SOLIDIFICATION SURFACE SPEED CONTROL OF FERROMAGNETIC PIECES USING EDDY CURRENT HEATING

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Eddy-currents generate heat sources in the melted volume of a ferromagnetic material placed in a moving crucible. The solidification is produced by applying a cooling system on the crucible boundary. The liquid-solid transition surface evolution and its shape define the quality of the crystallization microstructure obtained in the casting process. The computation of the nonlinear eddy current problem coupled with that of thermal diffusion field for the moving ferromagnetic pieces is required in order to obtain the temperature field distribution and the liquid-solid transition surface location at each time step. This paper presents an efficient procedure for solving this coupled field problem and obtaining the time evolution of the melted piece speed, such that the solidification surface shape preserves the imposed parameters.

1. INTRODUCTION

When metallic pieces are obtained using a casting process, the "free" solidification starts often form centres with impurities and advances in an arborescent manner, resulting a non-uniform microstructure and micro-cracks that prejudice the mechanical properties. To achieve a uniform microstructure it is necessary to impose a controlled solidification of the melted conducting material during the casting process in a controlled thermal environment, defined by a phase change surface which gradually evolves in time.

Eddy-current heating is the most efficient way to locally heat the piece, by which one can realize the volume distribution of losses inside the piece. The cooling system acts on one side of the melted shape in which the melted metal can be found. The solidification is controlled by moving both the eddy-current producing system and the cooling system.

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The temperature field computation and, therefore, the solidification surface computation, is a very complicated problem, involving the necessity of eddy current problem computation, coupled with thermal field problem, when the piece is moving. The temperature strongly modifies the material parameters, which are required for the electromagnetic field computation. For the solidifying ferromagnetic pieces, small changes of the temperature below Curie point lead to a dramatic change of the **B-H** relationship, which becomes strongly nonlinear. The changes near the liquid-solid transition surface necessitate taking into account the solidification latent heat.

The electromagnetic field regime is periodic, the piece moving speed being much smaller than the field variation speed. The nonlinearity of the **B-H** relationship is treated, most frequently, by using the static permeability method, iteratively corrected by different procedures [1]. The big advantage of this procedure consists in the possibility to use the phasor representation of the electromagnetic field quantities. This is why this method is frequently adopted by the most well-known commercial programs (FLUX, VECTOR FIELDS etc). Unfortunately, the static permeability method is not always convergent and the computed solution can be very different compared to the real one in the case of strong nonlinearities. The analysis in time domain leads to a convenient solution, but the computation time needed to evaluate the asymptotic evolution in order to determine the permanent regime solution can be huge. The harmonic balance method [2] assumes the representation of the electromagnetic field quantities using a finite number of harmonics that are introduced, in this form, in the field equations. The coupling between harmonics, given by the nonlinear relation **B-H**, leads to a huge nonlinear system. Moreover, the Newton-Raphson iterative technique used to treat the nonlinearities is not always convergent. Recently in [3], there has been proposed a very efficient computation method for electromagnetic field periodic regime in nonlinear media. Based on fixed point polarization method [4, 5], this method allows the de-coupling of the harmonics and the eddy current problem is solved using a phasor representation for each harmonic. The convergence is always ensured for any choice for the retained number of the harmonics in the Fourier decomposition. One can start the computation only for the fundamental, having all the advantages given by the static permeability method, and then by increasing the number of the harmonics taken into account we obtain a higher accuracy of the result. The convergence speed can be increased in a spectacular manner by using and over-relaxation method described in [6].

The space numerical discretization is frequently done using the finite element method. In the case of a relative movement between bodies, the discretization mesh is rebuild for each time step, being necessary reallocation procedures for the material parameters and the system equation matrices need to be recomputed. More appropriate are the integral methods [3, 7, 8] or finite element-boundary element hybrid methods [9, 10, 11]. The use of eddy current integral equation is very efficient for ferromagnetic bodies heating problems computation, for which the **B-H** relationship is strongly dependent on temperature [12].

