A GENETIC ALGORITHM-BASED APPROACH FOR A GENERAL STEADY-STATE ANALYSIS OF THREE-PHASE SELF-EXCITED INDUCTION GENERATOR

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Key words: Induction generator, Self-excitation, Genetic algorithm, Renewable energy.

This paper proposes a procedure for a steady-state analysis of a three-phase self-excited induction generator (SEIG) supplying balanced/unbalanced loads. The symmetrical component theory is used for the transformation of a complex three-phase generator-capacitance-load system to a simple equivalent passive circuit. A genetic algorithm (GA) has been employed for the determination of unknown variables by minimizing the impedance module of the equivalent circuit. This leads to a complete steady-state analysis of a three-phase SEIG for any combination of loads and excitation capacitances. By means of this procedure a 2.2 kW three-phase SEIG is analyzed under different load conditions.

1. INTRODUCTION

An induction generator can operate in parallel with the distribution network [1, 2]. Better utilization of renewable energy may be achieved by developing small-scale autonomous power systems like wind and mini hydro power plants, which use the SEIGs [3].

A huge number of journal publications has so far been dedicated to the steady-state analysis of the three-phase SEIG with three-phase balanced static load [5–12]. Analysis of the SEIG feeding power to a dynamic load (induction motor) is presented in [4], [13] and [14]. Steady-state performances of the SEIGs are usually determined on the basis of their equivalent circuits. An overwhelming majority of the aforementioned researchers used either the loop impedance approach [4], [5], [12] or the nodal admittance approach [6–11] for the determination of SEIGs performances from their equivalent circuits. Irrespective of the representation manner, the analysis of three-phase SEIGs requires a computation of the generated frequency (\(F\)) and magnetizing reactance (\(X_m\)) for the given operating conditions. Knowing the values of \(X_m\) and \(F\), the steady-state performances of the SEIGs can...
easily be determined through their equivalent circuits in conjunction with appropriate magnetization characteristics.

A steady-state analysis of three-phase SEIGs with static unbalanced loads has been presented in [15] and [16]. In order to determine the excitation frequency and magnetizing reactance, the pattern search minimization technique of Hooke and Jeeves is employed in reference [15]. Valid simplifying assumptions in the sequence equivalent circuit, which are applied in [16], make the final equation convenient in order to obtain the solution for $X_m$ and $F$.

In this paper, a GA-based optimization procedure is applied to the analysis of SEIG feeding balanced/unbalanced loads. The proposed approach enables practically all cases of unbalanced operation of the SEIG to be analyzed.

2. INDUCTION GENERATOR MODEL

If three-phase induction machines feed an unbalanced/asymmetrical external network, then a natural analysis mode is to resort the system to symmetrical components and rotating field concepts [3], [16]. Therefore, the induction generator is modelled with its positive and negative sequence circuits as it is shown in Fig. 1 [3].

![Sequence equivalent circuits](image)

Where $F$ is the per-unit frequency, $\nu$ is the per-unit speed, $R_s$ is the stator resistance per phase in p.u., $R_r$ is the rotor resistance per phase referred to the stator in p.u., $R_c$ is the core loss resistance per phase in p.u., $X_s$ is the stator reactance per phase in p.u., $X_r$ is the rotor reactance per phase referred to the stator in p.u., and $X_m$ is the magnetizing reactance per phase at the rated frequency in p.u.

In these circuits all parameters refer to the rated frequency, and it is assumed that all inductive reactances are proportional to the rated frequency; where $F$ is the per-unit frequency – the ratio of the generated frequency to the rated frequency, and $\nu$ is the ratio of the actual rotor speed to the synchronous speed corresponding to the rated frequency. In these circuit models, it is also assumed that all
parameters, except the magnetizing reactance $X_m$, are constant and independent of the saturation. The value of $X_m$ depends on the core flux which in turn depends on the ratio of the air-gap voltage to the frequency. The variation of the positive-sequence air-gap voltage with magnetizing reactance $X_m$ can be expressed by an appropriate equation based on the synchronous speed test data fitting. According to Fig. 1, the positive-sequence ($Z_p$) and negative-sequence ($Z_n$) impedances of the generator can be easily determined.

