THE CO-CHANNEL INTERFERENCE EFFECT ON AVERAGE ERROR RATES IN NAKAGAMI-Q (HOYT) FADING CHANNELS

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In this paper influence of co-channel interference (CCI) on the digital mobile radio systems operating over Nakagami-q (Hoyt) fading channels is investigated. Useful expressions are derived for average error rates over coherent and non-coherent frequency and phase- shift keying modulation schemes in interference limited system. The effect of selection diversity on the performance improvement for various values of fading severity and input signal-to-interference ratio (SIR) unbalance is also examined.

1. INTRODUCTION

Wireless communication systems performances are remarkably affected by multipath fading phenomena, which is a major source of performance impairment in a mobile environment. Frequency reuse, necessary for increasing land–mobile radio systems spectrum efficiency of many land–mobile radio systems also causes co-channel interference (CCI) which, in some systems, are more significant than the front-end Gaussian noise. Also, the CCI is subject to fading and it is necessary to incorporate this effect in assessing the performance of the wireless system. In interference limited fading environment systems, signal-to-interference ratio (SIR) based performance analysis is the most effective performance criterion, since SIR can be measured in real time both in base stations and in mobile stations using specific SIR estimators.

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While the Rayleigh and Rice distributions can be indeed used to model the envelope of fading channels in many cases of interest, it has been found experimentally, that the Nakagami distribution offers a better fit for a wider range of fading conditions in wireless communications. Nakagami-m fading describes multipath scattering with relatively large delay-time spreads, with different clusters of reflected waves In addition to Nakagami-m fading, the Nakagami-q (Hoyt) is a channel fading distribution that has received attention recently [3–6].

Well-known measure for evaluating digital mobile radio system performances over channel condition is the average bit error probability. The results known in the literature for the average error performance of M-ary coherent, M-ary differentially coherent, and noncoherent correlated binary modulation schemes over Nakagami-q channels are given in integral forms based on the moment generating function (MGF) approach [7, 8]. In [9], exact-form solutions for the integral-based formulas resulting in new expressions for the average error performance of different M-ary modulation schemes are derived. It is shown that the derived expressions reduce to the previously known results for Rayleigh fading as special cases. However, to the best of author's knowledge, performance studies of interference limited systems in the Nakagami-q fading environment has not so far been reported in the literature.

In this paper an approach to the CCI influence analysis on the average error rates of digital mobile radio systems operating over Nakagami-q (Hoyt) fading channels is presented. Closed form expressions are provided for the average probability of error of various modulation schemes in interference limited system. The effect of selection diversity on the performance improvement for various values of fading severity and input signal-to-interference ratio (SIR) unbalance is also examined.

2. SYSTEM MODEL

The desired signal envelope in a Nakagami-q fading channel follows the probability density function (pdf) given by [7] as:

$$p_R(R) = \frac{(1+q_d^2)R}{q_d\Omega_d}\exp\left(-\frac{(1+q_d^2)^2 R^2}{4q_d^2\Omega_d}\right)I_0\left(\frac{(1-q_d^4)R^2}{4q_d^2\Omega_d}\right), \tag{1}$$

where $\Omega_d = \mathrm{E}\,[R^2]$ is the desired signal average power, $I_0\,(x)$ is the zero-th order modified Bessel function of the first kind and $0 \leq q_d \leq 1$ is the desired signal Hoyt fading parameter. Hoyt distribution spans the range from one-sided Gaussian fading ($q_d = 0$) to Rayleigh fading ($q_d = 1$). It is typically observed on satellite links subject to strong ionospheric scintillation.

Similarly the CCI envelope in a Nakagami-q fading channel follows:

$$p_r(r)=\frac{(1+q_c^2)r}{q_c\Omega_c}\exp\left(-\frac{(1+q_c^2)^2r^2}{4q_c^2\Omega_c}\right)I_0\left(\frac{(1-q_c^4)r^2}{4q_c^2\Omega_c}\right), \tag{2}$$

with parameters defined in similar manner.

