DECENTRALIZED ROBUST SLIDING MODE CONTROL
FOR A CLASS OF INTERCONNECTED NONLINEAR SYSTEMS
WITH STRONG INTERCONNECTIONS

YACINE DEIA1, MADJID KIDOUCHE1, MOHAMED BECHERIF2

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In this paper, a new decentralized fuzzy sliding mode control (DFSMC) strategy for a class of large-scale nonlinear systems (LSNS) with strong unknown interconnections is proposed. The main objective of our contribution is to reduce the used switching gains in the decentralized sliding mode controller (DSMC) to decrease the chattering amplitude in the presence of strong interconnections in each sub-system. To achieve this objective, the global system is decomposed into sub-systems, then, two fuzzy logic approximations are constructed, the first one is used to approximate the unknown interconnections terms in order to provide a better system modelling, and the second is used to generate the discontinuous part of the control law in the (DSMC). Finally, a Quadrotor attitude angle is presented to show the effectiveness of the proposed control strategy.

1. INTRODUCTION

In the centralized control theory, it is well-known that the controlled system is driven by a single controller that has all the available information about the system, with the technologies development, most of the industrial systems become complex, that is they lead to many difficulties concerning the tasks of analyzing, designing, and implementation of control strategies due to the well-known reasons [1]: dimensionality, information structure constraints, uncertainty, and delays.

Many methods are proposed to prove the global stability of the whole system like the absolute stability [2], relative stability [3] and a construction of Lyapunov function for large scale systems is proposed by [4].

In the last few decades, there have been large interesting works addressed to the development of large scale systems theory [5–8], the main treated problem in this theory is how to decompose a given control problem to smaller sub-problems weakly interconnected one to each other that can be treated separately; as a result, the global system is not any more controlled by a one centralized controller but by many separated controllers represent together a decentralized controller.

Earlier versions of the sliding mode control methods [9–12] were focused on multi-inputs multi-outputs (MIMO) and large-scale systems control where all information about the system are available.

Decentralized sliding mode control has been developed by many works [12–15], even so, strict conditions are imposed on sub-systems with some assumptions on the interconnections (as the linearity assumptions, assumed bounded…), but in facts, most of the physical systems are natively nonlinear and some information about the system are not always available.

On the other hand, the used switching gain values in the DSMC depend on the bounds of uncertainties and interconnections in the system thus, systems with large uncertainties and interconnections bounds need higher switching gain and that produce a higher chattering amplitude in control signals, the boundary layer (BL) Approach is proposed in [16, 17] to reduce the chattering amplitude by replacing the discontinuous function sign by a continuous one but this solution can drive the system states only to the boundary layer but not to the sliding surface, as consequences, system with large uncertainties and interconnections need a larger boundary layer bound size due to large interconnections, and if we continue increasing the boundary layer thickness, we are pushing the control system to a system with no sliding mode.

In [18], a neural network is used to predict the inter-connections among sub-systems to solve this problem the sub-systems outputs have driven with success to a desired trajectory but the proposed adaptation mechanism is so slow to adapt the control law with the new conditions (new set point, uncertainty…) and this is not tolerated in too many applications.

In this work, a new control design based on the combination of the sliding mode with the fuzzy logic to control a class of nonlinear interconnected systems with strong interconnections is proposed, the designed control law is divided into two parts: the equivalent and the robust (discontinuous) one [17]; the fuzzy identification capability is used to approximate the unknown interconnection in each sub-system; thus the robust part is responsible to compensate only the system uncertainties and the fuzzy approximation error which means a lower switching gains to be used, then the robust part of the DSMC is generated by a fuzzy logic controller in order to eliminate the chattering phenomena.

This paper is organized in five sections starting with an introduction, in section two, the problem of a class of large scale nonlinear systems is formulated, the proposed DFSMC design methods are developed in section three, in section four, the simulation results show the effectiveness of the proposed control design and, finally, a conclusion is presented in the last section.