3. STEADY STATE EQUATIONS

The circuit connection of a three-phase delta connected SEIG feeding a three-phase unbalanced load is shown in Fig. 2. Assigning appropriate values for the terminal impedances $Z_A$, $Z_B$ and $Z_C$, a specific unbalanced operating condition can be simulated.

![Fig. 2 – Three-phase SEIG feeding a three-phase unbalanced load.](image1)

![Fig. 3 – A simplified circuit for the three-phase SEIG feeding a three-phase unbalanced load.](image2)

The delta connection is deliberately selected herein due to the fact that zero-sequence quantities do not exist. However, the presented concept can easily be extended to the star connection with the isolated neutral point applying the star-delta transformation. The circuit shown in Fig. 2 has no active voltage sources. Accordingly, the SEIG may be regarded as a passive circuit when viewed across any two stator terminals [15]. For convenience, A – phase is chosen as the reference and the input impedance of the SEIG across terminals 1 and 3 in Fig. 2 will be considered. In this manner, the equivalent scheme shown in Fig. 2 can be transformed into a simple circuit given in Fig. 3.

According to the marks given in Fig. 2, the current and voltage balance equations may be written as follows
Steady-state analysis of three-phase induction generator

\[ \begin{align*}
L_A + L_{LA} - L_B - L_{LB} &= 0, \\
L_A + L_{LA} - L_C - L_{LC} &= 0, \\
L_{LA} &= \frac{V_A}{Z_A} = Y_A Y_A, \\
L_{LB} &= \frac{V_B}{Z_B} = Y_B Y_B, \\
L_{LC} &= \frac{V_C}{Z_C} = Y_C Y_C. 
\end{align*} \]  \(3, 4, 5\)

Based on the symmetrical component theory, the following equations are developed for the delta connected system

\[ \begin{align*}
0 &= V_{VA} + V_{VB} + V_{VC}, \\
npA VVV &= +, \\
npB VVV &= 2, \\
npC VVV &= 2, \\
nppA YVYVI &= +, \\
nppB YVVCI &= 2, \\
nppC YVVCI &= 2. 
\end{align*} \]  \(7, 8, 9\)

where \( a \) is the complex operator \( e^{(j2\pi/3)} \).

This follows from equations (1) and (2)

\[ \begin{align*}
L_{LA} &= -L_A + L_B + L_{LB}, \\
L_{LA} &= -L_A + L_C + L_{LC}. 
\end{align*} \]  \(13\)

Substituting equations (4–5) and (10–12) into (13–14), it is obtained that

\[ \begin{align*}
L_{LA} &= A \cdot V_p + B \cdot V_n, \\
L_{LA} &= C \cdot V_p + D \cdot V_n, 
\end{align*} \]  \(15\)

where:

\[ \begin{align*}
A &= (a^2 - 1)V_p + a^2 V_n, \\
B &= (a - 1)V_n + aV_C, \\
C &= (a - 1)V_p + aV_C, \\
D &= \left( a^2 - 1 \right) V_n + a^2 V_C. 
\end{align*} \]

From equations (15) and (16), it is obtained that

\[ V_n = \frac{A - C}{D - B} V_p. \]  \(17\)

According with Fig. 3, the phase voltage \( V_A \) is

\[ V_A = -Z_{in} \cdot L_{LA}. \]  \(18\)

Finally, by substituting equations (7), (15) and (17) into equation (18), for the SEIG input impedance it is obtained that

\[ Z_{in} = \frac{(A + D) - (B + C)}{BC - AD}. \]  \(19\)
The positive-sequence and negative sequence admittances of the SEIG, that is, the admittances $Y_p$ and $Y_n$, are functions of the $F$ and $X_m$. Therefore, the SEIG input impedance may be expressed as

$$Z_{in} = R_{in}(X_m, F) + jX_{in}(X_m, F).$$

(20)

In accordance to Fig. 3, the following voltage balance equation can be written

$$Z_{loop}I_{eA} = 0,$$

(21)

where: $Z_{loop} = Z_A + Z_{in}$.