The solidification of moving ferromagnetic pieces, under the action of eddy current has been studied in [13], being determined the evolution of the solidification surface, under the known moving speed. This paper presents the computation procedure of a more important problem from the technological point of view: the determination of the piece's moving speed, such that the shape of the solidification surface to preserve some imposed parameters.

2. EDDY CURRENT PROBLEM COMPUTATION

For ferromagnetic bodies, **B-H** relation is strongly nonlinear and strongly depends on the temperature θ: $\mathbf{H} = \mathbf{F}(\mathbf{B}, \theta)$. This dependence is drastically close to the Curie temperature, and when this point is over-passed the material has the free space **B-H** relationship. The fixed point polarization method allows the replacement of the nonlinear media with a linear media, having the free space magnetic permeability μ_0 and a magnetization $\mathbf{M}$ that is corrected as a function of magnetic induction [3]

$$\mathbf{M} = \nu\mathbf{B} - \mathbf{F}(\mathbf{B}, \theta) \equiv \mathbf{G}(\mathbf{B}, \theta). \tag{1}$$

An important advantage results: one can use the eddy current integral equation for electromagnetic field problem computation. If we decompose the magnetization in Fourier series, retaining only a finite N number of harmonics, for each harmonic, n = 1…N, one can use the phasor $\mathbf{M}_n = \mathbf{M}'_n + \mathrm{j}\mathbf{M}''_n$. For the two-dimensional problems, the integral equation [3] has the form

$$\begin{aligned} \rho J_n(\mathbf{r}) + \frac{\mu_0}{2\pi}\mathrm{j}\omega_n \int_{\Omega} J_n(\mathbf{r}')\ln\frac{1}{R}\mathrm{d}S' = -\frac{\mu_0}{2\pi}\mathrm{j}\omega_n \int_{\Omega_0} J_{0n}(\mathbf{r}')\ln\frac{1}{R}\mathrm{d}S' - \\ -\frac{\mu_0}{2\pi}\mathrm{j}\omega_n \int_{\Omega_f} \mathbf{k}\cdot\left(\nabla'\times\mathbf{M}_n(\mathbf{r}')\right)\ln\frac{1}{R}\mathrm{d}S' , \end{aligned} \tag{2}$$

where ρ and J_n are, respectively, the resistivity and the electric current density induced in the conducting regions Ω, J_{0n} is the given current density in the non-ferromagnetic coil regions Ω_0, Ω_f is the region occupied by ferromagnetic materials, i.e. the solidifying material, **r** and **r'** are the position vectors of the observation and the source points, respectively, $R = |\mathbf{r} - \mathbf{r}'|$, and **k** is the longitudinal unit vector.

The numerical computation of equation (2) is done by employing a $\Re_j$ polygonal sub-domains as discretization mesh of the Ω domain, where one admits that the current density is constant over each sub-domain. In the discretization mesh of $\Re_M$ sub-domains used also for Ω domain, which can be different from $\Re_j$, one admits that the magnetization is constant over each sub-domain. Because the domains Ω and Ω_0 are in a relative motion the first integral from the right side is modified for each time step of the thermal diffusion problem. If the system contains also a ferromagnetic yoke for magnetic field concentration [13], then the domain Ω_f is different from Ω_0 and the yoke's area Ω_f-Ω_0 is in relative motion compared to Ω_0. In this case, also the second term from the right side of equation (2) is modified at each time step. To increase the computation speed of the eddy current problem is important to have the analytic form of the integrals used to evaluate the coefficients of all the matrices in the numerical form of equation (2).