2. PROBLEM FORMULATION

Let us consider the large-scale nonlinear system composed of N interconnected sub-systems, each sub-system \( S_i \) is described as:

\[
\begin{align*}
\dot{x}_{id} &= f(x_i) + b_i u_i + \Delta_i (x_1, x_2, ..., x_N) \\
y_i &= x_{i1}
\end{align*}
\]
where \( x_i = [x_{i1}, x_{i2}, \ldots, x_{il}]^T \) is the state vector, \( u_i \in \mathbb{R} \) is the control signal input and \( y_i \in \mathbb{R} \) is the output of the sub-system \( S_i \).

It is assumed that the nominal part \( f_{ni}(x_i) \) of the nonlinear function \( f(x_i) = f_{ni}(x_i) + \Delta f(x_i) \) is known and the interconnections \( \Delta(x) \in \mathbb{R} \), among sub-systems \((i = 1, 2, \ldots, N) \) are not available, these interconnections are considered as weak interconnections if they could be taken as system disturbances in each sub-system in the controller synthesis; and they considered as strong if they are large enough to impact the control and stabilization of each sub-system.

To this end, we consider the following assumption:

**Assumption 1.** For the sub-system described in (1) the unknown part \( \Delta f(x) \) of the nonlinear function \( f(x) \) is bounded by a positive value, that:

\[
|\Delta f(x_i)| \leq F_{in} .
\]  

(2)

**Assumption 2.** For the sub-system described by (1) the unknown interconnections \( \Delta(x) \) are bounded by a positive value, that:

\[
|\Delta_i(x)| \leq I_{in} .
\]  

(3)

Assume that the given reference \( y_{ir} \) is bounded and has up to \( d_i - 1 \) bounded derivatives. The desired output vector is:

\[
Y_i = [y_{ir}, y_{ir}, y_{ir}, \ldots, y_{ir}^{(d_i - 1)}]^T .
\]

The tracking error of the \( i^{th} \) sub-system is defined as:

\[
e_i = y_i - y_{ir} .
\]  

(4.a)

Then the error vector of the \( i^{th} \) sub-system is given by:

\[
e_i = [e_i, e_i, \ldots, e_i^{(d_i - 1)}]^T .
\]  

(4.b)

Our objective is to design a decentralized fuzzy sliding mode controller for each sub-system which will drive the output \( y_i \) to track a desired output trajectory \( y_{ir} \) (i.e: \( e_i \to 0 \)) in the presence of strong unknown interconnections and uncertainties using only a local measurements.

The sliding variable for the \( i^{th} \) sub-system is defined as [19]:

\[
S_i = \sum_{j=0}^{d_i-1} \lambda_{ij} e_i^{(j)} ,
\]  

(5)

with \( \lambda_{i0} = 1 \) and the selection of \( \lambda_{ij} \) must satisfy the following Hurwitz polynomial [19]:

\[
p_i^{d_i} + \sum_{j=0}^{d_i-1} \lambda_{ij} p_i^j = 0 .
\]  

(6)

### 3. CONTROL DESIGN

It is assumed that the interconnections are unknown and the proposed control design must compensate them and drive the system outputs to a desired trajectory; to achieve that, a fuzzy approximation is used to approximate the interconnections \( \Delta_i(x_i) \) in each sub-system.

The sliding variable derivative is:

\[
\dot{S}_i = e_i^{(d_i)} + \sum_{j=0}^{d_i-1} \lambda_{ij} e_i^{(j+1)} = f_i(x_i) + \Delta_i(x_i) + b_i u_i + \sum_{j=0}^{d_i-1} \lambda_{ij} e_i^{(j+1)} .
\]  

(7)

To ensure the existence of the sliding mode and its reachability occurrence in finite time, the control laws \( u_i \) must satisfy the \( \eta \) reachability condition given by [17]:

\[
S_i \dot{S}_i < -\eta ||S_i|| ,
\]  

