HARMONIC ANALYSIS OF CIRCUITS WITH NONLINEAR RESISTIVE ELEMENTS

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Key words: Harmonic analysis, Nonlinear resistive circuits, Steady state computation.

An efficient analysis of circuits with nonlinear resistive elements can be performed using the equivalent sources method. The nonlinear resistors are replaced by sources which are iteratively corrected based on their terminal voltages or currents. The internal resistances of the sources remain unchanged during the iterative process and their values can be chosen so that the iterative procedure convergence is always ensured. The equivalent circuit remains unchanged and only the sources values are iteratively corrected. That allows the development of a fast procedure for circuit analysis which computes on each harmonic the transfer immitance matrix between the controlled sources and their control variables. The computation time is further decreased by iteratively selecting a specific number of harmonics in terms of their weight.

1. INTRODUCTION

The most often technique used to obtain the steady state solution of nonlinear circuits is time domain analysis, which evaluates the asymptotic evolution characterized by a sufficiently small variation of the state variables. The nonlinearity treatment can be performed using the Newton-Raphson method [1] or the equivalent source method [2, 3]. If the independent sources have a broad range of frequencies then the time step is taken to be smaller than the smallest period. That leads to a huge computation time. Shooting algorithms are used in order to accelerate the computation [1, 4]. Introducing more variables of time ($t_k = \omega_k t$) has not yet produced convincing results [5]. Using the harmonic balance method is an option, but the system of equations can be very large in the case of signals with high content of harmonic components. The Newton-Raphson procedure does not always ensure the convergence of the iterative process. An equivalent sources method based method is proposed in [6]. The nonlinear resistors are replaced by sources iteratively corrected in terms of their terminal voltages or currents. It can

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be shown that the internal resistances of the equivalent sources can be chosen so that the iterative technique to be convergent. This procedure has some major advantages, namely: the convergence is always ensured, the rate of convergence can be speed up by using an overrelaxation technique, the error to the exact solution can be precisely evaluated, the linear circuit computation for each iteration can be performed only once, independently, for each harmonic component, the summation of harmonics being performed at each iteration for the equivalent source values correction, the circuit remaining unchanged.

In this paper the computation technique presented in [6] is developed in order to accelerate the computations at each iteration, using the above mentioned advantages.

2. EQUIVALENT SOURCES METHOD

For the sake of simplicity, we assume that a nonlinear resistor is voltage controlled and has the constitutive relationship $i = F(u)$. This relationship is replaced on each resistor port with a voltage source having the characteristic

$$u_k = r_k i_k + s_k \,, \tag{1}$$

where the source s_k nonlinearly depends on the port voltage

$$s_k = u_k - r_k F_k(u) \equiv G_k(u) \,. \tag{2}$$

If the function $F : R^p \to R^p$ is Lipschitzian

$$\left\| F(u') - F(u'') \right\|_{R^p} \le \Lambda \left\| u' - u'' \right\|_{R^p} \ , \ \forall u', u'' \in R^p \tag{3}$$

and monotonic

$$\left\langle F(u') - F(u''), u' - u'' \right\rangle_{R^p} \ge \lambda \left\| u' - u'' \right\|_{R^p}^2 , \ \forall u', u'' \in R^p \,, \tag{4}$$

with $\lambda > 0$, then one can choose r_k so that the function G, with the components G_k, to be a contraction

$$\left\| G(u') - G(u'') \right\|_{R^p} \le \theta \left\| u' - u'' \right\|_{R^p} \,, \tag{5}$$

with $\theta < 1$ [2]. The inner product is defined as $\left\langle x, y \right\rangle_{R^p} = x^{\mathrm{T}} r\, y$, where p denotes the number of nonlinear resistor ports and r is the diagonal matrix of the resistances r_k.

The iterative method of equivalent sources consists in the following steps:

a) at the first iteration the sources are initialized with arbitrary values $e^{(0)}$;
b) at each (n) iteration, a linear steady state circuit is solved

$$u^{(n)} = L(s^{(n-1)}) + u_0 , \tag{6}$$

where u_0 is the voltage vector at nonlinear resistor ports corresponding to the independent sources;

c) the equivalent sources values are corrected using: $e^{(n)} = G(u^{(n)})$;

d) when the error $\left\| s^{(n)} - s^{(n-1)} \right\| = \sqrt{\frac{1}{T}\int_0^T \mathrm{d}t \sum_{k=1}^{p} r_k \left(s_k^{(n)} - s_k^{(n-1)} \right)^2}$ is sufficiently small, the iterations repeated from b) are stopped.