The n-th harmonic of flux density is computed with the realtion

$$\mathbf{B}_n\left(\mathbf{r}\right)=\frac{\mu_0}{2\pi}\left[\mathbf{k}\times\int_{\Omega}\frac{J_n(\mathbf{r}')\mathbf{R}}{R^2}\mathrm{d}S'+\mathbf{k}\times\int_{\Omega_0}\frac{J_{0n}(\mathbf{r}')\mathbf{R}}{R^2}\mathrm{d}S'x+\right.$$
$$\left.+\int_{\Omega_f}\frac{\nabla'\times\mathbf{M}_n(\mathbf{r}')}{R^2}\times\mathbf{R}\mathrm{d}S'\right]. \quad (3)$$

For the numeric form of relation (3) one takes the average value of the magnetic induction in the sub-domains of the discretization mesh $\Re_M$. From $\mathbf{B}_n$ harmonics, we obtain the magnetic induction value in the time domain and we correct the magnetization in $\Re_M$, according to relation (1).

3. THERMAL DIFFUSION PROBLEM

The temperature distribution is obtained by solving the thermal diffusion equation

$$-\nabla\cdot\left(\lambda\,\nabla\theta\right)+c_v\frac{\partial\theta}{\partial t}=p\,, \quad (4)$$

where λ is the thermal conductibility, c_v is the thermal capacity of the ferromagnetic material, both parameters depending on temperature, and p are specific losses obtained from eddy current problem computation. For the liquid-solid phase transition, one adopts a caloric capacity which results from the solidification latent heat $c' = \frac{s}{\delta\theta}$, where s is the solidification latent heat and $\delta\theta$ is a temperature interval associated to the transition.

The boundary condition is

$$\lambda \frac{\partial\theta}{\partial n} + \alpha(t)(\theta - \theta_e) = 0 , \quad (5)$$

where θ_e is the outer temperature, α is the thermal convection coefficient, which is time dependent, because of the crucible's movement. The time discretization of equation (4) is done using the trapezoidal method and the space discretization is done using the finite element method, using a triangular discretization mesh. The thermal conductibility and the caloric capacity are iteratively corrected as a function of temperature.

4. THE MOVING SPEED OF THE CRUCIBLE

In the controlled solidification technology, the moving speed of the crucible is decisive in order to establish a correct solidification surface shape evolution. A too rapid movement can distort this surface to a non acceptable level, while a too slow speed leads to an increase of the technological process time and, sometimes, it can distort the solidification surface. In order to establish the moving speed of the crucible we impose a maximum deviation from the plane shape for the solidification surface, which when it is over-passed, the speed is reduced. At the same time, we have an imposed a minimum deviation, which if it is not obtained, the speed is increased. Moreover, the speed is controlled such that to obtain a largely enough temperature gradient, modelled by the maximum temperature difference from the test. The speed adaptation takes place during the Crank-Nicholson computation of the thermal diffusion problem.

5. NUMERICAL EXAMPLE

A coil having 15×60 mm, with a sinusoidal current density of $4\ \text{A/mm}^2$, at 50 Hz, induces eddy currents in a melted ferromagnetic piece, with section 20×60 mm (Fig. 1a), having, at $20\ °\text{C}$, the resistivity $10^{-7}\,\Omega\cdot\text{m}$, the thermal conductibility $46\,\text{W}/(\text{K}\cdot\text{m})$, the thermal capacity $4\times10^6\ \text{J}/(\text{K}\cdot\text{m}^3)$, all depending on the temperature (Table 1). Near the coil's thermal insulation, we admit the value $\alpha = 0.2\,\text{W}/(\text{K}\cdot\text{m}^2)$ for the thermal convection coefficient. In order to obtain a good temperature coefficient, we adopt a thermal convection coefficient

$\alpha = 500\,\mathrm{W/(K \cdot m^2)}$ under the thermal insulation and $\alpha = 800\,\mathrm{W/(K \cdot m^2)}$ at the crucible's basis.

Table 1

Temperature dependence of resistivity, thermal conductibility and thermal capacity for the ferromagnetic material (taken from [13])

θ (°C)	ρ ($10^{-7} \times \Omega$m)	λ (W/(K · m))	c_v ($10^6 \times$ J/(K · m^3))
100	1.04	40.0	3.97
500	1.20	35.0	3.80
1000	1.45	30.0	3.63
1300	1.60	27.0	3.50
1500	1.72	25.0	3.34
1800	1.89	22.0	3.00
2000	2.00	20.6	2.86

The melting temperature is $1300\,°\mathrm{C}$ and the latent heat is $2.142 \cdot 10^9$ J/m^3. Below Curie temperature, of $780\,°\mathrm{C}$, **B-H** relation is nonlinear and depends on temperature (Fig. 1).

