RECONFIGURABLE COMPLEX DIGITAL DELTA-SIGMA MODULATOR SYNTHESIS FOR DIGITAL WIRELESS TRANSMITTERS

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Digital generation of radio-frequency signals is a key concept of software defined radio. One of the main difficulties for multi-standard transmitter design based on Delta-Sigma ($\Delta\Sigma$) modulators is to respect the different specifications in terms of out-of-band spurious emission. Focusing on the TX path of a UMTS/DCS1800 mobile phone, we propose hereinafter an efficient method aiming at synthesizing a complex 5th-order $\Delta\Sigma$ modulator which respects the targeted standards spurious specifications. The use of a complex noise transfer function allows a better optimization of noise shaping. In addition, this approach eases the reconfiguration of the modulator, and, it is a powerful tool for all order (real and complex) $\Delta\Sigma$ modulators synthesis, as it will be illustrated.

1. INTRODUCTION

The context of this work is the digital generation of radio-frequency (RF) signals for multi-standard transmitters for software defined radio (SDR) where a high level of integration is needed. Furthermore, digital CMOS circuits are more cost effective than other technologies. Designing RF and analog parts in CMOS is a real challenge because supply voltages are constantly decreasing and isolation from digital clocks is still an issue. Moreover, the digital part can be easily reconfigured to support different standards, each of them with the corresponding RF frequency, signal bandwidth, modulation scheme, etc. That is the reason why investigations on digital RF signal generation are of great interest. The objective of using $\Delta\Sigma$ modulation is to generate a high-speed 1-bit signal that contains information in the transmit band defined by the targeted standard. The main asset

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of the ΔΣ modulation is the possibility to move quantization noise outside the band of interest, which is called noise-shaping. This is done by creating a feedback from the quantizer. Hence, two different linearized transfer functions can be expressed: one for the noise (*i.e.*, Noise Transfer Function (NTF)) and the other for the signal (*i.e.*, Signal Transfer Function (STF)) [1, 2]. The performance of a ΔΣ modulator mainly depends on its noise-shaping filter order and its over sampling ratio (OSR), which is the ratio of the sampling frequency to twice the signal bandwidth.

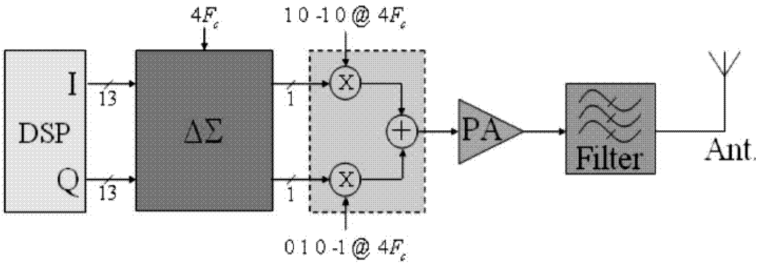

Fig. 1 – Simplified digital transmission chain.

The study presented here lies within the general scope of designing a reconfigurable digital RF transmitter as illustrated on Fig. 2. The digital-to-analog (D/A) conversion (which uses a ΔΣ modulation and a 1-bit switching-mode power amplifier (PA) for achieving performances required by the targeted standards) is put as close as possible to the antenna. Thus, the modulator block is the core of the circuit, and the output PA signal can be easily filtered, e.g., with bulk acoustic wave (BAW) filters. The use of complex signal processing for wireless applications has blossomed recently. This is especially true for high-bit-rate standards, and for multi-standard transceivers. A complex signal consists of a pair of real signals at an instance in time or sample. If one denotes the discrete complex signal, $X(k)$, as $x(k)=x_I(k)+\mathrm{j}X_Q(k)$, where $\mathrm{j}=\sqrt{-1}$, then a Hilbert Space can be defined using appropriate definitions for addition, multiplication, and inner-product and norm. The great interest of a complex ΔΣ modulator is the possibility to perform a noise transfer function which is not symmetric. To realize such a transfer function it is necessary to use non-conjugate complex zeros/poles [3, 4, 5]. This involves crossing signals between the I and Q channels, *i.e.* to put in a complex gain in the system as shown in Fig. 1 which illustrates the state-space representation of the proposed complex modulator. Moreover, this modulator schematic allows a more efficient non-conjugate out-of-bands zeros/poles placement. In any transmitter, stringent emission requirements are not only inside the band of the standard, but also out of band in order to avoid interference with other systems, especially for the dual-band UMTS/DCS 1800 transmitter considered herein this paper. Fig. 2 shows the output spectrum of the 3rd-order and a novel 5th-order ΔΣ modulator after the filter, and compares them with ETSI requirements at the antenna connector [6, 7]. As it can be seen, the most difficult

