SELECTION DIVERSITY RECEIVER OVER CORRELATED RAYLEIGH FADING CHANNELS IN THE PRESENCE OF MULTIPLE INTERFERERS

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An approach to the performance analyses of signal-to-interference (SIR) based selection combining (SC) operating over the correlated Rayleigh fading channels experiencing an arbitrary number of correlated multiple, co-channel interferers is presented. General analysis of multibranch SC where each correlated branch experiences an arbitrary number of multiple correlated co-channel interferers. Infinite series expressions are obtained for the output SIR's probability density function (PDF), cumulative distribution function (CDF) and an important measure of the system's performances, the outage probability (OP). An average bit error probability (ABER) is efficiently evaluated for modulation schemes such as binary frequency-shift keying (BFSK) and binary differentially phase-shift keying (BDPSK).

1. INTRODUCTION

The main design goals in the cellular mobile terrestrial and satellite communication systems is providing required quality of service in combination with high capacity [1]. Because of that, any commercial or military mobile-radio system tends to conserve the available spectrum by reusing allocated frequency channels in areas that are geographically located as close to each other as possible. However, it appears when, due to frequency reuse, signals from two or more channels operating at the same frequency, but from different locations, interfere. Co-channel interference is defined as the interfering signal that has the same carrier frequency as the useful information signal. Now, crucial issue is the determination

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methodologies for analysing the influence of any kind of co-channel interference on these systems [2]. The main objective is then to analyse how the interference as a general distortion (in the form of signal-to-interference ratio, SIR) affects well-accepted criteria of performance of wireless systems, such as outage probability, and average bit-error probability in order to the practical system implementation which satisfies the predetermined minimum performance levels. SIR-based measurements and analyses, can be performed in real time both in base stations and in mobile stations using specific SIR estimators as well as those for both [3–4] analog and digital wireless systems (*e.g.*, GSM, IS-54).

Wireless communication system performances are also remarkably affected by fading phenomena. Various techniques for reducing fading effect and influence of co-channel interference are used. Space diversity reception is an effective remedy that exploits the principle of providing the receiver with multiple faded replicas of the same information-bearing signal. Their goal is to upgrade transmission reliability without increasing transmission power and bandwidth and to increase channel capacity. Several principal types of combining techniques can be performed by their dependence on complexity restriction put on the communication system and amount of channel state information available at the receiver. While other combining techniques require all or some of the amount of the channel state information of received signal and separate receiver chain for each branch of the diversity system, which increase its complexity, selection combining (SC) receiver process only one of the diversity branches, and is much simpler for practical realization, in opposition to these combining techniques [5].

In fading environments where the level of the co-channel interference is sufficiently high as compared to thermal noise (*i.e.* cellular systems), SC selects the branch with the highest SIR (SIR-based selection diversity) [6].

The analysis of co-channel interference effects on the wireless communication systems performance metrics has been extensively analysed [7]. Performance analysis of optimum combining over Rayleigh fading channels with multiple co-channel interferers was presented in [8].

Moreover to the best author's knowledge, no analytical study of SIR based selection combining involving assumed correlated Rayleigh fading channels experiencing an arbitrary number of multiple, correlated Rayleigh co-channel interferers, has been reported in the literature.

Firstly, for proposed environment model, closed form expressions for cumulative distribution function (CDF) and probability distribution function (PDF) of the SC output SIR are derived. Capitalizing on this, important performance measures such as outage probability (OP) and average bit error probability (ABER) over some modulation techniques are efficiently evaluated. Outage probability is shown graphically for different system parameters. Numerical results are graphically presented in order to show the effects of the number of multiple interferers, diversity order, and the level of correlation between received desired signals and multiple interferences and input SIR unbalance to the system performances. This analysis is important since in designing a cellular mobile system, one may wish to determine optimal values of system parameters in order to achieve reasonable influence of interferers on the outage and bit-error rate occurrence.

2. SYSTEM MODEL

Let us consider the case of M correlated Rayleigh distributed interferers over k-th branch of the SC diversity system with N branches, having the same average power. This assumption is suitable for the two limiting cases that can bound the performances of any interference-limited systems, that correspond to the scenario when the interferers are on the cell edges closest to the desired user cell (worst interference case scenario) or where they are at the furthest edges (best interference case scenario) [9]. Also, in practice, there are several wireless systems that could be adequately modelled using this assumption as explained in [10], such as an interfering cluster of co-located terminals or a single multi-antenna interferer. When diversity system is applied on small terminals with multiple antennas, correlation arises between branches. We will discuss the following case considering proposed model of constant correlation between the branches for the Nakagami-*m* model, given in [11]. The constant correlation model [12] can be obtained by setting $\Sigma_{i,j} \equiv 1$ for $i = j$ and $\Sigma_{i,j} \equiv \rho$ for $i \neq j$ in correlation matrices, for both desired signal and interference signal anvelopes.