For a successful self-excitation it is required that $I_{eA} \neq 0$, hence

$$Z_{loop} = 0.$$  

(22)

4. SOLUTION METHOD

Equation (22) can be solved by using the numerical Newton-Raphson or polynomial method. The necessary mathematical manipulations and the lengthy derivations of nonlinear equations required for the unknown variables in the methods are tedious, time consuming and liable to human error. These disadvantages can be overcome by using the optimization techniques [9–11] and [15]. The genetic algorithm is one of the techniques, that is, one optimization procedure based on the natural evolution process imitation [17]. Through a series of GA operations a new population is obtained and its individuals are engendered by the individuals from the previous population according to the natural evolution principles: the choice of parents, crossover and mutation. Basic operations of the GA are:

1. Representation of individuals. All data (variables) that make an individual are written in a string. A string is composed of substrings. Each substring represents a binary encoded variable on which the process of optimisation is carried out.

2. Initialization. Individuals with random strings are generated that set up the initial population.

3. Fitness function calculation. It is used to rate the quality of an individual and it represents an equivalent of the function that should be optimised, that is, objective function.

4. Selection. During the selection process the individuals that will participate in the reproduction (parents) are selected. The point of the selection is to store and transfer good individuals to the next generation.
5. Crossover. The way in which coded column parts (substrings) are crossed over actually makes a GA. Crossover is an exchange process of column parts between two individuals, that is, "parents". One or two new individuals engender by the crossover, that is, a "child". The possibility of inheriting the first parent's characteristics by a child is introduced during this process.

6. Mutation. Mutation is a way to give a new piece of information to an individual. Mutation represents an accidental bit variation of an individual, generally with a constant probability for each bit within a population.

7. Ending conditions. The GA is an iterative process which ends when a maximum number of generations is achieved or when another criterion is fulfilled, such as a minimum offset from the best fitness value and medium fitness value of all individuals in a current population. Otherwise, return to 3.

In this paper, the MATLAB realization of the GA is applied. The GA parameters implemented in this paper are shown in Table 1. The GA enables a simultaneous computation of the $X_m$ and $F$ values for any given circuit configuration and operating conditions such as the speed, excitation capacitances, and load impedances. This is achieved by minimizing the impedance module $|Z_{loop}|$, which is a function of the unknowns ($X_m$ and $F$) and all other known parameters. The evaluation of the objective (fitness) function is performed by the MATLAB program based on the mathematical model that is presented in paragraphs 2 and 3. The objective function is

$$\min |Z_{loop}(X_m, F)| = \min \sqrt{\text{Re}(Z_{loop})^2 + \text{Im}(Z_{loop})^2},$$

with the constraints: $X_m^{\text{min}} \leq X_m \leq X_m^{\text{max}}$ and $F^{\text{min}} \leq F \leq F^{\text{max}}$.

Having thus determined the $X_m$ and $F$, the next step is to determine the normalised air-gap voltage $E_p/F$ for the value of $X_m$ based on the relevant magnetizing characteristic of a induction machine. The magnetizing characteristics relating the $E_p/F$ with the $X_m$ can be obtained experimentally by a synchronous speed test. Knowing the air-gap voltage $E_p$, the positive-sequence current and voltage, can be computed in accordance with Fig. 1.a. The negative-sequence voltage $E_n$ can be determined by Eq. (17). Finally, the terminal voltages and stator currents of the SEIG can be calculated by Eq. (7–9) and (10–12), respectively; and the line currents by Eq. (3–5).