(8)

with \( \eta \) is a small positive constant.—

The decentralized sliding mode control law that satisfies (8) can be given by:

\[
u_i = \frac{1}{b_i} \left[ -f_i(x_i) - \sum_{j=0}^{d_i-1} \lambda_{ij} e_i^{(j+1)} - k_i \text{sign}(S_i) \right] ,
\]  

(9)

and the positive switching gain \( k_i \) must be chosen large enough to compensate the uncertainties and the interconnections terms, so \( k_i \) must satisfy the following:

\[
k_i \geq F_{im} + I_{im} + \eta .
\]  

(11)

In this case, when the uncertainties and the interconnection terms are large, \( k_i \) become large and produce a higher chattering amplitude, the standard sliding mode with boundary layer can solve only the problem of system with small uncertainties and interconnections because the thickness of the boundary layer is proportional to the chattering amplitude and if we continue increasing the boundary layer thickness, we actually drive the system states to a system with no sliding mode.

For the system with large uncertainties and interconnections, this control design is proposed, a fuzzy logic is used to approximate the interconnections \( \Delta_i(x_i) \) using only local information and a fuzzy logic controller is used to approximate the sign function by a smoothing way in order to reduce the switching frequency.

Let us denote the fuzzy approximation error by:

\[
e_{\Delta, i}(x_i) = \Delta_i(x_i) - \hat{\Delta}_i(x_i) \quad \text{with} \quad \left| e_{\Delta, i}(x_i) \right| < e_{\Delta, \text{um}} ,
\]  

(12)

where \( e_{\Delta, \text{um}} \) is the upper bound of the fuzzy approximation error and \( \hat{\Delta}_i(x_i) \) is the approximated interconnection for the \( i^{th} \) sub-system.

### 3.1.FUZZY SYSTEMS

The interconnections \( \hat{\Delta}_i(x_i) \) among the sub-systems are considered as a MISO Takagi-Sugeno fuzzy system mapping an input vector \( z = [z_1, z_2, \ldots, z_N]^T \in U \in \mathbb{R}^n \) to a scalar output \( y_f \in \mathbb{R} \) with \( U = U_1 \times U_2 \times U_3 \times \ldots \times U_n \),
Decentralized robust control for an interconnected systems class

Let \( \mu_{A_k}(z_i) \) be the membership function associated to fuzzy set \( A_k \), the choice of this membership function is based on C-means clustering algorithm of an experimental input/output data vectors; as result of this algorithm, a Gaussian membership function is constructed.

The output can be rewritten as:

\[
y_i = w^T(z)\Theta_i,
\]

where \( \Theta_i = [\mu_{f_1}, \mu_{f_2}, \ldots, \mu_{f_p}]^T \) is the vector of fuzzy system parameters, and \( w(z) = [w_1(z), w_2(z), \ldots, w_p(z)]^T \) is a basis function defined as [20]:

\[
w_i(z) = \mu_i(z) / \sum_{i=1}^{p} \mu_i(z).
\]

In sliding mode control, the control signal is divided into two components: equivalent \( u_{ie} \) and robust \( \Delta u_{ir} \).

\[
u_i = u_{ie} + \Delta u_{ir}.
\]

In this paper, the robust component of the controls laws is generated by a Takagi-Sugeno fuzzy logic controller (FLC) [21] with the surface \( S_i \) as an input and the robust part of the control signal \( \Delta u_{ir} \) as an output [22] as shown in Fig. 1. For each \( i^{th} \) controller, the membership functions of the two variables \( S_i \) and \( \Delta u_{ir} \) are illustrated in Fig. 2.

Fig. 1 – Fuzzy sliding mode structure of the \( i^{th} \) controller.

Fig. 2 – Membership functions of \( S_i \) and \( \Delta u_{ir} \), where \( \Delta u_{ir} \) is the FLC output.

The rules of the fuzzy controller are given by:

\[
R_j: \text{If } s \text{ is } A^j \text{ THEN } u \text{ is } B^j,
\]

where \( s \) is the input variable, \( A^j \) and \( B^j \) are fuzzy sets defined on \( U_i \) and \( Y_j \) are constants.