It can be shown that the linear function L which corresponds to the solution of the linear circuit is nonexpansive

$$\left\| u' - u'' \right\| = \left\| L(s') - L(s'') \right\| \le \left\| s' - s'' \right\| . \tag{7}$$

Therefore the above described iterative procedure $s^{(n-1)} \xrightarrow{L} u^{(m)} \xrightarrow{G} s^{(n)}$ is convergent.

3. CIRCUIT STEADY STATE HARMONIC ANALYSIS

The time periodic sources are expanded into Fourier series, adopting the approximate values given by

$$s_{a_l k}(t) = (s_k)_0 + \sqrt{2} \sum_{m \in \Phi_l} \left((s'_k)_m \sin(m\omega t) + (s''_k)_m \cos(m\omega t) \right), \tag{8}$$

where Φ_l is a set of harmonics orders having weights higher than a lower limit χ_l. The limit weight χ_l is chosen such that the set to contain fewer harmonics than an imposed value μ_l. The approximation $s_{a_l} = Y(s)$, introduced by the limitation of sources values to a finite number of harmonics, accelerates the convergence of the iterative procedure as

$$\left\| s_{a_l} \right\|^2 = \sum_{k=1}^{p} \left((s_k)_0^2 + \sum_{m \in \Phi_l} \left((s'_k)_m^2 + (s''_k)_m^2 \right) \right) r_k \le \left\| s \right\|^2 \tag{9}$$

and, therefore, emphasizes the contraction of the approximate iterative procedure

$$s^{(n-1)} \xrightarrow{Y} s_{a_l}^{(n-1)} \xrightarrow{L} u^{(n)} \xrightarrow{G} s^{(n)} . \tag{10}$$

This property can be exploited starting with a small number of harmonics μ_1, then raising it up to a value which, obviously, can be less than the total spectrum of harmonics.

4. LINEAR CIRCUIT SOLUTION

In order to solve the linear circuit corresponding to each harmonic a phasor representation is used. Both DC and harmonic solutions are obtained using the modified nodal method. We have

$$(P + \mathrm{j}\omega_m Q)(X_m' + \mathrm{j}X_m'') = (B_m' + \mathrm{j}B_m''), \tag{11}$$

where $\omega_m = \omega \cdot m$, P is the nodes conductances matrix, bordered with values +1 and –1 associated whit the nodes that surrounds the inductances or ideal voltage sources, Q is the capacitances, the self and mutual inductances matrix, $\underline{X}_m = X_m' + \mathrm{j}X_m''$ is the unknown vector, formed by nodes potentials, inductances and ideal voltage sources currents and $\underline{B}_m = B_m' + \mathrm{j}B_m''$ is the vector of short circuit currents of nodes, bordered with the values of ideal voltage sources.
From equation (11) the following system results

$$PX_m' - \omega_m Q X_m'' = B_m', \tag{12′}$$

$$PX_m'' - \omega_m Q X_m' = B_m'', \tag{12″}$$

which can be written in the form

$$(I + \omega_m^2 P^{-1} Q P^{-1} Q) X_m' = P^{-1} B_m' + \omega_m P^{-1} Q P^{-1} B_m'', \tag{13′}$$

$$(I + \omega_m^2 P^{-1} Q P^{-1} Q) X_m'' = P^{-1} B_m'' - \omega_m P^{-1} Q P^{-1} B_m', \tag{13″}$$

where the matrices $M = P^{-1}$, $N = P^{-1} Q P^{-1}$ and $Z = P^{-1} Q P^{-1} Q$ are evaluated only once, before starting the iterative process. Only for the selected harmonics components we determine the nodes potentials that surrounds the ports of the nonlinear resistors and then the immitance matrix $\underline{W}_m = W_m' + \mathrm{j}W_m''$. We have

$$u_m' = W_m' s_m' - W_m'' s_m'' + u_{m0}', \tag{14′}$$

$$u_m'' = W_m' s_m'' + W_m'' s_m' + u_{m0}''. \tag{14″}$$

The immitance matrix is evaluated only once for each set of selected harmonics and are used for all iterations. The computation procedure consists of the following steps:

1. The data are introduced (*e.g.*: defining elements and nodes that surround them).
2. The matrices *P* and *Q* are numerically evaluated and stored.
3. The matrices *M, N* and *Z* are numerically evaluated and stored.
4. We adopt an imposed number of total harmonics that will be taken into account (μ_T) and a time step, smaller than the period of the greater harmonic.
5. The harmonic analysis of the independent sources is performed, retaining all the resulted harmonics.
6. The ports voltages u_0 (see equation (6)) are determined choosing a zero value for the equivalent sources.
7. The equivalent sources are corrected using (2), and the initial value $s^{(0)}$ is obtained.
8. The harmonic analysis of equivalent sources is performed, retaining a small number μ_1 of the harmonics which have the greater weight. For these harmonics the immitance matrix $\underline{W}_m$ is evaluated.
9. The iterations presented in relation (10) are made. Each harmonic component of the ports voltages is computed using (14), plus the corresponding harmonics of u_0. To correct the equivalent sources, ports voltages are determined in the time using (8), summing only harmonics from the set Φ_l. The harmonic analysis of equivalent sources is also done according to (8). Finally, we obtain the values of the equivalent sources with a convenient error.
10. Return to step 8, adopting a larger number of harmonics, having the greater weight.
11. The computation process is stopped at a convenient number of harmonics, much smaller than μ_l.

The Fourier decomposition of the selected harmonics is made using special techniques which severely reduce the computation time.

5. ILLUSTRATIVE EXAMPLE

We consider the circuit in Fig. 1, with $r_c = 1\text{k}\Omega$, $r_1 = 1\Omega$, $C = 1\mu\text{F}$, $L = 0.1\text{mH}$, $e(t) = 10\sin(2\pi f_1 t)\sin(2\pi f_2 t)$ V, $f_1 = 1\text{kHz}$, $f_2 = 1\text{MHz}$ and the diode *u-i* relationship given in Fig. 2.

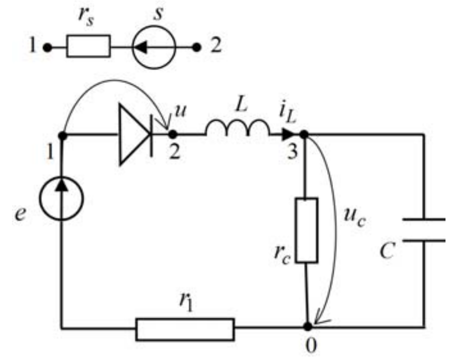


Fig. 1 – Nonlinear circuit.

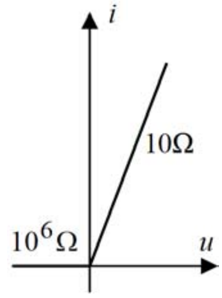


Fig. 2 – Diode *u-i* relationship.

The nonlinear element is replaced by a 10 Ω resistor in series with a voltage source, whose dependence on the diode branch voltage u is

$$s = G(u) = \begin{cases} 0, & \text{for} \quad u \geq 0 \\ 0.99999u, & \text{for} \quad u < 0 \end{cases}. \tag{15}$$

The matrices in (12) are

$$P = \begin{pmatrix} g_s + g_1 & -g_s & 0 & 0 \\ -g_s & g_s & 0 & 1 \\ 0 & 0 & g_c & -1 \\ 0 & 1 & -1 & 0 \end{pmatrix}, \quad Q = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & C & 0 \\ 0 & 0 & 0 & -L \end{pmatrix},$$
$$B_m' = \begin{pmatrix} s_{m1}' \\ -s_{m1}' \\ 0 \\ 0 \end{pmatrix}, \quad B_m'' = \begin{pmatrix} s_{m1}'' \\ -s_{m1}'' \\ 0 \\ 0 \end{pmatrix}, \quad X_m' = \begin{pmatrix} V_{m1}' \\ V_{m2}' \\ V_{m3}' \\ I_{Lm}' \end{pmatrix}, \quad X_m'' = \begin{pmatrix} V_{m1}'' \\ V_{m2}'' \\ V_{m3}'' \\ I_{Lm}'' \end{pmatrix}, \tag{16}$$

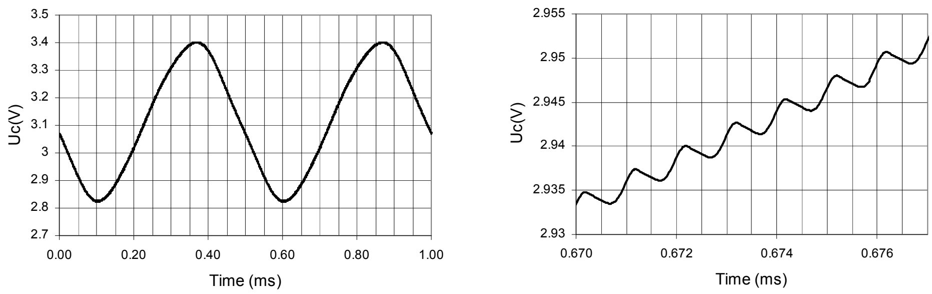
where $g = 1/r$. The total number of harmonics taken into consideration is μ_T = 8 000, and the number of equal time steps over a period 48 000. Three sets of harmonics with the most important weights are used. For each set the maximum number of harmonics (Table 1) is imposed. The weights are reported to the maximum value of the independent source $E_{\max} = 10V$. The imposed maximum error for stopping the iterations, reported to $E_{\max}$ is 10^{-4}.

