TRANSIENT STABILITY CONSTRAINED OPTIMAL POWER FLOW USING IMPROVED PARTICLE SWARM OPTIMIZATION APPROACH

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Key words: Power system, Transient stability constrained optimal power flow (TSCOPF), Improved particle swarm optimizer (IPSO), Optimal power flow (OPF), Contingencies.

Stability is an important constraint in power system operation; the transient stability constrained optimal power flow (TSCOPF) has a considerable attention in recent years. The solution obtained from the conventional optimal power flow (OPF), which considers only the static constraints, does not guarantee transient stability in the system against possible contingencies such as line fault. In this paper, a novel OPF is proposed by adding the transient stability constraints into the conventional OPF problem using the improved particle swarm optimizer (IPSO). It is so called transient stability constrained optimal power flow, which helps to return the system to a normal operating condition after a disturbance. The basic idea of the proposed method is to model the transient stability as an objective function rather than an inequality constraint and consider classic transient stability constrained optimal power flow (TSCOPF). The proposed method is tested on the IEEE 30-bus system. The objective function is to minimize the total cost of fuel for all generators.

1. INTRODUCTION

Optimal power flow (OPF) is an essential tool for power system operation and planning. The main purpose of an OPF program is to calculate the optimal operating point of a power system and setting the variables for the economic and secure operation of the system. Transient stability studies the test of the optimal solution obtained from the OPF under credible disturbances to ensure the stability of the system. If the system is transiently unstable under one of the disturbances, the OPF solution must be modified. In the conventional OPF, the solution takes into consideration the static constraints that it is not always powerful in the case of the occurrence of some contingencies. Besides, a large amount of financial loss is due to the non-synchronism in the power system that has been reported in many countries [1]. Recently, due to several blackouts caused by transient instability, dynamic constraints are added into OPF to guarantee the transient stability of the power system against possible contingencies. As a result, a novel OPF is proposed by adding the transient stability constraints into the conventional OPF problem called TSCOPF [1]. It is a fundamental issue for power system dynamic security. It involves the dynamic security and economic operation of power systems, which has been extensively investigated and addressed in [2, 3]. Since the TSCOPF is a large-scale nonlinear optimization problem with differential-algebraic equations (DAE) it cannot be solved by employing directly the standard mathematical programming methods such as interior point methods (IPM) [4]. Hence, the key issue in the TSCOPF problems is to handle the differential equations from transient stability constraints. The transient stability constraints consist of additional equality constraints i.e., a set of differential-algebraic equations (DAEs) that describes the dynamic behavior of rotor angle after undergoing severe disturbances and the additional inequality constraints i.e. stability criteria to indicate whether or not the system is stable after the contingency. The widely used criterion is the maximum allowable deviation of the rotor angle with respect to the centre of inertia (COI). To handle the transient stability equality constraints, in [1–5] a set of DAEs is first converted into numerically equivalent algebraic equations using some numerical methods such as modified Euler’s method, and then step-by-step simulation (time domain simulation) is performed to observe the rotor angle deviation. Intelligent methods such as differential evolution (DE) [6], artificial neural network (ANN) [7], evolutionary programming [8], genetic algorithm (GA) [9] and particle swarm optimization (PSO) [10, 11] are population-based stochastic optimizations which do not rely on the derivatives and therefore could handle the non-derivative and non-convex problems. In [12], ant colony optimization (ACO) is used in an industrial problem of the Algerian west network to optimize the cost. Moreover, hybrid artificial bee colony algorithm (ABCA) to obtain a near global solution and the micro-genetic algorithm (mGA) to determine the optimal global solution is used in [13]. The results founded in [12, 13] have presented a great precision. However, they treated only the OPF problem in a statically regime. Hence, in this paper, we introduced the transit stability as a constraint to the OPF. We obtained then a dynamical regime named TSCOPF. Practically, PSO, which imitates the social behaviors of bird flocking, is an excellent stochastic optimization algorithm with only a few parameters needing to be tuned [14]. Coupled with the encouraging performance for some hard optimization problems with faster and stable convergence, PSO has been widely and successfully applied in power system optimization, such as the one with constriction factor method in [10]. Due to the highly non-linear inherence of power system transient stability, the computation of TSCOPF is very difficult. Generally, the difficulties appear in two sub-problems:

1) How to add the transient stability constraints to the conventional OPF model, i.e. how to express the transient stability limit.

2) How to solve the optimization problem after the transient stability constraints are included.