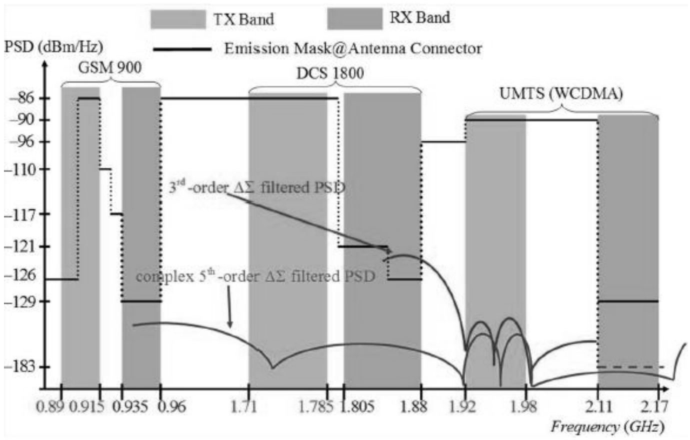

Fig. 2 – Expected complex representation of ΔΣ modulator.

requirements to meet are spurious emissions in the UMTS and DCS 1800 receive bands. The 5th-order modulator outperforms the 3rd-order one [7] by placing the additional NTF zeros in or close to these bands. Unfortunately, this can not be achieved by classical modulator design tools. The key concept of the proposed method is to optimize the placement of NTF out-of-band non-conjugate zeros in order to respect both standards spurious emissions, and to keep the STF response flat in the pass band. Section 2 will present the proposed approach that uses linear algebra and control engineering theories. Section 3 will deal with the application, and conclusions will be drawn in Section 4.

2. **PROPOSED APPROACH**

Fig. 3 shows the state-space representation of the proposed complex 5th-order ΔΣ modulator, which is equivalent to crossing two similar modulators loop filter. Accordingly, the following parameters are the same within each loop filter: a_i are the feedback coefficients from the quantizer, c_i are the integrator inter-stage coefficients, $i = 1, L, 5$; g_2 is the resonator coefficient, b_i is the feed-in coefficient at the modulator input, and $N = \eta_i + \eta_Q$ is the complex quantization noise, $\{\alpha_2, \alpha_3\}$ are cross-coupling coefficient between the two paths, and $\{\beta_2, \beta_3, \beta\}$ are terms aiming at implementing each cross term and ensuring the modulator stability as explained in the sequel.

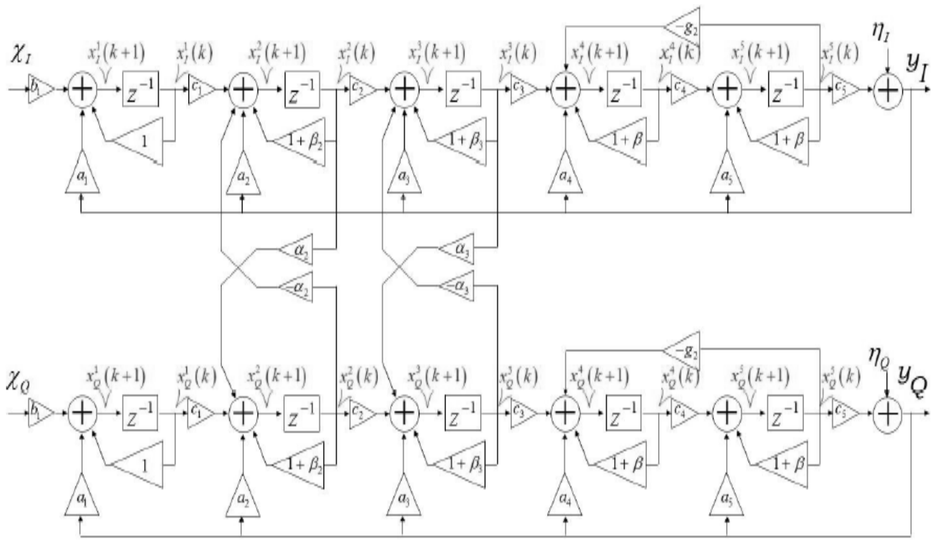

Fig. 3 – State space representation of the proposed ΔΣ modulator.