Table 1

Different imposed number of harmonics
and details of the obtained results

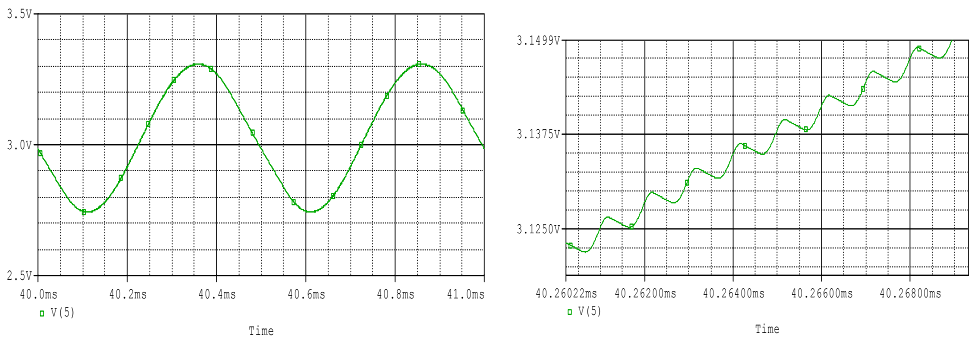
Selection	Imposed number of harmonics μ_l	Obtained number of harmonics Φ_l	Lower weight χ_l	Number of iterations
1	10	6	5.000×10^{-4}	344
2	50	47	4.500×10^{-4}	1 292
3	100	65	4.050×10^{-4}	1 964
4	200	73	3.645×10^{-4}	886

The capacitor voltage is presented in Fig. 3. In Fig. 4 the result obtained with a SPICE software for the same voltage and using the same imposed maximum error is shown.



(a) (b)

Fig. 3 – Capacitor voltage in the steady state, computed using the proposed method: a) over the period $T_1 = 1/f_1$; b) detail for $t \in [0.67,\ 0.677]$ ms..



(a) (b)

Fig. 4 – Capacitor voltage in the steady state, computed using SPICE: a) over the period $T_1 = 1/f_1$; b) Detail for $t \in [40.26,\ 40.269]$ ms.

6. CONCLUSIONS

This paper presents an efficient procedure for harmonic analysis of nonlinear circuits with resistive elements in steady state regime. This method allows the separation of the harmonics for solving the circuit at each iteration. Hence an important advantage results: transfer immitances between the control variables and the nonlinearly controlled sources are easily obtained based on three matrices which are determined only once, at the first iteration. The proposed procedure is recommended in cases when the circuit independent sources have very different frequencies. In these cases, the time domain analysis requires going through a large number of steps to achieve the asymptotic solution. The selection of harmonics with the largest weights permits the usage of a much smaller number of harmonics than the full spectrum and greatly reduces the computation effort. In the proposed illustrative example, although the number of harmonics in the spectrum is 8000, 73 harmonics were sufficient to obtain a good result.

Received on July 13, 2012

REFERENCES

1. K. Kundert, *Introduction to RF Simulation and its Application*, Journal of Solid State Circuits, **34**, *9*, pp. 1928–1320, 1999, updated on January, l, 2004, http://www.designers-guide.com.
2. F.I. Hantila, *A method for solving nonlinear resistive networks*, Rev. Roum. Sci. Techn., Electrotechn. et Energ., **24**, *2*, pp. 217–226, 1979.
3. C. Fluerasu, *RESEL – Simulation interactive sur ordinateur personnel des régimes transitoires dans les réseaux électriques. Deuxième partie : Algorithmes principaux*, Rev. Roum. Sci. Techn. – Electrotechn. et Energ., **36**, *2*, 1991.
4. * * *, *PAN simulator*, to be downloaded free from http://brambilla.elet.polimi.it
5. F. Constantinescu, M. Nitescu, *A multi-rate method for finding the periodic steady-state of nonlinear high-speed circuits*, Proceedings of ECCTD'99, pp. 767–770, 1999.
6. F.I. Hantila, F. Constantinescu, A. G. Gheorghe, M. Nitescu, M. Maricaru, *A new algorithm for frequency domain analysis of nonlinear circuits*, Rev. Roum. Sci. Techn. – Electrotechn. et Energ., **54**, *1*, pp. 57–66, 2009.