

PERFORMANCE ANALYSIS OF COOPERATIVE SYSTEM IN NAKAGAMI-M FADING CHANNEL

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Key words: Amplify-and-forward relay, Nakagami-M fading, Co-channel interference, Outage probability, Average bit error rate.

In this paper we consider performance of a cooperative system consisting of dual hop amplify-and-forward relay system. We consider channel state information assisted system in Nakagami-m channel. Also, within the analysis we considered the existence of co-channel interference at relay. The influence of the interference, as well as the influence of other system's parameters, such as the fading parameter m and outage probability threshold γ_{th} on the system's performance is considered.

1. INTRODUCTION

Dual-hop relaying communication has a number of advantages over direct-link transmission in terms of connectivity, power saving, and channel capacity for the high data-rate coverage required in future cellular and ad-hoc networks. Relay processing can be classified as either amplify-and-forward (AF) or decode-and-forward (DF). There are two types of AF relaying schemes considering two different power constraints at the relay: fixed-gain [1], also called APS in [2] and channel state information (CSI)-based, also called IPS in [2]. Although AF relaying is well studied in literature [1–5], for example, most of the existing work assumed only thermal noise, without any interference. However, it was shown in [6] that the interference can cause a severe performance degradation.

Few works that have studied the impact of interference on the AF and the decode-and-forward (DF) relaying performance have assumed interference either at the relay(s) or the destination(s) [6, 7]. In [6], an AF cooperative strategy for interference limited networks is considered. Their asymptotic analysis has shown that interference limits the diversity gain of the system and the related channel capacity is bounded by a stationary point. The authors of [7], have analyzed the performance gains of a half-duplex multi-user cooperative network where the relay

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destination slot is reused. Thus that slot becomes an interfering scenario at each destination. It was shown that cooperative transmission is able to provide significant gains in terms of throughput over the direct transmission under round-robin packet scheduling and uniform distribution of terminals by controlling the rate of each transmission. In [8] and [9], the performance of DF opportunistic relaying in the presence of interference has been investigated. In [10], the outage probability of a fixed gain AF relay system in Rayleigh fading with interference at the destination has been investigated. Recently, in [11] authors studied the outage probability and the average bit error rate (BER) of the CSI-assisted AF protocol with interference at the relay in Rayleigh fading channel.

In this paper we consider performance of a cooperative dual hop amplify-and-forward relay system. We considered a multiple interferer scenario at relay and derived the outage probability and the average BER expressions. The influence of the interference, as well as the influence of other system's parameters, such as the fading parameter m and outage probability threshold γ_{th} on the system's performance is considered.

2. SYSTEM MODEL

We consider a communication system, where a source S communicates with a destination D through a relay R , as shown in Fig. 1 [11]. All nodes are equipped with a single antenna. It is assumed that S does not have a direct link to D . The communication in the system is divided into two orthogonal time intervals. In the first time interval, S sends its symbol s_0 to R . In the second time interval, R communicates with D . It is assumed that R operates in an interference limited environment, and that the interference at D is negligible.

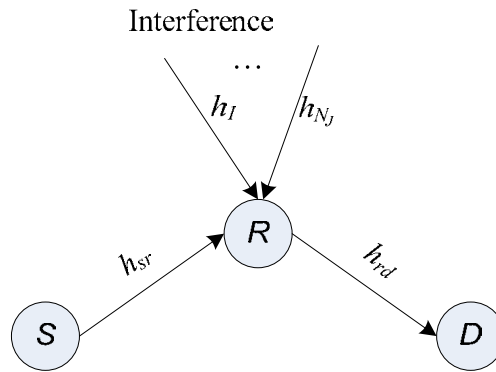


Fig. 1 – Model of the considered communication system.

At R , the received signal in the presence of N_J interferers, each with an average power P_j , and additive white Gaussian noise (AWGN) can be expressed as

$$y_r = \sqrt{P_s} h_{sr} s_0 + \sum_{j=1}^{N_J} \sqrt{P_j} h_j s_j + n_r, \quad (1)$$

where P_s is the transmit power, h_{sr} is the complex channel between S and R with average fading power Ω_{sr} , h_j ($j = 1, \dots, N_J$) is the channel from interference to R with average fading power Ω_j independent of h_{sr} , the transmitted symbols, s_0 and interfering symbols s_j are assumed to have zero mean and unit variance and n_r is the AWGN at R with variance σ_r^2 . N_J is the number of interfering signals. All links are assumed to be subject to Nakagami- m fading.