Using the modified architecture highlighted in Fig. 3, the discrete modulator state-space description can be expressed as:

$$\begin{cases} \boldsymbol{X}(k+1) = \boldsymbol{A}X(k) + \boldsymbol{B}U(k) \\ \boldsymbol{Y}(k) = \boldsymbol{C}X(k) + \boldsymbol{D}U(k) \end{cases}, \tag{1}$$

where $X = X_I + \mathrm{j}X_{\mathrm{Q}} = [x_I^1 + \mathrm{j}x_{\mathrm{Q}}^1 x_I^2 + \mathrm{j}x_{\mathrm{Q}}^2 x_I^3 + x_{\mathrm{Q}}^3 x_I^4 + x_{\mathrm{Q}}^4 x_I^5 + \mathrm{j}x_{\mathrm{Q}}^5]^{\mathrm{T}}$ is the complex state vector, $\boldsymbol{U}$=($\boldsymbol{X}$, $\boldsymbol{N}$) is the complex input vector (signal and noise),

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & a_1c_5 \\ c_1 & 1+\alpha_1+\beta_2 & 0 & 0 & a_2c_5 \\ 0 & c_2 & 1+\beta_3+\mathrm{j}\alpha_{31} & 0 & a_3c_5 \\ 0 & 0 & c_3 & 1+\beta & a_4c_5-g_2 \\ 0 & 0 & 0 & c_4 & 1+\beta+a_5c_5 \end{bmatrix},$$

is

$$\boldsymbol{B}=\begin{bmatrix} b_1 & a_1 \\ 0 & a_2 \\ 0 & a_3 \\ 0 & a_4 \\ 0 & a_5 \end{bmatrix} \mathrm{C}=\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ c_5 \end{bmatrix}^{\mathrm{T}} \quad \boldsymbol{D}=\begin{bmatrix} 0 \\ 1 \end{bmatrix}^{\mathrm{T}},$$

$\boldsymbol{Y} = (\boldsymbol{Y}_I + \mathrm{j}\boldsymbol{Y}_Q)$ is the complex output signal.

It is obvious to verify that matrix A is not singular, hence it is invertible. Then, using control engineering theory and assuming null initial conditions 0, we show that:

$$\left[\frac{\mathrm{STF_{num}}}{\mathrm{TF_{den}}} \cdot \frac{\mathrm{NTF_{num}}}{\mathrm{TF_{den}}}\right] = \boldsymbol{C}(z\boldsymbol{I}-\boldsymbol{A})^{-1}\boldsymbol{B}+\boldsymbol{D}, \tag{2}$$

$$\mathrm{TF_{den}} = \det(z\boldsymbol{I}-\boldsymbol{A}), \tag{3}$$

where $\boldsymbol{I}$, $\mathrm{STF_{num}}$, $\mathrm{TF_{den}}$ and det stand for identity matrix, STF numerator, NTF numerator, both STF and NTF denominator, and the determinant of a matrix, respectively. Expanding (3), $\mathrm{STF_{num}}$ becomes:

$$\mathrm{STF_{num}} = b_1 c_1 c_2 c_3 c_4 c_5. \tag{4}$$

Also from(3), since we are interested in the NTF zeros, we derive their expressions as:

$$\begin{cases} z_1 = 1,\ z_2 = (1+\beta_2) + \mathrm{j}\alpha_2,\ z_3 = (1+\beta_3) + \mathrm{j}\alpha_3, \\ z_4 = (1+\beta) + \mathrm{j}\sqrt{g_2 c_4},\ z_5 = (1+\beta) - \mathrm{j}\sqrt{g_2 c_4}. \end{cases} \tag{5}$$