This rest of the paper is structured as follows. Section 2 gives a brief review of the Improved PSO method (IPSO). Section 3 explains the transient stability assessment. Section 4 deals with the TSCOPF problem formulation. Section 5 highlights the main components of the proposed method. Section 6 provides the simulation results and discussions. The last section is devoted to the conclusion.

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2. OPTIMAL POWER FLOW (OPF)

The OPF problem is an extension problem of economic load dispatch (ELD). In OPF, the solution is not only power generation but also other controllable parameters in a power system such as voltages at generator bus, transformer tap settings, line flow limit etc. The constraints in ELD are also extended. The real loss in the system is calculated and added into power flow equation whereas in ELD the system loss is obtained by approximation [12, 13]. All additional variable limits in the system are considered as inequality constraints, for instance, voltage limit at load bus, line flow limit on the transmission line, stability limit… etc. Various objective functions can be considered in OPF, for example, fuel cost minimization, system loss minimization, optimal voltage profile, power transfer maximization and load shedding minimization under an emergency condition and in some cases the multi-objective is taken into account. Therefore, the OPF problem is a large scale of non-linear optimization problem requiring a powerful tool to solve it [15]. PSO is applied to solve the OPF problem with different objectives that include fuel cost minimization, voltage profile improvement, and voltage stability enhancement [15]. The results confirm the potential of PSO and show its effectiveness and superiority over the classical gradient and GA techniques. However, the main issue of the OPF problem is that only the static constraints without TS are studied, which causes the divergence of the objective function in the case of the occurrence of perturbations and faults [1]. Hence, for the global stability, the transient stability must be added to the OPF problem and the minimization algorithm such as IPSO should take into consideration this constraint [5].

2.1. IMPROVED PARTICLE SWARM OPTIMIZATION

PSO is an evolutionary computation technique developed by Eberhart and Kennedy [16] and was inspired by the social behavior of bird flocking and fish schooling. It uses a population of individuals, called particles, which fly through the problem hyperspace with some given initial velocities. At each iteration, the velocities of the particles are stochastically adjusted considering the historical best position of the particles and their neighborhood best position; where these positions are determined according to some predefined fitness function. Then, the movement of each particle naturally evolves to an optimal or near optimal solution [16].

2.2. BASIC CONCEPT

PSO is an optimization algorithm where, one must have at a given iteration, a set of solutions or alternatives called “particles”. From one iteration to the next one, each particle Xi moves according to a rule that depends on some factors. In order to understand this rule, one must also keep a record of the best point pbest, found by the particle in its past life and the current global best point gbest, find by the swarm of particles in their past life as shown in Fig.1.

\[
X_i^{\text{(new)}} = X_i + V_i^{\text{new}}, \tag{1}
\]

where \(V_i\) is called the \(i^{\text{th}}\) particle velocity defined by:

\[
V_i^{\text{(new)}} = w_{i0}V_i + w_{i1}\text{Rnd}(p_{\text{best}} - X_i) + \frac{w_{i2}}{\text{Iter}}(g_{\text{best}} - X_i), \tag{2}
\]

where the first term of the summation represents inertia or habit (the particle keeps moving in the direction it had previously moved), the second represents memory (the particle is attracted to the best point in its trajectory) and the third represents cooperation or information exchange (the particle is attracted to the best point found by all particles). The parameters \(w_{i0}\) and \(w_{i2}\) are weights fixed in the beginning of the process. Rnd are random numbers sampled from an uniform distribution in \([0, 1]\). The following weighting function is usually used in determining \(w_{i0}\):

\[
w_{i0} = W_{\text{max}} - \frac{W_{\text{min}} - W_{\text{iter}}}{{\text{iter}}} , \tag{3}
\]

\(W_{\text{max}}, W_{\text{min}}\) – initial and final weights, \(\text{iter}_{\text{max}}\) – maximum number of iterations, \(\text{iter}\) – current iteration number.

This function supports the speed of the first iterations.

In addition, the according parameters must be fixed:

\(W_{\text{max}}, W_{\text{min}}, w_{i0}, w_{i2}\), \(\text{iter}_{\text{max}}\) and \(T_{\text{pop}}\), population size.

2.3. MODIFIED PARTICLE SWARM OPTIMIZATION

PSO algorithm presents some limitations and can fall into local minimum problem especially in constraint optimization problem. Hence, to overcome this problem and in order to speed up convergence and improve solution quality, some recent articles introduced effective changes on the classic method of particle swarm optimization [17].