In an interference-limited system the effect of noise may be ignored. In this case it can be shown that the instantaneous signal-to-interference ratio (SIR), $\lambda = R^2/r^2$, has the pdf [2]:

$$p_\lambda(t)=\frac{1}{2\sqrt{t}}\int_0^\infty p_R(r\sqrt{t})p_r(r)r\mathrm{d}r\ . \tag{3}$$

After substituting (1) and (2) in (3), we derive:

$$p_\lambda(t)=\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\left(\frac{S^{2l+1}\left(1-q_d^4\right)^{2k}\left(1-q_c^4\right)^{2l}\left(1+q_d^4\right)\left(1+q_c^2\right)}{2^{2k+2l-2}\Gamma(k+1)k!\Gamma(l+1)l!}\frac{\Gamma(2k+2l+2)q_d^{4l+3}q_c^{4k+3}\lambda^{2k}}{\left(\lambda\left(1+q_d^2\right)^2q_c^2+S\left(1+q_c^2\right)^2q_d^2\right)^{2k+2l+2}}\right) \tag{4}$$

where the ratio $S = \Omega_d/\Omega_c$ is the average-signal to average interference power ratio (SIR), which is useful in determining the co-channel reduction factor in systems with frequency reuse. The derivation is explained in Appendix 1.

The cumulative distribution function (cdf) of λ can be obtained like in [2] as:

$$F_\lambda(t)=\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\left(\frac{\left(1-q_d^4\right)^{2k}\left(1-q_c^4\right)^{2l}q_dq_c!\Gamma(2k+2l+2)}{2^{2k+2l-2}\Gamma(k+1)k!\Gamma(l+1)l!\left(1+q_d^2\right)^{4k+1}\left(1+q_c^2\right)^{4l+1}}\frac{\left(\dfrac{\lambda\left(1+q_d^2\right)^2q_c^2}{\lambda\left(1+q_d^2\right)^2q_c^2+S\left(1+q_c^2\right)^2q_d^2}\right)^{2k+1}}{2k+1}\right.$$
$$\left.{}_2F_1\left(2k+1,-2l;2k+2;\frac{\lambda\left(1+q_d^2\right)^2q_c^2}{\lambda\left(1+q_d^2\right)^2q_c^2+S\left(1+q_c^2\right)^2q_d^2}\right)\right). \tag{5}$$

The derivation is explained in Appendix 2.

3. AVERAGE PROBABILITY OF ERROR

In an interference-limited environment, the average bit error probability can be derived by averaging the conditional error probability P_e over the pdf of SIR:

$$\bar{P}_e=\int_0^\lambda P_e(\lambda)p_\lambda(\lambda)\mathrm{d}\lambda\ . \tag{6}$$

For a noncoherent frequency shift keying (NCFSK) or a differentially coherent phase-shift keying (DPSK) system, the conditional probability of error for a given SIR, λ , is given by $P_e = 1/2 \exp(-\alpha\lambda)$, with $\alpha = 1$ for binary PSK, and with $\alpha = 1/2$ for binary FSK. After substituting (4) into (6), by using [10] previous can be easily written as :

$$\bar{P}_e = \sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\left(\frac{\left(1-q_d^4\right)^{2k}\left(1-q_c^4\right)^{2l} q_d q_c \Gamma(2k+2l+2)\Gamma(2k+1)}{2^{2k+2l-1}\Gamma(k+1)k!\Gamma(l+1)l!\left(1+q_d^2\right)^{4k+1}\left(1+q_c^2\right)^{4l+1}}\psi\left(2k+1;-2l;\frac{\alpha S\left(1+q_c^2\right)^2 q_d^2}{\lambda\left(1+q_d^2\right)^2 q_c^2}\right)\right) \tag{7}$$

where $\psi(a, b, x)$ is the confluent hypergeometric function of the second kind [10]. The derivation is explained in Appendix 3.

For a coherent system, the conditional probability of error may be expressed in terms of the confluent hypergeometric function [10] as:

$$P_e = \frac{1}{2}\mathrm{erfc}(\sqrt{\alpha\lambda}) = \frac{1}{2}\left(1-2\sqrt{\frac{\alpha\lambda}{\pi}}\ {}_1F_1\left(\frac{1}{2};\frac{3}{2};-\alpha\lambda\right)\right). \tag{8}$$

Similarly as previous after substituting (4) into (8), we obtain:

$$\bar{P}_e = \frac{1}{2}-\sqrt{\frac{\alpha\lambda}{\pi}}\sum_{k=0}^{\infty}\sum_{l=0}^{\infty}\sum_{p=0}^{\infty}\left(\frac{S^{p+\frac{1}{2}}\left(1-q_d^4\right)^{2k}\left(1-q_c^4\right)^{2l} q_d^{\,2p+2}\Gamma\left(2k+p+\frac{3}{2}\right)}{2^{2k+2l-2}\left(1+q_d^2\right)^{4k+1}\left(1+q_c^2\right)^{4l+1} q_c^{\,2p}}\right.$$
$$\left.\frac{\alpha^p\left(\frac{1}{2}\right)_p\Gamma(2k+2l+2)}{\left(\frac{3}{2}\right)_p p!\Gamma(k+1)k!\Gamma(l+1)l!}\psi\left(2k+p+\frac{3}{2};p+\frac{1}{2}-2l;\frac{\alpha S\left(1+q_c^2\right)^2 q_d^2}{\lambda\left(1+q_d^2\right)^2 q_c^2}\right)\right) \tag{9}$$

where $(a)_p$ is the Pochhammer symbol [10] and α was previously defined for required modulation techniques.