The output can be expressed as:

\[
y_f = \sum_{k=1}^{p} \mu_k(z) y_f^k = \sum_{k=1}^{p} \mu_k(z),
\]

where

\[
\mu_k(z) = \prod_{i=1}^{k} \mu_{A_k}(z_i)
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\]

where

\[
\mu_k(z) = \prod_{i=1}^{k} \mu_{A_k}(z_i)
\]
By using the inequalities (2) and (12) we obtained:
\[ \dot{V}_i(S_i) < \left| S_i \left( F_{im} + e_{i\Delta m} - k_i \Delta u_i \right) \right| \]
\[ \dot{V}_i(S_i) < \left| S_i \left( F_{im} + e_{i\Delta m} - k_i \Delta u_i \right) \right|. \]  
(25)

We have \( -1 \leq \Delta u_i \leq 1 \) that means \( |\Delta u_i| \leq 1 \) thus:
\[ \dot{V}_i(S_i) < \left| S_i \left( F_{im} + e_{i\Delta m} - k_i \right) \right|. \]  
(26)

By comparing the inequality (26) with the \( \eta \)-reachability condition given in (8) the gain \( k_i \) must satisfy the following:
\[ k_i \geq F_{im} + e_{i\Delta m} + \eta. \]  
(27)

The switching gain \( k_i \) must compensate the uncertainties \( \Delta f_i \) and the interconnections approximation error \( e_{i\Delta m} \). Therefore, by applying (27), the convergence of the tracking error to zero in finite time is guaranteed.

4. SIMULATION RESULTS

To show the effectiveness of the proposed control design a quadrotor attitude angles is considered, this system is composed of three strongly interconnected angles [23] as shown in Fig 3.

The equations that describe the motion of the Quadrotor attitude after the system decomposition are given by [24]:

\[
\begin{aligned}
S_1 : & \quad \dot{x}_{11} = x_{12} \\
& \quad \dot{x}_{12} = \left( \frac{-K_{a}}{J_{a}} \right) x_{12}^2 + \left( \frac{l}{J_{x}} \right) u_{i1} + \\
& \quad + \left( \frac{J_{y} - J_{z}}{J_{x}} \right) x_{22} x_{32} + \left( \frac{J_{y}}{J_{x}} \right) x_{32} \Omega,
\end{aligned}
\]  
(28.a)

\[
\begin{aligned}
S_2 : & \quad \dot{x}_{21} = x_{22} \\
& \quad \dot{x}_{22} = \left( \frac{-K_{a}}{J_{a}} \right) x_{22}^2 + \left( \frac{l}{J_{y}} \right) u_{i2} + \\
& \quad + \left( \frac{J_{z} - J_{y}}{J_{y}} \right) x_{12} x_{32} + \left( \frac{J_{z}}{J_{y}} \right) x_{12} \Omega,
\end{aligned}
\]  
(28.b)

\[
\begin{aligned}
S_3 : & \quad \dot{x}_{31} = x_{32} \\
& \quad \dot{x}_{32} = \left( \frac{-K_{a}}{J_{a}} \right) x_{32}^2 + \left( \frac{l}{J_{z}} \right) u_{i3} + \\
& \quad + \left( \frac{J_{x} - J_{z}}{J_{z}} \right) x_{12} x_{22},
\end{aligned}
\]  
(28.c)

where the system outputs \( y_1 = x_{11} = \phi \), \( y_2 = x_{21} = \theta \) and \( y_3 = x_{31} = \psi \) are the roll, the pitch and the yaw angles respectively. \( u_i \) are the control signals. \( J_{x}, J_{y}, J_{z}, J_{a} \) are the system inertia parameters, and \( \Omega = w_1 - w_2 + w_3 - w_4 \) where \( w_i \) is the angular speed of the \( i^{th} \) rotor (\( i = 1–4 \)).