In this work, we have limited ourselves to three proposed changes which are:

2.3.1. Velocity limit of particles

After initializing the positions of particles randomly between their limits, the initial velocities of the particles are randomly selected within the limits [17]:

\[
\epsilon \leq x_i^0 - \epsilon \leq v_i^0 \leq x_i^0 + \epsilon \leq x_i^0 + \epsilon , \tag{4}
\]

with \(\epsilon\), a chosen small positive real number.

In addition, at each iteration \(k+1\), we must limit the velocity of the element \(j\) of the particle \(i\) calculated by [18]:

\[
\]
\begin{equation}
V_{ij}^{k+1} = \begin{cases} 
V_{ij}^{\text{max}} & \text{if } V_{ij}^{k+1} \geq V_{ij}^{\text{max}} \\
V_{ij}^{k} & \text{if } -V_{ij}^{\text{max}} \leq V_{ij}^{k+1} \leq V_{ij}^{\text{max}} \\
-V_{ij}^{\text{max}} & \text{if } V_{ij}^{k+1} \leq -V_{ij}^{\text{max}} 
\end{cases},
\end{equation}

with 
\begin{equation}
V_{ij}^{\text{max}} = \frac{X_{j}^{\text{max}} - X_{j}^{\text{min}}}{R},
\end{equation}

where \(R\) is an integer chosen between 5 and 10.

### 2.3.2. Reducing the search space

In order to accelerate the speed of convergence of the method, a strategy to reduce the search space was introduced in [19]. In this case, the search space is dynamically adjusted; this will allow us to intensify our research in promising areas. Initially, the search limits for the control variable \(j\) in the particle \(i\) are the minimum and maximum control variables:

\begin{equation}
X_{j}^{\text{max}} = X_{j}^{\text{max}} \quad \text{and} \quad X_{j}^{\text{min}} = X_{j}^{\text{min}}
\end{equation}

and at iteration \(k+1\), the search limits become:

\begin{equation}
\begin{aligned}
X_{j}^{k+1,\text{max}} &= X_{j}^{\text{max}} - (X_{j}^{k,\text{max}} - g_{\text{best}}^{k}) \Delta \\
X_{j}^{k+1,\text{min}} &= X_{j}^{\text{min}} - (g_{\text{best}}^{k} - X_{j}^{k,\text{min}}) \Delta
\end{aligned}
\end{equation}

where \(g_{\text{best}}^{k}\) is the best position of particle \(j\) until iteration \(k\) and \(\Delta\) is a chosen step (eq \(= 0.1\)).

The new position of the particle is determined by:

\begin{equation}
X_{j}^{k+1} = \begin{cases} 
X_{j}^{k} + V_{ij}^{k+1} & \text{if } X_{j}^{k,\text{min}} \leq X_{j}^{k} + V_{ij}^{k+1} \leq X_{j}^{k,\text{max}} \\
X_{j}^{k,\text{min}} & \text{if } X_{j}^{k} + V_{ij}^{k+1} < X_{j}^{k,\text{min}} \\
X_{j}^{k,\text{max}} & \text{if } X_{j}^{k} + V_{ij}^{k+1} > X_{j}^{k,\text{max}}
\end{cases}
\end{equation}

### 2.3.3. Crossover operation

To ensure the diversity of the population and to accelerate the convergence of the method, a crossover operation to the conventional PSO algorithm is added [17]. At each iteration \((k+1)\), the new position of each particle \(\bar{x}_{i}^{k+1}\) is replaced by:

\begin{equation}
\bar{x}_{i}^{k+1} = (x_{j}^{i,k+1}, x_{j}^{i+1,k+1}, \ldots, x_{j}^{i,n,k+1}),
\end{equation}

with

\begin{equation}
x_{j}^{i,k+1} = \begin{cases} 
x_{j}^{i,\bar{k}} & \text{if } \bar{r}_{ij}^{k+1} \leq C_{\text{g}} \\text{pbest}_{ij} \quad \text{otherwise.}
\end{cases}
\end{equation}

\(\bar{r}_{ij}^{k+1}\) generates a random number between 0 and 1 inclusive. \(C_{\text{g}}\) is the probability of crossing; the empirically determined optimum value is 0.5 and \(\text{pbest}_{ij}\) is the best position of the particle.

