_{\alpha 0} = B_\alpha(0)$ and $B_{\beta 0} = B_\beta(0)$ are the initial conditions for the flux densities, they must correspond to the initial magnetic field strengths.
Voltage frequency doubler based on nonlinear $B-H$ characteristic

\[ H_{\alpha 0} = H_{\alpha}(0) = N_0 J_0/l_f \text{ and } H_{\beta 0} = H_{\beta}(0) = -N_0 J_0/l_f \]  
(11)

respectively. That implies first numerically solving the following transcendental equations

\[ H_{j0} = B_{j0} \left[ k_1 \exp(k_2 B_{j0}^2) + k_3 \right] \text{ with } j \in \{\alpha', \beta'\} \]  
(12)

for variables $B_{\alpha 0}$ and $B_{\beta 0}$. Obviously, the constants $k_1$, $k_2$ and $k_3$ specific to the considered ferromagnetic material are supposed to be previously determined using a fitting procedure, for example the one proposed in [4]. Note that the two functions defined in (8) are not invertible, i.e. there is no analytical formula for the functions $B_{\alpha}(H_{\alpha})$ and $B_{\beta}(H_{\beta})$.

To conclude, after determining the time-variation functions of the flux densities $B_{\alpha}(t)$ and $B_{\beta}(t)$, the electromagnetic induction law is applied along the oriented closed paths ($\Gamma_{\alpha}$) and ($\Gamma_{\beta}$), as depicted in Fig. 3. Similarly to (4), the output voltage of dominant frequency $2 f_1$ then becomes

\[ u_2(t) = u_{2\alpha}(t) - u_{2\beta}(t) = -w_2 A_{\Gamma_{\alpha}} \frac{d}{dt} \left[ B_{\alpha}(t) - B_{\beta}(t) \right]. \]  
(13)

Refining the proposed approach may take into account flux leakage and core laminations stacking factor, as performed in [6]. Also in [6], the $B-H$ characteristic is modelled using the Ollendorff formula which can represent an alternate solution to (8). An even more accurate model could use the hysteretic cycle for the soft magnetic material instead of the magnetizing curve [7]. Finally, the transformer dedicated field analysis numerical procedure proposed in [8] can find direct application to the present configuration.

4. ILLUSTRATIVE EXAMPLE

A numerical implementation of the calculus described in the previous chapter considers the following numerical values which are consistent to a generic single-phase low-voltage transformer (for each magnetic circuit ($\alpha$) and ($\beta$)): $U = 230 \, \text{V}$, $\gamma = 0$, $R_1 = 1 \, \Omega$, $w_1 = 300$ turns, $w_2 = 30$ turns, $l_f = 0.6 \, \text{m}$ and $A_{\Gamma} = 25 \cdot 10^{-4} \, \text{m}^2$. After introducing the additional resistor $R_\alpha$, the total primary resistance will be considered $R_{\alpha 0} = 1 \, \text{k}\Omega$.

The pre-magnetizing dc windings have both $w_0 = 25$ turns and each carry a current $I_0 = 2 \, \text{A}$. With the above data, the initial magnetic field strength values then become $H_{\alpha 0} = 83.3 \, \text{A/m}$ and $H_{\beta 0} = -83.3 \, \text{A/m}$, respectively.
Fig. 4 – Output voltage time-variation graph over an input voltage period ($T_1$) after 8 s time from commutation and the corresponding harmonic content for:

a) $f_1 = 50$ Hz ($T_1 = 20$ ms); b) $f_1 = 500$ Hz ($T_1 = 2$ ms); c) $f_1 = 1$ kHz ($T_1 = 1$ ms).
For the actual iron cores material the GO (grain oriented) 35ZH135 electrical steel was chosen for which the \((H, B)\) points of the characteristic were available from the data sheet. Although, a simple interpolation of these points might be an alternate solution to (8), the compactness of the former is preferable. The performed fitting procedure provides the Brauer formula constants: 
\[k_1 = 0.061 \text{ m/H}, \quad k_2 = 4.524 T^{-2} \quad \text{and} \quad k_3 = 11.998 \text{ m/H}.\]