The simulation results are obtained by using MATLAB/SIMULINK 7, the initial values of the system states are given by:
\[
(x_{11}, x_{12}, x_{21}, x_{31}, x_{32})^T = (0.5, 0, 0.5, 0, 1, 0)
\]

The model parameter values of the Quadrotor system are adopted from [24] and listed in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>( g )</td>
<td>Gravity</td>
<td>9.81</td>
<td>m/s²</td>
</tr>
<tr>
<td>( m )</td>
<td>Mass</td>
<td>0.5</td>
<td>kg</td>
</tr>
<tr>
<td>( l )</td>
<td>Distance</td>
<td>0.2</td>
<td>m</td>
</tr>
<tr>
<td>( J_{x} )</td>
<td>Roll inertia</td>
<td>4.85×10⁻³</td>
<td>kg·m²</td>
</tr>
<tr>
<td>( J_{y} )</td>
<td>Pitch inertia</td>
<td>4.85×10⁻³</td>
<td>kg·m²</td>
</tr>
<tr>
<td>( J_{z} )</td>
<td>Yaw inertia</td>
<td>8.81×10⁻³</td>
<td>kg·m²</td>
</tr>
<tr>
<td>( K_{a} )</td>
<td>Aerodynamic factor (( X, Y ))</td>
<td>5.5670×10⁻⁴</td>
<td>/</td>
</tr>
<tr>
<td>( K_{z} )</td>
<td>Aerodynamic factor (( Z ))</td>
<td>6.3540×10⁻²</td>
<td>/</td>
</tr>
<tr>
<td>( J_{a} )</td>
<td>Rotor inertia</td>
<td>2.8385×10⁻⁵</td>
<td>kg·m²</td>
</tr>
</tbody>
</table>

Figure 4 shows the simulation results of the proposed control design. The desired positions are \( x_{11}, x_{21}, x_{31} = (0; 0; 0) \), it clearly observed in Figs. 4a–c, that the proposed DFSMC can drive the system outputs \( x_{11}, x_{21}, x_{31} \) to the desired position in finite time.

The used switching gains in each controller \( u_{i1}, u_{i2}, u_{i3} \) are \( k_1, k_2, k_3 = (1; 1.1; 1.19) \) respectively; on the other hand, to achieve the same performances in the classical sliding mode control the switching gains must be increased to \( k_1, k_2, k_3 = (12; 11.7; 13.1) \).
Figures 4d–f show the control signal for each sub-system. Those figures show that the proposed control design has no chattering in the control signals $U_{ij}$.

In order to show the effectiveness of the proposed control design, we try to stabilize the roll angle ($x_{11}$) = (0) in the presence of large variation (i.e. large interconnections) in the two others sub-systems with the trajectory ($x_{21}$, $x_{31}$) = ($\sin(2.07t)$, $3\sin(0.75t)$), the simulations results are shown in Fig. 5.

Figure 5 show that the proposed DFSMC design achieves good performances in the presence of large variations in $x_{21}$, $x_{31}$ by using the same switching gains shown above, in the other part, as can be seen from the simulation results of Fig. 5a, the standard decentralized sliding mode control (the dotted line in Fig. 5a) failed to control the system in the presence of these large variations.

Figures 5d–f show the control signals for each sub-system, those figures show that the proposed control design has no chattering in the control signals $U_{ij}$.
5. CONCLUSION

In this paper, a decentralized fuzzy sliding mode control for a class of large-scale nonlinear interconnected systems with strong interconnections is presented. The designed method is a combination of sliding mode control with a fuzzy logic. The latter is employed to approximate the unknown interconnection terms, and to provide a lower switching gain despite the existence of uncertainties and interconnections. As a result, the convergence of the tracking errors to zero in finite times is obtained without chattering behavior in the control signals. The simulation results prove that the proposed DFSMC can achieve good performances compared to the classical sliding mode control. Another advantage of the proposed design is that the used switching gains are reduced without losing the desired systems performances.

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