The roots of TF_{den} are the poles of both signal and noise transfer functions. Hence, using the determinant properties and the LU factorization, the coefficients of matrix ($z\boldsymbol{I}$–$\boldsymbol{A}$) are factorized into the product of an upper triangular matrix and then the poles can be determined [7]. Moreover, (5) shows that z_2 and z_3 are complex non-conjugate, and the two last zeros are complex conjugate. This is an important result since it allows a great degree of freedom in the NTF out-of-band zeros placement which are complex non-conjugate. Before analyzing zeros, let us recall that, from filter design theory, the modulator will be stable if all zeros are on the unit circle and all poles inside the unit disc. However, from (5) it is obvious that both conditions are not satisfied. In addition, for high-order $\Delta\Sigma$ modulator, the stability would depend on the input signal level [1, 5]. Hence, we propose in the following a method aimed at optimizing zeros and poles placement and at keeping the modulator stable.

The novelty of the proposed method is to simultaneously treat the zeros and poles placements. For a better understanding, we focus here on NTF zeros, the poles placement method can be found in [7]. Keeping zeros on the unit circle is equivalent to satisfy $|z| = 1$, $n = 1, L, 5$. An original way to check this relation is to introduce integrator feedback coefficients β_2, β_3 and β as illustrated in Fig. 2. Since we are interested in the TX path of either UMTS or DCS 1800, a zero is placed at their respective center frequency band (i.e., one of them has to be equal to 1), which is given by the first zero in. From (5), computing the condition to keep z_4 and $z_5$on the unit circle led us to write:

$$z_4 = \sqrt{1 - g_2 c_4} + \mathrm{j}\sqrt{g_2 c_4},\ z_5 = \sqrt{1 - g_2 c_4} - \mathrm{j}\sqrt{g_2 c_4}, \tag{6}$$

$$z_2 = (1 + \beta') + \mathrm{j}\alpha_2,\ z_3 = (1 + \beta') + \mathrm{j}\alpha_3. \tag{7}$$

Therefore, we compute again the condition to keep zeros z_2 and z_3 on the unit circle under the constraint: $|\beta|<1$, $|\alpha_n|$, $n = 2, 3$. Finally, the optimized zeros expressions are given by:

$$\begin{cases} z_1 = 1,\ z_2 = \sqrt{1 - \alpha_2^2} + \mathrm{j}\alpha_2, \\ z_3 = \sqrt{1 - \alpha_3^2} + \mathrm{j}\alpha_3,\ z_4 = \sqrt{1 - g_2 c_4} + \mathrm{j}\sqrt{g_2 c_4}, z_5 = \overline{z_4} \end{cases} \tag{8}$$

Relation (8) allows to easily set the zeros in the unit circle, with a very low computation cost, since there are just four coefficients to choose for both targeted standards, as illustrated on Fig. 4 a,b. The next step of the method is to optimize poles placement in order to keep the STF flat in the pass band. Since $z_0 = 1$ is a zero of NTF and using the simplifications detailed above, we prove that the simplified NTF is given by:

$$\mathrm{NTF} = \frac{\sum_{n=1}^{4} \lambda_n z^n - 1}{\sum_{n=1}^{4} \xi_n z^n + a_5 c_5 - 1}, \tag{9}$$

where the coefficients λ_n and ζ_n depend on all the coefficients of the modulator.

Analyzing the results obtained in [1, 2, 5], and using filter synthesis theory led to write the poles optimization relations detailed in [7]. Let us recall that in this configuration, one of the poles must be real and not equal to 0 which is equivalent to set set $a_5 c_5 \neq 1$ in (9). Let us note set $p_1$as this pole. As proved in [7], all poles are equispaced on the circle $C(z_1, |p_1 - z_1|)$ centered on z_1 and of radius $\rho = |p_0 - z_0|$, where ρ is a constant. Finally, using the entire optimization method led us to satisfy the stability condition and to keep the STF flat in the pass band, as it will be highlighted in the following. Let us briefly analyze (8). As detailed in [8] and highlighted on Fig. 4 a, the complex zeros compensate the dominating poles (*i.e.*, those which are closest to the unit circle). Thus, the real pole p_1 becomes the dominating one. Consequently, the STF response is similar to a first -order filter, *i.e.*, its response is flat in the pass band.