In the second time interval, R forwards y_r to D after multiplying it with gain G . Hence, the received signal at D is given by

$$y_d = h_{rd} G \left(\sqrt{P_s} h_{sr} s_0 + \sum_{j=1}^{N_J} \sqrt{P_j} h_j s_j + n_r \right) + n_d, \quad (2)$$

where n_d is the AWGN at D with variance σ_d^2 , and h_{rd} is the complex channel between R and D with average fading power Ω_{rd} , independent of h_{sr} and h_j . In the presence of interference, G can be expressed as [6]

$$G = \frac{\sqrt{P_r}}{\sqrt{P_s |h_{sr}|^2 + \sum_{j=1}^{N_J} P_j |h_j|^2 + \sigma_r^2}}. \quad (3)$$

As a result, the SINR of the decision variable can be written as

$$\gamma_{eq} = \frac{P_s |h_{sr}|^2 |h_{rd}|^2}{|h_{rd}|^2 \left(\sum_{j=1}^{N_J} P_j |h_j|^2 + \sigma_r^2 \right) + \sigma_d^2 / G^2}. \quad (4)$$

Having in mind that the effect of n_r can be neglected, by from (3) and (4) we get SINR as [11]

$$\gamma_{eq1} = \frac{\gamma_1 \gamma_2}{\gamma_{INF} (\gamma_2 + 1) + \gamma_1}, \quad (5)$$

where $\gamma_1 = P_s |h_{sr}|^2$, $\gamma_2 = \frac{P_r}{\sigma_d^2} |h_{rd}|^2$, and $\gamma_{INF} = \sum_{j=1}^{N_J} P_j |h_j|^2$.

3. PERFORMANCE ANALYSIS

The outage probability and the average bit error rate will be determined for the considered cooperative system.

The outage probability, P_o is defined as the probability that γ_{eq1} drops below an acceptable SNR threshold γ_{th}

$$P_o = \Pr(\gamma_{eq1} < \gamma_{th}) = F_{\gamma_{eq1}}(\gamma_{th}), \quad (6)$$

where $\Pr(\cdot)$ denotes probability, and $F_{\gamma_{eq1}}(x)$ is the cumulative distribution function (cdf) of random variable γ_{eq1} which may be written as [11]

$$F_{\gamma_{eq1}}(\gamma_{th}) = 1 - \int_0^\infty \int_0^\infty \Pr\left(\gamma_1 > \frac{\gamma_{th}(w + \gamma_{th} + 1)z}{w}\right) \times \\ \times f_{\gamma_2}(w + \gamma_{th}) f_{\gamma_{INF}}(z) dw dz, \quad (7)$$

where $f_{\gamma_2}(x)$, and $f_{\gamma_{INF}}(x)$ are probability density functions (pdf) of γ_2 and γ_{INF} , respectively. Since h_{sr} , h_{rd} , and h_j are Nakagami-M random variables, γ_1 , γ_2 and γ_{INF} are Gamma random variables with the following pdfs

$$f_{\gamma_1}(x) = \frac{m_1^{m_1} x^{m_1-1}}{\Gamma(m_1) \bar{\gamma}_1^{m_1}} e^{-\frac{x \cdot m_1}{\bar{\gamma}_1}}, \quad (8)$$

$$f_{\gamma_2}(x) = \frac{m_2^{m_2} x^{m_2-1}}{\Gamma(m_2) \bar{\gamma}_2^{m_2}} e^{-\frac{x \cdot m_2}{\bar{\gamma}_2}}, \quad (9)$$

$$f_{\gamma_{INF}}(x) = \frac{m_3^{N_J m_3} x^{N_J m_3-1}}{\Gamma(N_J m_3) \bar{\gamma}_{INF}^{N_J m_3}} e^{-\frac{x \cdot m_3}{\bar{\gamma}_{INF}}}, \quad (10)$$