4. **DIVERSITY EFFECT**

The mitigation of fading and co-channel effects could be reached through the space diversity techniques. Their goal is to upgrade transmission reliability without increasing transmission power and bandwidth and to increase channel capacity. There are several principal types and selection combining (SC) is one of the least complicated combining methods. In fading environments as in cellular systems where the level of the cochannel interference is sufficiently high as compared to the thermal noise, SC selects the branch with the highest signal-to-interference ratio (SIR-based selection diversity). The average probability of error for coherent and noncoherent binary FSK and PSK can be easily extended to include the effect of SC. In multibranch SC diversity system with independent statistics on the branches, the pdf of the SC output SIR is given by:

$$p_\lambda(t) = \sum_{j=1}^{N} p_{\lambda_j}(t) \prod_{\substack{k=1 \\ k \neq j}}^{N} F_{\lambda_k}(t), \tag{10}$$

with N denoting the number of branches.

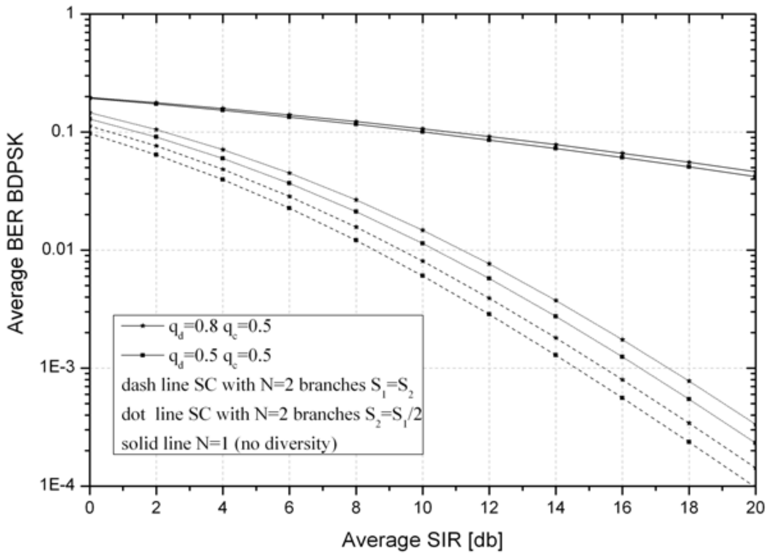

Fig. 1 – Average error probability versus average SIR for binary DPSK in a Nakagami-*q* fading environment for various values of system parameters.

In Fig. 1 and Fig. 2 diversity effect on the average error probability versus average SIR for binary DPSK and NCFSK in a Nakagami-*q* fading environment for various values of system parameters are presented. Here we can observe obvious upgrading of system performances when diversity is present (case of two diversity branches). Also it is evident that for the case of balanced input SIR (S_1=S_2), system has better performances (smaller value of average error probability) than for the case of unbalanced input SIR (S_2 = S_1/2). Finally we can conclude from the figures that higher values of fading severity parameter q_d deteriorate system performances.