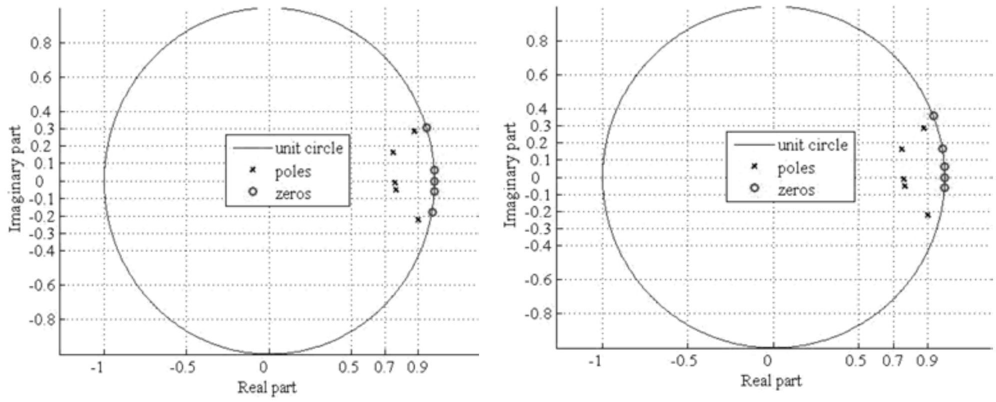

Fig. 4 – Optimized zeros and poles: a) UMTS TX path; b) DCS TX path.

Finally, the position of p_1, thus, the value of ρ allows the static gain regulation. Since we are interested in designing a very high speed digital circuit, all coefficients must be written as powers of 2, that allows replace all multiplications by additions. In the next section we show that the reconfigurability of the structure consists in changing only a few coefficients.

3. APPLICATION

Let us first note that setting $\alpha_2 = \alpha_3 = 0$, and introducing a resonator coefficient g_0 on Fig. 1, allows to find the particular case of real modulators [7]. The modulator is optimized so as to match both UMTS and DCS 1800 standards out-of-band spurious emissions, with different constraints on out-of-bands zero relative positions. Moreover, another constraint is added on the coefficients in order to ease the architecture implementation. Thus, only a few coefficients need to be changed to move from the UMTS configuration to the DCS 1800 one, as highlighted below.

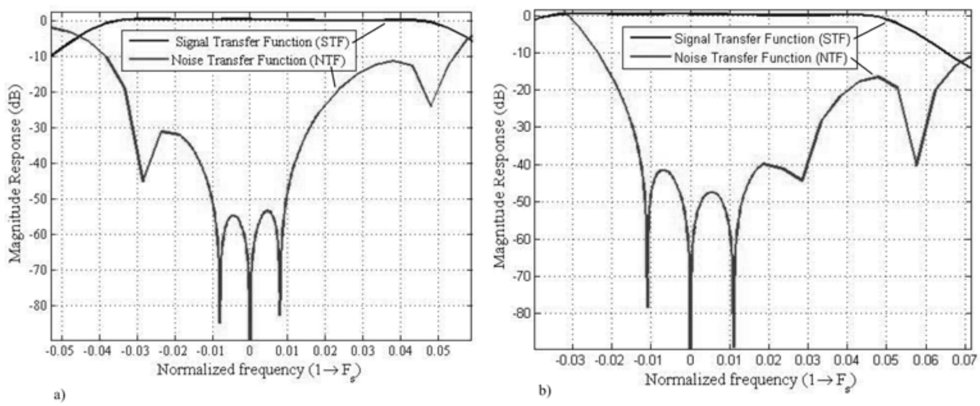

Fig. 5 – Optimized complex transfer functions in the a) UMTS TX path;
b) DCS TX path.

Fig. 5 a illustrates the output of this complex ΔΣ modulator in UMTS configuration. It shows 2 conjugate NTF zeros in the UMTS TX band, and the real zero at the center frequency band. The two others, which are complex non-conjugate, are placed in the DCS1800 TX band and in the UMTS RX band, respectively. This complex 5^{th}-order ΔΣ modulator can be reconfigured to satisfy the spurious emissions attenuations of the DCS standard (DCS TX band: 1710-1785 MHz, channel bandwidth: 200 kHz). The sampling frequency is set to $F_S =$ $= 4F_C = 6.99$ GHz and only 3 coefficients need to be changed.