### 3. TRANSIENT STABILITY CONSTRAINTS

Transient stability is the ability of a power system to maintain synchronism when subject to a severe transient disturbance such as a fault on the transmission network and loss of generator and load etc. [20]. After the disturbance, if the generator rotor angle separation between the machines in the system remains within the certain bounds (limits), it can be said that the system maintains synchronism. The transient stability constraints consist of equality constraint i.e. swing equations and inequality constraint i.e. transient stability limits. In this study, the classical model of a synchronous generator is used. This model basically represents a synchronous machine by a constant voltage source behind a transient reactance and the mechanical power input remains constant during the transient period. The load is modeled by constant impedance. The transient stability analysis of the \(i\)-th synchronous generator can be done by solving a set of DAEs (swing equation) that describes the motion of rotor angle as follows:

\begin{equation}
\begin{aligned}
\delta_{i} &= \omega_{i} - \omega_{R} \\
2H_{i}\dot{\omega}_{i} &= \omega_{R} \left( P_{mi} - P_{el} - D_{i}\omega_{i} \right) \quad \text{(12)}
\end{aligned}
\end{equation}

where \(P_{el} = E_{i}^{e}G_{a} + \sum_{j=1}^{NG} E_{i}E_{j}G_{ij}\cos(\delta_{i} - \delta_{j})\).

The transient stability limit, which is inequality constraints, can be formulated as follows:

\begin{equation}
\left| \delta_{i}^{i} - \delta_{i}^{\text{col}} \right| \leq \delta_{\text{max}}
\end{equation}

where \(\delta_{\text{max}}\) is the maximum allowable deviation of rotor angle with respect to COI. This stability limit is acceptable and used widely for utility engineering [2].
4. TSCOPF PROBLEM FORMULATION

A standard OPF problem can be formulated as expressed in [15] with different objectives.

4.1. OBJECTIVE FUNCTION

The main objective of the TSCOPF problem is to minimize the TSCOPF objective function (the total fuel cost) expressed in (16).

\[ F = \sum_{i=1}^{NG} f_i (P_{Gi}) , \]  

where \( F \) is the total fuel cost; \( f_i (P_{Gi}) \) is the fuel cost function of the \( i \)-th generator; \( P_{Gi} \) is the active power output of the \( i \)-th generator. To demonstrate the effectiveness of the proposed method, the generation cost function is represented by a quadratic function as follows:

\[ f (P_{Gi}) = a_i + b_i P_{Gi} + c_i P_{Gi}^2 \quad [\$/h], \]  

where \( a_i, b_i, \) and \( c_i \) are the cost coefficients of the \( i \)-th generators.

4.2. CONSTRAINTS

Equality and inequality constraints are defined as follows:

\[ P_{Gi} - P_{Di} - \sum_{j=1}^{N} V_j (G_{ij} \cos \theta_{ij} + B_{ij} \sin \theta_{ij}) = 0 \quad \text{for } i = 1, 2, \ldots , N \]  

(18)

\[ Q_{Gi} - Q_{Di} - \sum_{j=1}^{N} V_j (G_{ij} \sin \theta_{ij} + B_{ij} \cos \theta_{ij}) = 0 \quad \text{for } i = 1, 2, \ldots , N \]  

(19)

\[ P_{Gi,\text{min}} \leq P_{Gi} \leq P_{Gi,\text{max}} \quad \text{for } i = 1, 2, \ldots , NG \]  

(20)

\[ Q_{Gi,\text{min}} \leq Q_{Gi} \leq Q_{Gi,\text{max}} \quad \text{for } i = 1, 2, \ldots , NG \]  

(21)

\[ V_{i,\text{min}} \leq V_i \leq V_{i,\text{max}} \quad \text{for } i = 1, 2, \ldots , N \]  

(22)

\[ T_{i,\text{min}} \leq T_i \leq T_{i,\text{max}} \quad \text{for } i = 1, 2, \ldots , NT \]  

(23)

\[ |S_L| \leq S_{L,\text{max}} \quad \text{for } i = 1, 2, \ldots , NL \]  

(24)

\[ \dot{\delta} = \omega_i - \omega_R \]  

(25)

\[ 2H_i \dot{\omega}_i = \omega_R (P_{mi} - P_{ei} - D_i \omega_i) \quad \text{for } i = 1, 2, \ldots , NG \]  

(26)

\[ \sum_{i=1}^{NG} H_i (\delta_i - \delta_{\text{col}}) \leq \delta_{\text{max}} \quad \text{for } i = 1, 2, \ldots , NG \]  

(27)

where (18) and (19) are active and reactive power balance equations; Eqs. (20)–(24) are limits of active and reactive power output, voltage magnitude, transformer tap setting, and line loading respectively. Eq. (25) is swing equation that describes transient behavior of a synchronous generator. Eq. (26) is the transient stability limit.