Depending on the specific problem, in addition to the $X_m$ and $F$, the unknowns could be the excitation capacitance, rotor speed, and load impedance as well. For example, the voltage control of the SEIG can be achieved by a change in
the excitation capacitance for a constant value of the speed and load impedance. In this case the optimization problem can be expressed as the objective function:

$$\min \left[ \left| Z_{\text{loop}} (X_m, F, C) \right| + \left| V - V_{\text{spec}} \right| \right],$$

(24)

with the constraints: $X_{m}^{\text{min}} \leq X_m \leq X_{m}^{\text{max}}$, $F_{n}^{\text{min}} \leq F \leq F_{n}^{\text{max}}$, $C_{\text{min}} \leq C \leq C_{\text{max}}$, where $V$ is the terminal voltage, and $V_{\text{spec}}$ is the specified terminal voltage (usually $V_{\text{spec}} = 1 \text{ p.u.}$).

Table 1

<table>
<thead>
<tr>
<th>Generation</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Population</td>
<td>20</td>
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<tr>
<td>Initial population</td>
<td>Feasible population</td>
</tr>
<tr>
<td>Scaling function</td>
<td>Top, Quantity: 0.4</td>
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<tr>
<td>Selection</td>
<td>Tournament, Tournament K: 4</td>
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<tr>
<td>Crossover</td>
<td>Heuristic, Ratio: 1.2</td>
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<tr>
<td>Mutation</td>
<td>Adaptive feasible</td>
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<td>Ending conditions</td>
<td>Max generation: 100</td>
</tr>
<tr>
<td></td>
<td>Termination tolerance on the function value: $10^{-6}$</td>
</tr>
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</table>

5. SIMULATION RESULTS

The proposed procedure is tested on a three-phase, four pole, 50 Hz, delta connected, squirrel cage induction machine of 2.2 kW, 230 V, and 8.6 A. In order to verify the mathematical model and methodology, the results obtained herein for the three-phase SEIG with balanced resistive loads and the results of analysis presented in [11] are comparatively given in Table 2. The comparison confirms the validity and accuracy of the proposed method.

The procedure proposed in [11] is applicable only for three-phase balanced load. On the other hand, the method that is presented in this paper is general, because it is applicable to both balanced and the unbalanced load, as shown in Tables 3 and 4. It is considered that the generator is driven at a constant speed of 1.0 p.u. (that is, 1500 rpm) by a regulated prime mover.

Table 2

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
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</thead>
<tbody>
<tr>
<td></td>
<td>$C, R$ (µF, Ω)</td>
<td>$v$ (p.u.)</td>
<td>Computed results</td>
<td>Experimental results</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$F$ (p.u.)</td>
<td>$V$ (p.u.)</td>
</tr>
<tr>
<td>1</td>
<td>36,160</td>
<td>0.95333</td>
<td>0.934899</td>
<td>0.583202</td>
</tr>
<tr>
<td>2</td>
<td>36,160</td>
<td>1.04666</td>
<td>1.033292</td>
<td>0.942325</td>
</tr>
<tr>
<td>3</td>
<td>51,220</td>
<td>0.85666</td>
<td>0.848210</td>
<td>0.765084</td>
</tr>
</tbody>
</table>
Table 3
Case studies of the three-phase SEIG for the different circuit configurations

<table>
<thead>
<tr>
<th>Case</th>
<th>$Z_A = R_A/C_A$ ($\Omega$)</th>
<th>$C_A$ (µF)</th>
<th>$Z_B = R_B/C_B$ ($\Omega$)</th>
<th>$C_B$ (µF)</th>
<th>$Z_C = R_C/C_C$ ($\Omega$)</th>
<th>$C_C$ (µF)</th>
<th>Connection</th>
<th>Results</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>150</td>
<td>50</td>
<td>150</td>
<td>50</td>
<td>150</td>
<td>50</td>
<td>Bal. three-ph.</td>
<td>0.9854</td>
</tr>
<tr>
<td>2</td>
<td>100</td>
<td>120</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>Plain sing.-ph.</td>
<td>0.9893</td>
</tr>
<tr>
<td>3</td>
<td>100</td>
<td>50</td>
<td>-</td>
<td>100</td>
<td>-</td>
<td>-</td>
<td>C-2C</td>
<td>0.9907</td>
</tr>
</tbody>
</table>