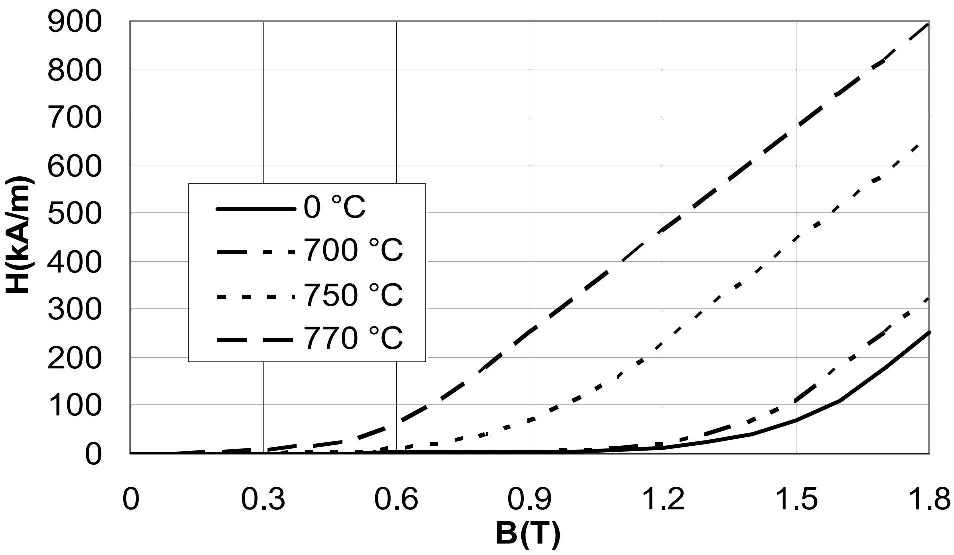


Fig. 1 – **H-B** relationship for different temperatures [13].

A maximum value of 4 mm has been imposed for the solidification surface deviation with respect to the plane surface, which, if it is over-passed, the speed is reduced. At the same time, a minimum value of 0.5 mm has been imposed, under which the speed can be increased.

In order to evaluate the temperature gradient, a minimum value of 500 ^{0}C has been imposed with respect to which the maximum temperature difference is compared and the speed modification takes place with respect to the weight of this ratio. In Fig. 2 there is depicted the evolution of the optimum speed.

The time evolution of the isotherms can be followed in Fig. 3. The time evolution $1300\,°C$ isotherm is given in Fig. 4. In Fig. 5 one can see the Curie isotherms evolution. The temperature decreased under Curie point leads to the apparition of the ferromagnetism (a drastically increase of the magnetic "permeability" accompanied by the nonlinear **B-H** relationship modification).

In Fig. 6 one can notice the change of the field line spectrum, together with the decrease of the temperature below Curie point. The field lines are attracted in the area with a nonlinear ferromagnetic characteristic of the material.