4.3. PROBLEM FORMULATION

According to subsections 4.1 and 4.2, the TSCOPF problem is now formulated as follows:

Minimize Eq. (17)

Subject to Eqs. (18) – (27).

5. SIMULATION RESULTS AND DISCUSSION

The proposed method is applied on the IEEE 30-bus system. In order to obtain the best results, PSO method is applied in two single goals, with and without TS by applying IPSO. The control settings of IPSO method are summarized in Table 1.

<table>
<thead>
<tr>
<th>Control settings of IPSO method</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_{\text{max}} ) 0.9</td>
</tr>
</tbody>
</table>

The prototype program is developed on MATLAB environment and implemented on a personal computer with Intel Core i5 2.6 GHz processor and 8 GB RAM. Numerical examples are divided into 2 cases as follows:

- **Case 1**: solve the optimal power flow problem using IPSO without Transient Stability limit.
- **Case 2**: solve the global problem using IPSO with Transient Stability limit (Transient Stability Constraints TSCOPF).

5.1. IEEE 30-BUS SYSTEM DESCRIPTION

The IEEE 30-bus system consists of 41 transmission lines, 6 generators, and 4 tap-changing transformers. In this system, shunt var compensations are installed at buses 10, 12, 15, 17, 20, 21, 23, 24, and 29. The single-line diagram of the system is shown in Fig. 2. The single-line diagram and bus data are given in [21] and [22].

5.2. CASE 1

The optimal settings of the control variables are given in Table 3. Initially, the total fuel cost was 901.59 $/h. The total cost obtained by IPSO is 797.477 $/h. It is clear that...
the total fuel cost is greatly reduced (12 %) reduction.

To assess the potential of the proposed approach, a comparison between the results of fuel cost obtained by the proposed IPSO approach and those reported in the literature has been carried out. Generally, these methods are metaheuristics with stochastic properties. The comparison is made using a number of successive runs of the same algorithm, starting from different initial solutions/populations. Finally, the best solution produced during these runs is chosen as the reference solution found by that algorithm that is compared to decide the best performance. The results of this comparison are given in Table 2. The obtained results validate the proposed methods and prove their performance from the point of view of the quality of the solution.

Table 2

<table>
<thead>
<tr>
<th>Method</th>
<th>Fuel cost ($/h)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Particle swarm optimization-based approach [13]</td>
<td>800.410</td>
</tr>
<tr>
<td>Differential evolution algorithm [23]</td>
<td>799.2891</td>
</tr>
<tr>
<td>Genetic algorithm [21]</td>
<td>800.805</td>
</tr>
<tr>
<td>Artificial bee colony algorithm micro-genetic [13]</td>
<td>800.419</td>
</tr>
<tr>
<td>Proposed approach</td>
<td>797.477</td>
</tr>
</tbody>
</table>

5.3. CASE 2

A single contingency is considered in the TSCOPF problem. The contingency A shown in Fig.2 is a three-phase grounding fault occurred near to the bus 2 at t = 0 s and cleared by removing the line 2–5 at t = 0.350 s. Transient stability analysis is based on time-domain methods where the non-linear sets of differential and algebraic equations are solved step-by-step using Runge-Kutta (RK45). For time-domain simulation, the integration time step is 0.01 s, the maximum integration time is 1.5 s and $\delta_{\text{max}} = 120$ degrees.

The effectiveness of IPSO will be tested again by more complicated optimization problems, i.e. TSCOPF. Table 3 shows the best solutions, run time in terms of the total generator fuel cost, the control variables and cost differences based on the best solution between TSCOPF and OPF are also given. From this table, the computational times for TSCOPF is highly increased from OPF, since the time domain simulation is performed for every individual during IPSO iterations to calculate the rotor angles of all generators. Figure 3 shows the simulation results of all generator rotor angles relative to the COI when transient stability constraints are not considered, that is, the base case. It is obvious that the system operated at the point suggested by the conventional OPF solutions fails to maintain the transient stability when it is subjected to the credible contingency. As indicated from the TSCOPF results of the IPSO method, the rotor angle curves of all generators are plotted in Fig. 4. It can be seen that, with the TSCOPF optimal solutions, the system is transiently stable. After the contingency A, the rotor angles with respect to COI of 2-nd generator (with the largest swing) based on TSCOPF and OPF solutions using IPSO are shown in Fig. 5. Obviously, TSCOPF can guarantee system stability after the considered contingency whereas OPF cannot. The fuel cost of TSCOPF is obviously higher than the OPF. This is because the additional transient stability constraints are imposed in the OPF. The solution of OPF is actually unstable in contrast to the solution of TSCOPF that tolerates the disturbance. It is clear that the TSCOPF solution is indispensable in the operation of the power system.