where $\bar{\gamma}_1 = P_s \Omega_{sr}$, $\bar{\gamma}_2 = P_r \Omega_{rd} / \sigma_d^2$, and $\bar{\gamma}_{INF} = \sum_{j=1}^{N_J} P_j \Omega_j$. Their cdfs are:

$$F_{\gamma_1}(x) = \frac{\gamma(m_1, x \cdot m_1 / \bar{\gamma}_1)}{\Gamma(m_1)}, \quad (11)$$

$$F_{\gamma_2}(x) = \frac{\gamma(m_2, x \cdot m_2 / \bar{\gamma}_2)}{\Gamma(m_2)}, \quad (12)$$

$$F_{\gamma_{INF}}(x) = \frac{\gamma(N_J m_3, x \cdot m_3 / \bar{\gamma}_{INF})}{\Gamma(N_J m_3)}, \quad (13)$$

where $\gamma(m, x)$ is the lower incomplete Gamma function defined as

$$\gamma(m, x) = \int_0^x z^{m-1} e^{-z} dz. \quad (14)$$

After substituting (7)–(13) in (6), the outage probability is

$$P_o = 1 - \int_0^\infty \int_0^\infty \left(1 - \frac{\gamma\left(m_1, \frac{\gamma_{th}(w + \gamma_{th} + 1)z}{w} \cdot \frac{m_1}{\bar{\gamma}_1}\right)}{\Gamma(m_1)} \right) \times \quad (15)$$

$$\times \frac{m_2^{m_2} (w + \gamma_{th})^{m_2-1}}{\Gamma(m_2) \bar{\gamma}_2^{m_2}} e^{-\frac{(w + \gamma_{th}) \cdot m_2}{\bar{\gamma}_2}} \frac{m_3^{N_J m_3} z^{N_J m_3 - 1}}{\Gamma(N_J m_3) \bar{\gamma}_{INF}^{N_J m_3}} e^{-\frac{z \cdot m_3}{\bar{\gamma}_{INF}}} dw dz.$$

The average BER of the considered system will be derived in this section. We will consider 4-QAM modulation format, but the results may be easily extended for any other modulation scheme. BER for the coherent reception of 4-QAM modulation is equal to [12]

$$P_b \approx E\left[Q\left(\sqrt{\gamma_{eq1}}\right)\right], \quad (16)$$

where $Q(\cdot)$ is the Gaussian Q-function.

Because of easier mathematical operations, γ_{eq1} will be replaced by

$$\gamma_{eq2} = \min\left(\frac{\gamma_1}{\gamma_{INF}}, \gamma_2\right) \quad (17)$$

as in [6]. Using γ_{eq2} , the average BER may be written as

$$P_b \approx \frac{1}{\sqrt{2\pi}} \int_0^\infty F_{\gamma_{eq2}}(x^2) \exp(-x^2/2) dx \quad (18)$$

where $F_{\gamma_{eq2}}(x)$ is the cdf of γ_{eq2} . It was shown [11] that $F_{\gamma_{eq2}}(x)$ is equal to

$$F_{\gamma_{eq2}}(x) = 1 - C_{\frac{\gamma_1}{\gamma_{INF}}}(x) C_{\gamma_2}(x), \quad (19)$$

where $C_{\frac{\gamma_1}{\gamma_{INF}}}(x)$ and $C_{\gamma_2}(x)$ are complementary cdfs of random variables γ_1/γ_{INF} and γ_2 , respectively. The pdf of γ_1/γ_{INF} is equal to

$$\begin{aligned}
f_{\frac{\gamma_1}{\gamma_{INF}}}(x) &= \int_0^\infty z \cdot f_{\gamma_1}(z \cdot x) f_{\gamma_{INF}}(z) dz = \\
&= \frac{\Gamma(m_1 + N_J m_3)}{\Gamma(m_1) \Gamma(N_J m_3)} \frac{m_1^{m_1} m_3^{N_J m_3}}{\bar{\gamma}_1^{m_1} \bar{\gamma}_{INF}^{N_J m_3}} \frac{x^{m_1-1}}{\frac{m_3}{\bar{\gamma}_{INF}} + \frac{m_1}{\bar{\gamma}_1} x}.
\end{aligned} \tag{20}$$