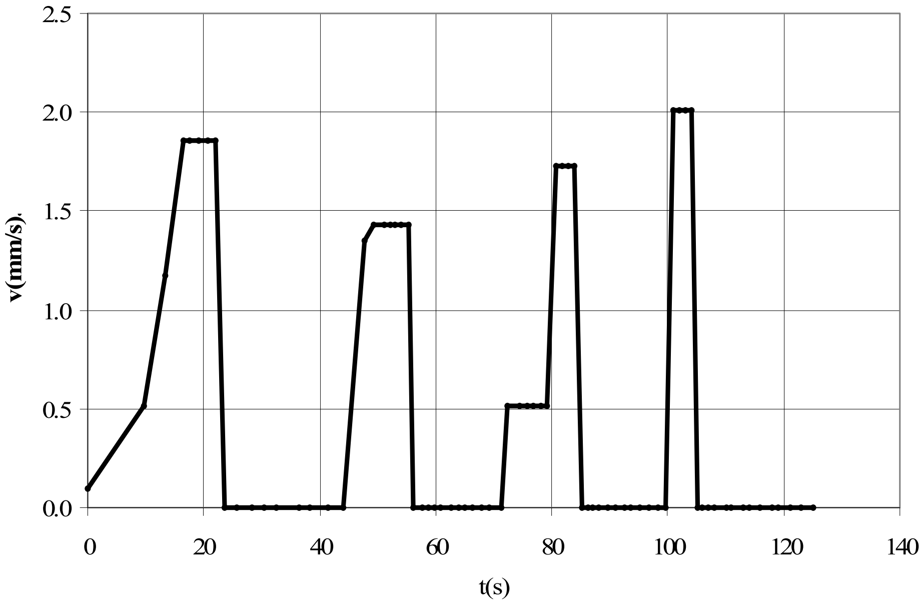
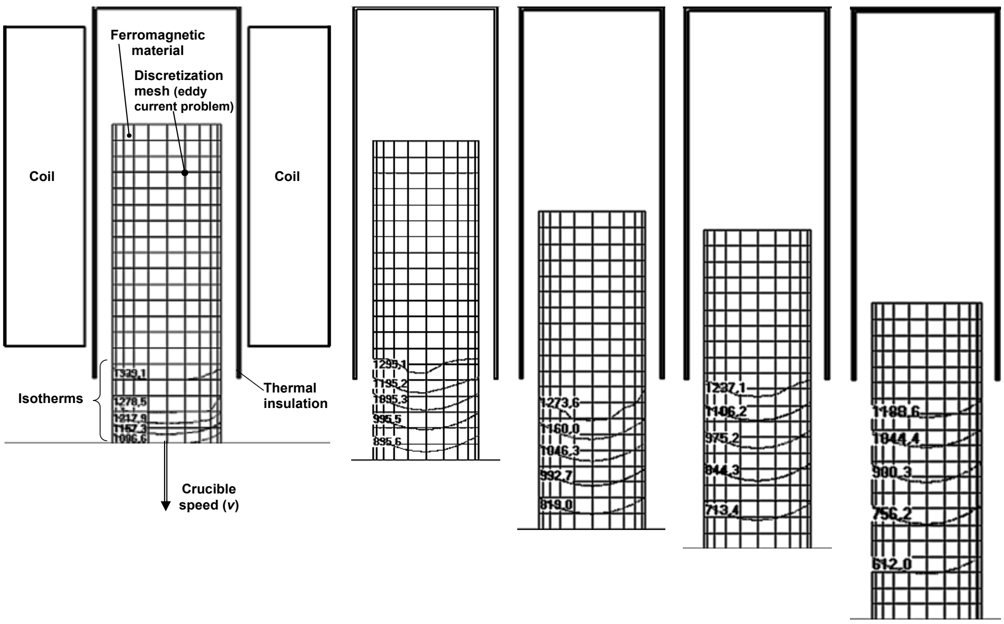
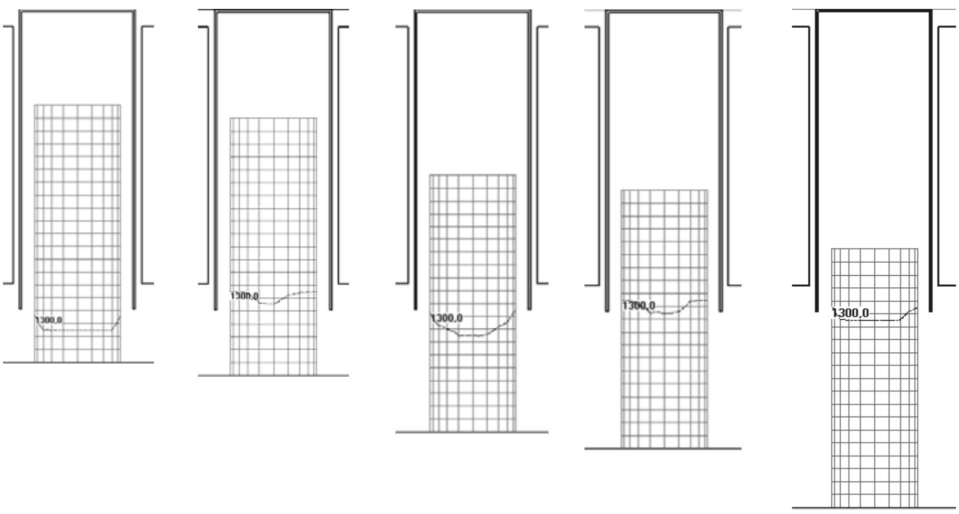