Since one variable Nakagami-*m* (and Rayleigh, obtained from Nakagami-*m* as the special case for $m = 1$) distribution could be derived from the central χ^2 (chi-square) distribution with $2L$ degrees of freedom (L independent complex Gaussian RVs) joint multivariate pdf distributions for both desired and total interfering signal correlated envelopes could be expressed by:

$$p_{R_1,\ldots,R_n}(R_1,\ldots,R_n) = \left(1-\sqrt{\rho_d}\right)\sum_{k_1=0}^{\infty}\ldots\sum_{k_n=0}^{\infty}\frac{2^n\Gamma(1+k_1+\ldots+k_n)\rho_d^{\frac{k_1+\ldots+k_n}{2}}}{\Gamma(1+k_1)\ldots\Gamma(1+k_n)k_1!\ldots k_n!}\left(\frac{1}{1+(n-1)\sqrt{\rho_d}}\right)^{1+k_1+\ldots+k_n} \tag{1}$$

$$\left(\frac{1}{\Omega_{d1}\left(1-\sqrt{\rho_d}\right)}\right)^{1+k_1}\left(\frac{1}{\Omega_{dn}\left(1-\sqrt{\rho_d}\right)}\right)^{1+k_n} R_1^{2k_1+1}\ldots R_n^{2k_n+1}\exp\left(-\frac{R_1^2}{\Omega_{d1}\left(1-\sqrt{\rho_d}\right)}\right)\exp\left(-\frac{R_n^2}{\Omega_{dn}\left(1-\sqrt{\rho_d}\right)}\right),$$

$$p_{r_1,\ldots,r_n}\left(r_{1,\ldots,}r_n\right) = \frac{\left(1-\sqrt{\rho_c}\right)}{\Gamma(M)}\sum_{l_1=0}^{\infty}\ldots\sum_{l_n=0}^{\infty}\frac{2^n\Gamma(M+l_1+\ldots+l_n)\rho_c^{\frac{l_1+\ldots+l_n}{2}}}{\Gamma(M+l_1)\ldots\Gamma(M+l_n)l_1!\ldots l_n!}\left(\frac{1}{1+(n-1)\sqrt{\rho_c}}\right)^{M+l_1+\ldots+l_n} \tag{2}$$

$$\left(\frac{M}{\Omega_{c1}\left(1-\sqrt{\rho_c}\right)}\right)^{M+l_1}\left(\frac{M}{\Omega_{cn}\left(1-\sqrt{\rho_c}\right)}\right)^{M+l_n} r_1^{2M+2l_1-1}\ldots r_n^{2M+2l_n-1}\exp\left(-\frac{Mr_1^2}{\Omega_{c1}\left(1-\sqrt{\rho_d}\right)}\right)\exp\left(-\frac{Mr_n^2}{\Omega_{cn}\left(1-\sqrt{\rho_d}\right)}\right).$$

The power correlation coefficient ρ_d for desired signal is defined as $\mathrm{cov}(R_i^2,R_j^2)/(\mathrm{var}(R_i^2)\mathrm{var}(R_j^2))^{1/2}$, while power correlation coefficient ρ_c for interfering signal is defined as $\mathrm{cov}(r_i^2, r_j^2)/(\mathrm{var}(r_i^2)\mathrm{var}(r_j^2))^{1/2}$, with R_k and r_k being the amplitudes of the desired and interference signals received at the k-th branch. $\Omega_{dk}=\overline{R_k^{\,2}}$ denotes the average desired signal power at k-th branch, while $\Omega_{ck}=\overline{r_k^{\,2}}$ is the total average interference signal power at k-th branch, where Ω_{cik}, $i=1\ldots M$ is the average power of the single co-channel interference, and stands $\Omega_{ck} = M\Omega_{cik}$, with M denotes the number of interferers over k-th branch.