The complementary cdf of the same random variable is now equal to

$$\begin{aligned}
C_{\frac{\gamma_1}{\gamma_{INF}}}(x) &= 1 - \int_0^x f_{\frac{\gamma_1}{\gamma_{INF}}}(x) dx = \\
&= 1 - \frac{\Gamma(m_1 + N_J m_3)}{\Gamma(m_1) \Gamma(N_J m_3)} \left(\frac{\bar{\gamma}_{INF} m_1}{\bar{\gamma}_1 m_3} \right)^{m_1} \frac{x^{m_1}}{m_1} \times \\
&\quad \times {}_2F_1 \left(m_1, m_1 + N_J m_3; 1 + N_J m_3; -\frac{\bar{\gamma}_{INF} m_1 x}{\bar{\gamma}_1 m_3} \right),
\end{aligned} \tag{21}$$

where ${}_2F_1(a, b; c; x)$ is the hypergeometric ${}_2F_1$ function. Complementary cdf of γ_2 is equal to

$$C_{\gamma_2}(x) = 1 - \frac{\gamma(m_2, x \cdot m_2 / \bar{\gamma}_2)}{\Gamma(m_2)}. \tag{22}$$

After substituting (19), (21), and (22) into (18), by numeric integration we get the average bit error probability.

4. NUMERICAL RESULTS

In this section we present some numerical results that illustrate the influence of different parameters on the outage probability and BER. In all the following figures we considered the case of single interferer ($N_J = 1$). The strength of the interference is studied using signal to interference ratio parameter $\rho = \bar{\gamma}_1 / \bar{\gamma}_{INF}$. It is assumed that $m_1 = m_2 = m_3 = m$.

The outage probability as function of γ^{th} is shown in Fig. 2. Outage probability threshold γ^{th} has stronger impact on the outage probability for lower γ^{th} and for higher m .

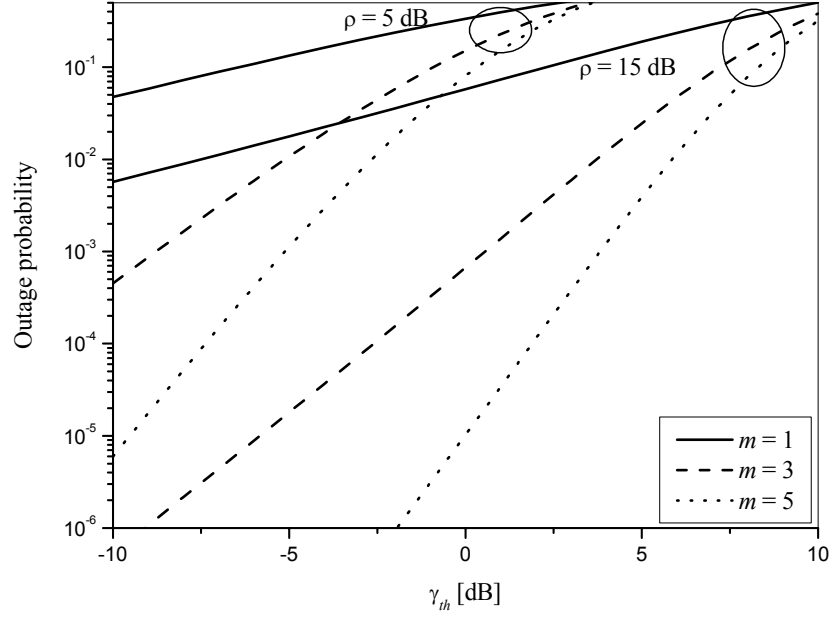


Fig. 2 – Outage probability as a function of γ_{th} for $\bar{\gamma}_2 = 10$ dB.