Fig. 2 – Time evolution of the crucible's speed.



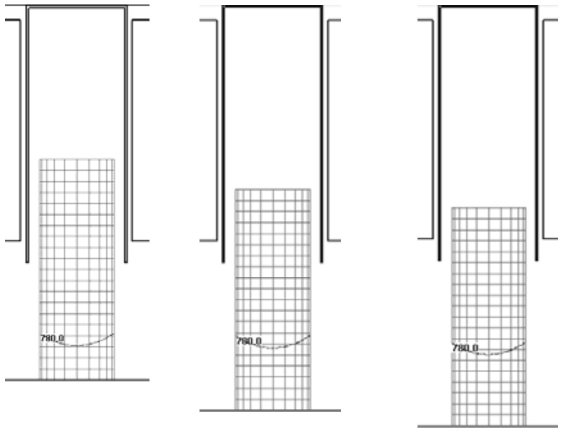
a) t = 21 s b) t = 44 s c) t = 58 s d) t = 78 s e) t = 103 s

Fig. 3 – Time evolution of the isotherms.



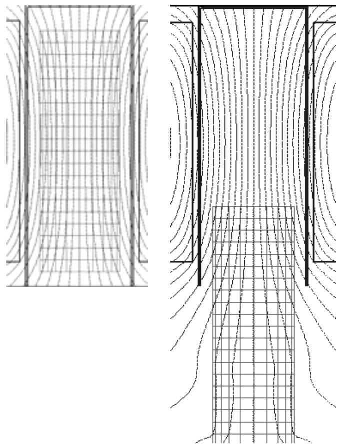
a) t = 21 s b) t = 44 s c) t = 58 s d) t = 78 s e) t = 103 s

Fig. 4 – Time evolution of 1300 °C isotherm.



a) t = 78 s b) t = 97 s c) t = 103 s

Fig. 5 – Time evolution of Curie isotherm (780 °C).



a) t = 9 s b) t = 103 s

Fig. 6 – Field lines.

6. CONCLUSIONS

A numerical method for the phase change surface position evaluation and the computation of the movement in the eddy current controlled solidification is proposed. For thermal diffusion problem computation it is necessary to take into account that the boundary conditions is changing during the crucible motion. To ensure the Crank-Nicholson procedure stability, it was necessary to adjust the time step both in the neighbourhood of the solidification temperature and in the neighbourhood of Curie point, where **B-H** relationship drastically changes. The time step modification takes place by imposing a sufficiently small temperature variation in all nodes of the thermal discretization mesh and it represents the main time consumer of the elaborated software programs. However, in order to reduce the computation time, when correcting the time step, one solved only the thermal diffusion problem, without solving the eddy current problem, too. An efficient solution to avoid the numerical thermal field problems computation in the neighbourhood of the solidification point is proposed in [14].

A very fast and accurate procedure for the computation of the electromagnetic field periodic problem in nonlinear media with moving bodies was developed, based on the fixed point polarization method. The nonlinear media is replaced by a linear one where the nonlinearity is transferred to the magnetization. This one can be decomposed using Fourier series, retaining a convenient number of harmonics, the field problem being computed for each harmonic, using the complex representation. The convergence of the method is always ensured. Using the free space permeability, allows the use of the eddy current integral equation, with its important advantages for crucible's movement modelling.

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