Table 4
Performance of the SEIG for the circuit configurations listed in Table 2

<table>
<thead>
<tr>
<th>Case</th>
<th>Phase voltages</th>
<th>Phase currents</th>
<th>Voltage unbalanced factor</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$V_A$ (p.u.)</td>
<td>$I_A$ (p.u.)</td>
<td>$V_B$ (p.u.)</td>
</tr>
<tr>
<td>1</td>
<td>1.0675</td>
<td>0.8342</td>
<td>1.0675</td>
</tr>
<tr>
<td>2</td>
<td>1.1154</td>
<td>0.6621</td>
<td>1.0512</td>
</tr>
<tr>
<td>3</td>
<td>1.1790</td>
<td>0.9067</td>
<td>1.0652</td>
</tr>
</tbody>
</table>

Fig. 4 – Excitation capacitance versus load current for $V = 1$ p.u. for: a) balanced three-phase connection; b) plain single-phase connection; c) C-2C connection.

Fig. 5 – Variation of $F$, $VUF$ and $\eta$ with load current for $V = 1$ p.u. for: a) balanced three-phase connection; b) plain single-phase connection; c) C-2C connection.

Fig. 4 shows the variation of the excitation capacitance required to maintain the constant terminal voltage of $V = 1$ p.u. with the change in the resistive load current at the constant generator speed of 1.0 p.u. The excitation capacitance has to vary continuously in order to keep the machine terminal voltage at a constant value. The excitation capacitance requirement increases: (a) from 38 µF at no load to 75 µF at 1 p.u. load current for the balanced three-phase connection, (b) from
96 µF to 130 µF for the plain single-phase connection, and (c) from 35.3 µF to 45 µF for the C-2C connection.

The variations of frequency, voltage unbalance factor and efficiency with load current at the corresponding values of the excitation capacitance for the constant terminal voltage of \( V = 1 \) p.u. are shown in Fig. 5. With an increase in load, the frequency decreases from 0.9991 p.u. at no load to 0.9561 p.u. at the nominal load for the balanced three-phase connection, from 0.9982 p.u. to 0.9805 p.u. and from 0.9987 p.u. to 0.9848 p.u. for the plain single-phase, and C-2C connection, respectively. The change in frequency is the highest in the case of the balanced three-phase connection due to the highest value of the load. The voltage unbalance factor increases from 0.1135 p.u. at no load to 0.1672 p.u. at the nominal load for the plain single-phase connection, and decreases from 0.0732 p.u. at no load to 0.0092 p.u. at the nominal load for the C-2C connection. It is obvious that the C-2C connection has better performances than the plain single-phase connection.

6. CONCLUSION

In this paper a procedure for general steady-state analysis of three-phase SEIGs feeding balanced/unbalanced loads has been presented. The proposed methodology is general since it can be applied to different configurations of excitation capacitances and consumers. Application of GA gives great freedom in the choice of objective function and control variables in the analysis of different modes of AG, which is the main advantage of the proposed procedure over other methodologies to deal with this problem. The voltage control of the SEIG feeding balanced/unbalanced loads has been investigated as well. The methodology proposed in this paper can be exploited in planning and development of power generation in autonomous small-scale hydro units with the SEIGs.

APPENDIX

The details of induction generator: three-phase, four-pole, 50 Hz, delta connected, squirrel cage induction machine, 2.2 kW, 230 V.

Parameters: \( R_s = 0.07224 \) (p.u.), \( R_r = 0.03795 \) (p.u.), \( X_s = X_r = 0.10459 \) (p.u.).

Variation of air gap voltage with magnetizing reactance at rated frequency:

\[
X_a < 1.77464 \Rightarrow \frac{E_a}{F} = 1.49744 - 0.007X_a ; \\
2.06096 > X_a \geq 1.77464 \Rightarrow \frac{E_a}{F} = 2.02226 - 0.01338X_a ; \\
2.32904 > X_a \geq 2.06096 \Rightarrow \frac{E_a}{F} = 2.52129 - 0.0186X_a ; \\
X_a \geq 2.32904 \Rightarrow \frac{E_a}{F} = 0. \]
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