Let us define the instantaneous value of SIR at the k-th diversity branch input as $\lambda_k=R_k^2/r_k^2$. Like it was mentioned above SC chooses and outputs the branch with the largest SIR. Joint probability density function of instantaneous values of SIR at the input of multibranch SC combiner could be obtained as in [13]:

$$p_{\lambda_1,\ldots,\lambda_n}(t_1,\ldots,t_n)=\frac{1}{2^n\sqrt{t_1\ldots t_n}}\int_0^\infty\ldots\int_0^\infty p_{R_1,\ldots,R_n}\left(r_1\sqrt{t_1},\ldots,r_n\sqrt{t_n}\right)p_{r_1,\ldots,r_n}(r_1,\ldots,r_n)r_1\ldots r_n\,\mathrm{d}r_1\ldots\mathrm{d}r_n\,. \tag{3}$$

And joint cumulative distribution function can be written as [13]:

$$F_{\lambda_1,\ldots,\lambda_n}(t_1,\ldots,t_n)=\int_0^{t_1}\ldots\int_0^{t_n}p_{\lambda_1,\ldots,\lambda_n}(x_1,\ldots,x_n)\,\mathrm{d}x_1\ldots\mathrm{d}x_n\,. \tag{4}$$

Let $S_\mathrm{k} = \Omega_{dk}/\Omega_{cik} = \Omega_{dk}/(\Omega_{ck}/M)$ be the average SIR's at the k-th input branch of the multi-branch SC. Cumulative distribution function of output SIR, could be derived from (4) by equating the arguments $t_1= \ldots =t_n = t$ as in [13]:

$$F_\lambda(t)=\underbrace{\sum_{k_1,\ldots k_n=0}^{\infty}\sum_{l_1,\ldots l_n=0}^{\infty}}_{2n}G_1 t^{n+k_1+\ldots+k_n}\frac{\prod_{i=1}^{n}{}_2F_1\left(1+k_i,1-M-l_i;2+k_i;\dfrac{t}{t+\dfrac{(1-\sqrt{\rho_d})}{(1-\sqrt{\rho_c})}S_i}\right)}{\left(t+\dfrac{(1-\sqrt{\rho_d})}{(1-\sqrt{\rho_c})}S_i\right)^{1+k_i}}, \tag{5}$$

with ${}_2F_1(u_1,u_2;u_3;x)$, being the Gaussian hypergeometric function [14], and:

$$G_1=\frac{(1-\sqrt{\rho_d})(1-\sqrt{\rho_c})^M\,\Gamma(1+k_1+\ldots+k_n)\Gamma(M+l_1+\ldots+l_n)\Gamma(1+M+k_1+l_1)\cdots\Gamma(1+M+k_n+l_n)}{\Gamma(M)(1+k_1)\cdots(1+k_n)\Gamma(1+k_1)\cdots\Gamma(1+k_n)\Gamma(M+l_1)\cdots\Gamma(M+l_n)k_1!\cdots k_n!l_1!\cdots l_n!}\,. \tag{6}$$
$$\rho_d^{\frac{k_1+..+.k_n}{2}}\rho_c^{\frac{l_1+..+.l_n}{2}}\left(\frac{1}{1+(n-1)\sqrt{\rho_d}}\right)^{1+k_1+\ldots+k_n}\left(\frac{1}{1+(n-1)\sqrt{\rho_c}}\right)^{1+l_1+\ldots+l_n}.$$

In Table 1, the number of terms to be summed in order to achieve accuracy at the desired significant digit is depicted. As we can see from the table, the values of these terms are strongly related to the correlation coefficients ρ_d and ρ_c and the number of interferers M.

Table1

Terms need to be summed in the expression for CDF of triple branch SC output SIR case to achieve accuracy at the 4th significant digit presented in the brackets

$M = 2$, $\rho_d = \rho_c = 0.2$, $S_1 = S_2$		$M = 3$, $\rho_d = \rho_c = 0.2$, $S_1 = S_2$	
$S/\lambda = -10$ dB	$S/\lambda = 0$ dB	$S/\lambda = -10$ dB	$S/\lambda = 0$ dB
15	16	17	17
19	19	22	20
22	21	24	22

PDF of the SC output SIR can be obtained easily from previous expression:

$$p_\lambda(t) = \frac{\mathrm{d}}{\mathrm{d}t}F_\lambda(t) = \underbrace{\sum_{k_1,\ldots k_n=0}^{\infty}\sum_{l_1,\ldots l_n=0}^{\infty}}_{2n} G_1(1+k_1)\cdots(1+k_n)t^{n+k_1+\ldots+k_n}(A_1(t)+\ldots+A_n(t)),$$

$$A_i(t) = \left(\frac{S_i}{t+\frac{(1-\sqrt{\rho_d})}{(1-\sqrt{\rho_c})}S_i}\right)^{M+l_i} \prod_{\substack{j=1\\ j\neq i}}^{n} \frac{{}_2F_1\left(1+k_j, 1-M-l_j; 2+k_j; \frac{t}{t+\frac{(1-\sqrt{\rho_d})}{(1-\sqrt{\rho_c})}S_j}\right)}{(1+k_j)}. \qquad (7)$$