Fig. 5b highlights the output of the ΔΣ modulator in DCS1800 configuration. On this curve, there are still a zero at the center and two others which are complex conjugate at the extremities of the DCS TX band. Thanks to the flexibility given by the use of this complex modulator, the two complex non-conjugate zeros are placed in the DCS RX band and in the UMTS RX band.

Fig. 6 shows the power spectral density (PSD) and the emission mask reported at the antenna connector in the UMTS configuration of the real 5^{th}-order modulator synthesized through the proposed approach and recently implemented. Simulation results were carried out in the UMTS TX path (1 920–1 980 MHz) in a 5 MHz channel bandwidth [6] (the 7^{th} here) with a sampling frequency F_S=4F_C=6.99 GHz, the maximum modulator stable input equal to –2.0 dB_F (the amplitude refers to 0 dB_F full-scale sine wave), the OSR equal to 40, and the power at the modulator output equal to 4 dBm.

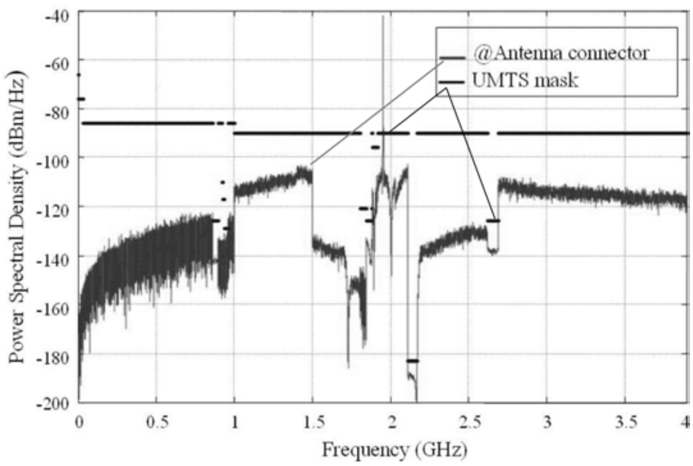

Fig. 6 – Performances analysis reported at the antenna connector in the UMTS TX path using the derived real ΔΣ modulator.

It clearly evidences significant spurious emissions attenuations in adjacent channels, with *e.g.*, a margin of 7 dB in the UMTS RX band. Nevertheless, the performances did not completely match both standards specifications, especially in the adjacent TX channels as also shown on Fig. 7, in the DCS TX path configuration.

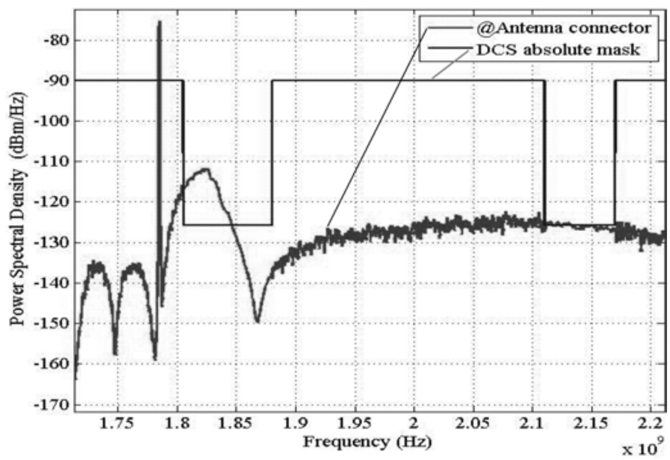

Fig. 7 – Performances analysis reported at the antenna connector in the DCS TX path using the derived real DS modulator.

4. **CONCLUSIONS**

We presented an original complex ΔΣ modulator synthesis method combining both linear algebra and control engineering theories. This approach allowed to directly derive zeros and poles expressions from the state matrix. Moreover, the proposed optimization scheme allowed designing a reconfigurable complex 5^{th}-order ΔΣ modulator which fulfils both UMTS and DCS1800 specifications. The implementation of this architecture in a digital CMOS 65nm technology is on progress.

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