Next, with (11), the two equations from (12) are solved numerically to get the initial conditions for the flux densities functions obeying the differential equations (9) and (10), namely \(B_{\alpha0} = 1.227 \text{T}\) and \(B_{\beta0} = -1.227 \text{T}\). In order to determine the variation trends of the output voltage characteristics, the following computation was performed for several input frequencies \(f_1\): 50 Hz, 500 Hz and 1 kHz. By using a general-purpose mathematics package, equations (9) and (10) are numerically solved and flux densities \(B_{\alpha}(t)\) and \(B_{\beta}(t)\) are obtained for the three chosen frequencies.

Finally, the corresponding output voltages are obtained with (13) and then plotted in Fig. 4 (over the input voltage period span and after reaching steady-state), along with a harmonic analysis comprising the 10 first harmonics of the Fourier series decomposition, in order to estimate the magnitude of the second order harmonic.

As expected, the second harmonic of the output voltage exhibits a larger magnitude than the other harmonics as one can notice by a simple visual assessment of the time-variation graphs, since an input voltage period span \((T_1)\) includes two periods of the output voltage. Another expected outcome is the practical inexistence of the uneven harmonics, validating thus the proposed approach.

For the case shown in Fig. 4a the use of a low-pass filter would be a necessary option in order to cut-off the even harmonics 4, 6, 10, ..., etc.

5. CONCLUSIONS

The proposed solution in which the two primary windings are connected in parallel avoids the preliminary stage of achieving resonance at the input voltage frequency by adjusting the inductor \(L_1\) and the capacitor \(C_1\). Instead, the additional resistance newly introduced in series drastically reduces the surged primary current when the no-loaded device is energized, but might increase Joule’s heat losses. Moreover, by examining Fig. 4, it turns out that the distortion factor (relative to \(2f_1\)) decreases with the increase of input voltage frequency \(f_1\) (under 5 % in the case shown in Fig. 4b, c, when no filtering process is necessary). Conversely, the output voltage magnitude becomes smaller for a higher input voltage frequency \(f_1\), issue that can be addressed by supplementing the number of turns in the secondary windings. In what concerns the influence of the total primary resistance \(R_{1t}\) over
primary Joule’s power losses and output voltage second harmonic level, an estimation at $f_1 = 50 \text{ Hz}$ was carried out. In particular, for $R_{1t} = 1 \text{ k}\Omega$ and $U = 230 \text{ V}$ the rms values for the primary windings non-sinusoidal currents are $I_{1\alpha} = 160.3 \text{ mA}$ and $I_{1\beta} = 303.9 \text{ mA}$, respectively. Thus, the total primary losses are $P_{11} = R_{1t} (I_{1\alpha}^2 + I_{1\beta}^2) = 118 \text{ W}$ with a second harmonic rms value of $U_{2,2} = 13.45 \text{ V}$.

Decreasing the $R_{1t}$ value to 0.5 k\Omega one gets $I_{1\alpha} = 203.4 \text{ mA}$, $I_{1\beta} = 328.7 \text{ mA}$, $P_{11} = 74.7 \text{ W}$ and $U_{2,2} = 9.52 \text{ V}$, showing in both cases moderate Joule’s power losses, but a decrease of the output voltage for the last example.

Therefore, operating the frequency doubler (under the proposed variant) is essentially a trade-off between output voltage magnitude and distortion factor, but, at the same time, the primary thermal losses introduced by the additional resistors must also be kept at an acceptable level. Future further analysis may concern the dc pre-magnetization, namely the initial point to start on the magnetization curve ensuring optimal output both in terms of magnitude and sinusoidal shape.

ACKNOWLEDGMENTS

The author is grateful to Emeritus Professor Ovidiu Centea, Doctor Honoris Causa of the Technical University of Civil Engineering Bucharest, for the valuable advice offered for improving this study.

Received on January 16, 2015

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