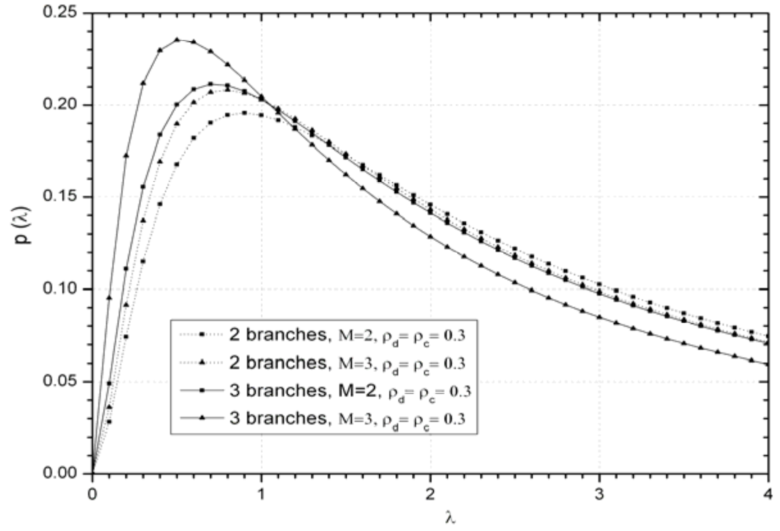

Fig. 1 – PDF of output SIR for various values of the number of multiple interferers and diversity branches.

Fig. 1 shows the PDF of output SIR for various values of the number of multiple interferers and diversity branches.

3. **OUTAGE PROBABILITY**

Outage probability P_{out} is standard performance criterion of communication systems operating over fading channels. This performance measure is also used to control the noise or co-channel interference level, helping the designers of wireless communications systems to meet the QoS and grade of service (GoS) demands.

Outage probability P_{out} is defined as the probability of falling of combined SNR below given outage threshold γ, also known as a protection ratio. Protection ratio depends on modulation technique and expected QoS. If the environment is interference limited, P_{out} is defined as the probability that the output SIR of used combiner will fall below protection ratio

$$P_{\text{out}} = P_R(\xi < \gamma) = \int_0^{\gamma} p_{\xi}(t)\,\mathrm{d}t = F_{\xi}(\gamma)\,. \tag{8}$$

Figs. 2 and 3 show the outage probability versus normalized parameter S_1/γ for balanced and unbalanced ratio of SIR at the input of the branches, various values of the number of multiple interferers and level of correlation. It can be seen from Fig. 2 how the outage probability increases when the number of multiple co-channel interferers increases due to growth of interference domination. Also for this dual SC diversity case it is evident how outage probability deteriorates when higher level of correlation between the diversity branches is presented. From Fig. 3 can be seen how outage probability behaviour improves as the diversity order (number of branches) increases. Similar conclusions could be drawn for the input SIR unbalance at the diversity branches. Outage probability deteriorates when input SIR unbalance is present and the degradation increases with number of branches.

Another important system parameters, which can be calculated based on outage probability are the coverage area, the area within which outage probability is guaranteed to be less than a given threshold, and the reuse distance, minimum distance between any two co-channel base stations which ensures a worst case outage probability no larger than the required value, also known as CCI reduction factor. Based on previous evaluation and presented numerical results one may determine optimal values of system parameters for achieving reasonable level of outage in practical wireless applications.

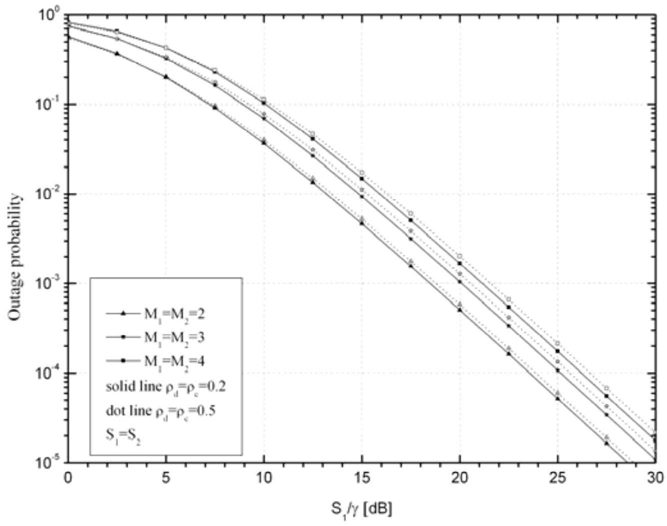

Fig. 2 – Outage probability for the two branch correlated case versus normalized parameter S_1/γ for various values of the number of multiple interferers and the level of correlation.