6. CONCLUSION

Transient stability constrained optimal power flow TSCOPF is a method that can simultaneously consider the static and dynamic constraints of the power system when optimizing the system. In this paper, the transient stability issue is treated as the additional constraints of the conventional OPF. The additional constraints are the swing equation, which describes the transient behavior of a synchronous generator, and transient stability limit, which is used to evaluate the stability status of the system. The swing equation is quite difficult to handle since it consists of a set of differential equations. Time domain simulation based on the Runge-Kutta (RK45) is adopted to cope with this swing equation.

The computational time of OPF problem is very short. On the other hand, the solution from TSCOPF can guarantee transient stability after a considered contingency. Certainly, increases in fuel cost and computational time are unavoidable.
Nomenclature

\( P_{DI}, Q_{DI} \) – active and reactive power loads at bus \( i \), respectively;
\( Q_{Gi} \) – reactive power generation at bus \( i \);
\( V_i, V_j \) – voltage magnitudes at buses \( i \) and \( j \), respectively;
\( G_{ij}, B_{ij} \) – real and imaginary parts of the \( ij \)-th element of the admittance matrix \( Ybus \) respectively;
\( \theta_g \) – difference of voltage angles between buses \( i \) and \( j \);
\( T_j \) – tap setting of the \( j \)-th transformer;
\( S_{ji} \) – line loading at line \( i \);
\( N \) – buses numbers;
\( NG \) – generating units number ;
\( NL \) – lines numbers;
\( NT \) – transformers numbers ;
\( \omega_R, \omega_i \) – rated speed and rotor speed, respectively;
\( P_{mio} \) – mechanical input and electrical output of the \( i \)-th generator, respectively;
\( D_j \) – damping coefficient of the \( i \)-th generator;
\( \delta_{COI} \) – position of the centre of inertia (COI);
\( \delta_{MAX} \) – maximum allowable deviation of the rotor angle with respect to COI.

Table 3

<table>
<thead>
<tr>
<th>Results</th>
<th>Case 1 Without TS</th>
<th>Case 1 With TS</th>
<th>Case 2 Without TS</th>
<th>Case 2 With TS</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{G1} ) (MW)</td>
<td>138.63</td>
<td>165.62</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( P_{G2} ) (MW)</td>
<td>46.89</td>
<td>35.02</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( P_{G3} ) (MW)</td>
<td>19.26</td>
<td>34.96</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( P_{G4} ) (MW)</td>
<td>24.15</td>
<td>24.83</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( P_{G5} ) (MW)</td>
<td>12.52</td>
<td>17.17</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( P_{G6} ) (MW)</td>
<td>12.00</td>
<td>12.00</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( V_1 ) (pu)</td>
<td>1.050</td>
<td>1.050</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( V_2 ) (pu)</td>
<td>1.050</td>
<td>1.006</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( V_3 ) (pu)</td>
<td>1.050</td>
<td>0.958</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( V_4 ) (pu)</td>
<td>1.050</td>
<td>0.974</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( V_5 ) (pu)</td>
<td>1.050</td>
<td>1.016</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( V_6 ) (pu)</td>
<td>1.050</td>
<td>1.022</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( T_1 ) (bus 6-9)</td>
<td>1.078</td>
<td>0.984</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( T_2 ) (bus 6-10)</td>
<td>1.100</td>
<td>1.053</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( T_3 ) (bus 4-12)</td>
<td>1.050</td>
<td>0.900</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( T_4 ) (bus 28-27)</td>
<td>0.906</td>
<td>1.017</td>
<td></td>
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</tr>
<tr>
<td>( Q_{C10} ) (Mvar)</td>
<td>4.730</td>
<td>2.500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Q_{C11} ) (Mvar)</td>
<td>5.000</td>
<td>5.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( Q_{C12} ) (Mvar)</td>
<td>0.570</td>
<td>2.00</td>
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REFERENCES