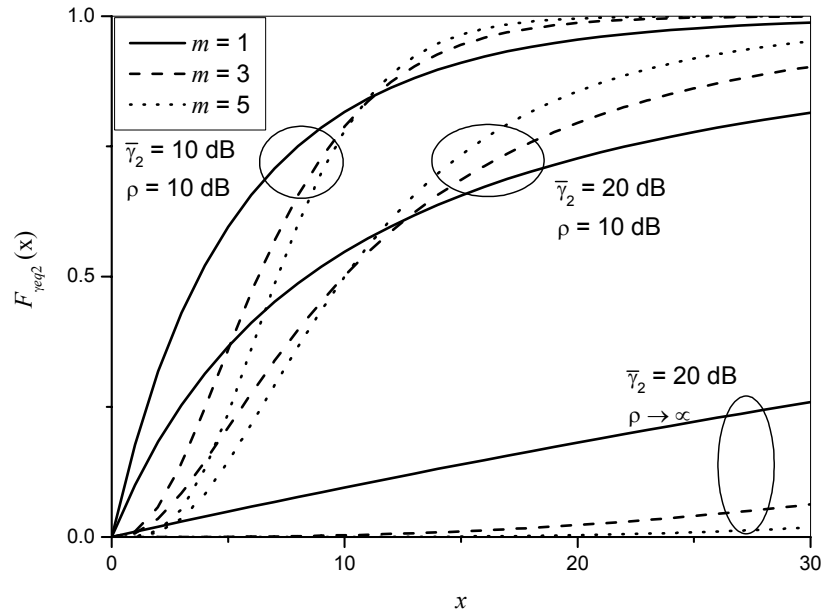


Fig. 3 – The cdf of γ_{eq2} , as a function of different parameters.

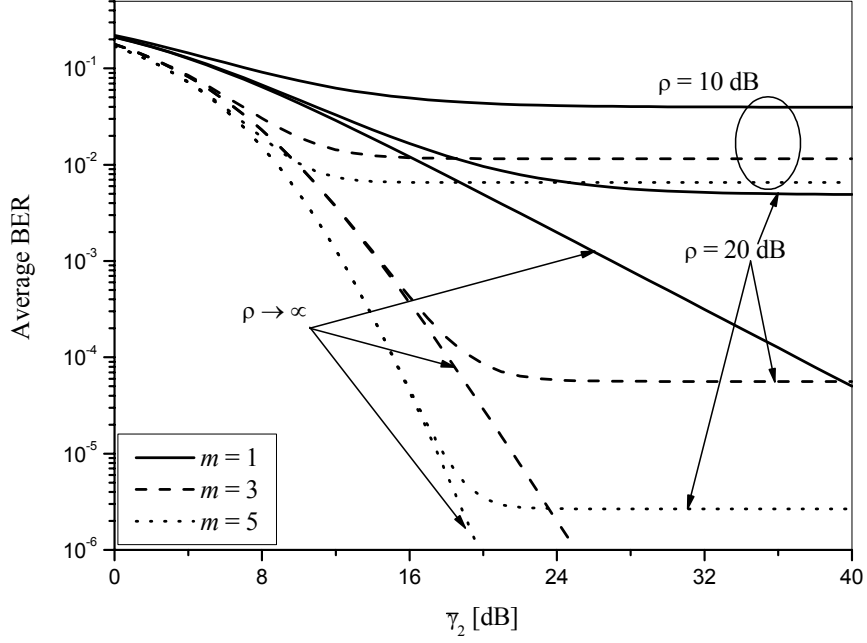


Fig. 4 – Average BER as a function of $\bar{\gamma}_2$.

Figs. 3 and 4 show the cdf $F_{\gamma_{eq2}}(x)$ and the average BER, as a function of different parameters. The average BER as a function of $\bar{\gamma}_2$, with m and ρ as parameters, is shown in Fig. 4. Due to impact of interference there is BER floor for higher signal to noise ratios. Considering case of no interference ($\rho \rightarrow \infty$), it can be seen that the interference may cause a significant performance loss for higher signal to noise ratios.

5. CONCLUSION

Performance of a dual hop amplify-and-forward system in Nakagami-M fading channel, with co-channel interference at relay is considered in this paper. The results show that the outage probability is more influenced by the Nakagami fading parameter m for higher signal to interference ratios. Also, the acceptable SNR threshold γ_{th} has stronger impact on the outage probability for lower values of γ_{th} . There is a BER floor for higher signal to noise ratios, due to impact of interference. Finally, the analysis showed that the interference may cause a variable performance loss. The higher signal to noise ratio, the higher loss exists.

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