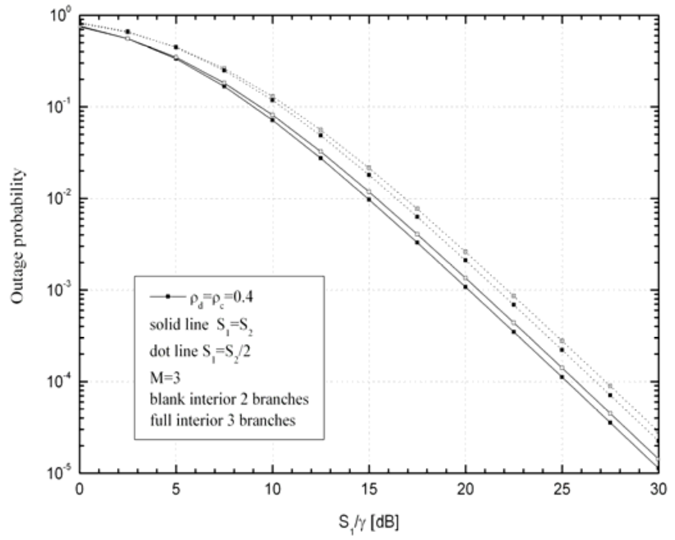

Fig. 3 – Outage probability for the correlated case versus normalized parameter S_1/γ for balanced and unbalanced ratio of SIR at the input of the branches, and diversity order.

4. AVERAGE BIT ERROR PROBABILITY

The average bit error probability (ABER) at the SC output is derived for noncoherent and binary signalling according to following expressions:

$$P_e = \int_0^\infty p_\xi(t)\,\frac{1}{2}\exp(-gt)\,\mathrm{d}t\,, \tag{9}$$

where g denotes modulation constant, *i.e.*, $g = 1$ for BDPSK and $g = 1/2$ for NCFSK. Substituting (7) in (9) numerically obtained average error probability is shown in Fig. 4 for dual branch SC combiner and various number of multiple interferers. A comparison of curves shows better performance of BDPSK modulation scheme against NCFSK modulation scheme. Also is more evident how ABER increases at both figures when the number of multiple correlated co-channel interferers increase due to growth of interference domination.

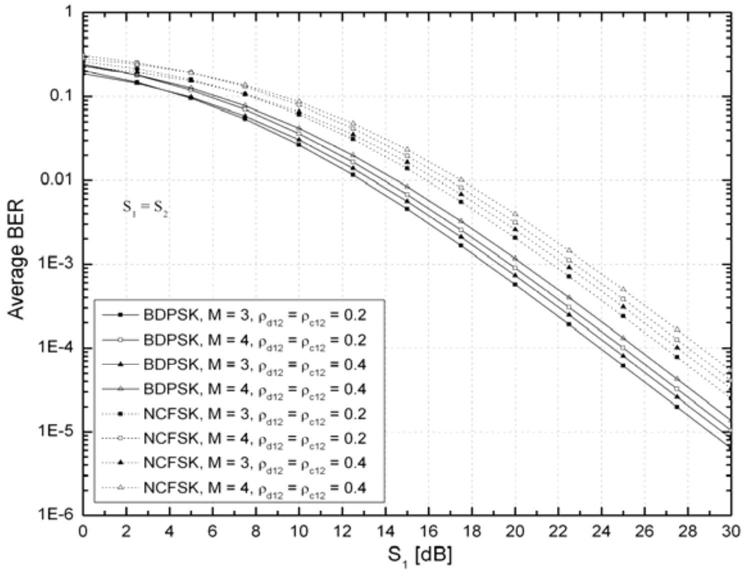

Fig. 4 – Average BER *versus* S_1 at dual branch SC output over BDPSK and NCFSK modulation schemes for various numbers of multiple interferers.

5. CONCLUSIONS

Multibranch SIR-based SC system operating over Rayleigh fading channels where each channel experiences an arbitrary number of multiple correlated co-channel interferers was studied. Closed form expressions for the SIR probability density function (PDF) and cumulative distribution function (CDF) at the output of the SC combiner were derived. Based on them, outage probability and average bit error probability were analysed, in order to show the effects of the number of multiple interferers, diversity order, level of correlation and input SIR unbalance on the system performances. This consideration could be taken into account during the design of various cellular mobile systems, for determining optimal values of system parameters, and achieving desired influence of interferers on the outage.

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