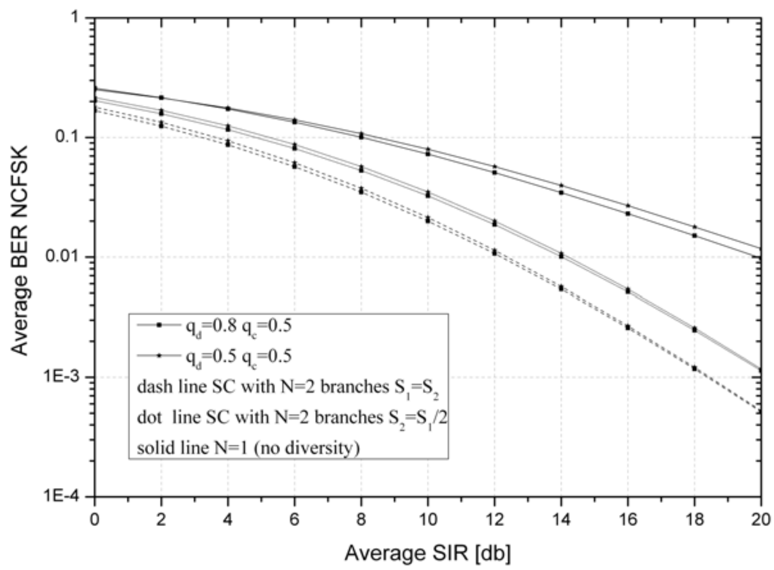

Fig. 2 – Average error probability versus average SIR for binary NCFSK in a Nakagami-*q* fading environment for various values of system parameters.

5. **CONCLUSIONS**

In this paper, average error probability of interference-limited digital cellular mobile radio communication system over frequency and phase-shift keying modulation schemes were investigated. The desired signal as well as the interference were assumed to be subject to Nakagami-q fading. The effect of SC diversity on system performance was also considered. The effects of various parameters such as fading severity and input SIR unbalance were also presented.

APPENDIX 1

Defining the instantaneous signal-to-interference ratio (SIR) as $\lambda = R^2/r^2$, the pdf of SIR can be obtained as [2]:

$$p_\lambda(t) = \frac{1}{2\sqrt{t}} \int_0^\infty p_R(r\sqrt{t}) p_r(r) r \mathrm{d}r \tag{A.1}$$

Substituting (1) and (2) in (A.1), with respect to well-known series representation of modified Bessel function:

$$I_v(z) = \sum_{k=0}^{\infty} \frac{z^{v+2k}}{2^{v+2k} k! \Gamma(v+k+1)} \tag{A.2}$$

results in:

$$p_\lambda(t) = \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} C_1 \int_0^\infty r^{4k+4l+3} \exp\left(-r^2 \lambda \frac{\left(1+q_d{}^2\right)^2}{4q_d{}^2\Omega_d}\right) \exp\left(-r^2 \frac{\left(1+q_c{}^2\right)^2}{4q_c{}^2\Omega_c}\right) \mathrm{d}r\,, \tag{A.3}$$

with:

$$C_1 = \frac{S^{2l+1}\left(1-q_d^4\right)^{2k}\left(1-q_c^4\right)^{2l}\left(1+q_d^4\right)\left(1+q_c^2\right)\lambda^{2k}}{2^{6k+6l+1}\Gamma(k+1)k!\Gamma(l+1)l!q_d^{4k+1}q_c^{4l+1}\Omega_d^{2k+1}\Omega_c^{2l+1}}\,. \tag{A.4}$$

Let $S_k = \Omega_d / \Omega_c$ denote the average SIR's value. Then, the following integrals from previous expression can be presented in the form:

$$I = \int_0^\infty r^{4k+4l+3} \exp\left(-r^2 \frac{\left(1+q_d{}^2\right)^2 q_c{}^2\lambda + \left(1+q_c{}^2\right)^2 q_d{}^2 S}{4q_d{}^2 q_c{}^2 \Omega_d}\right) \mathrm{d}r\,. \tag{A.5}$$

Now by using variable substitution:

$$t = r^2 \frac{\left(1+q_d{}^2\right)^2 q_c{}^2\lambda + \left(1+q_c{}^2\right)^2 q_d{}^2 S}{4q_d{}^2 q_c{}^2 \Omega_d} \tag{A.6}$$

and well-known definition of Gamma function:

$$\Gamma(a) = \int_0^\infty t^{a-1} \exp(-t)\,\mathrm{d}t \tag{A.7}$$

finally, (A.3) can be written as:

$$p_\lambda(t) = \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} \left(\frac{S^{2l+1}\left(1-q_d^4\right)^{2k}\left(1-q_c^4\right)^{2l}\left(1+q_d^4\right)\left(1+q_c^2\right)}{2^{2k+2l-2}\Gamma(k+1)k!\Gamma(l+1)l!} \frac{\Gamma(2k+2l+2) q_d^{4l+3} q_c^{4k+3} \lambda^{2k}}{\left(\lambda\left(1+q_d^2\right)^2 q_c^2 + S\left(1+q_c^2\right)^2 q_d^2\right)^{2k+2l+2}} \right) \tag{A.8}$$

APPENDIX 2

CDF of output SIR could be derived from:

$$F_\lambda(t) = \int_0^t p_\lambda(x)\mathrm{d}x \tag{A.9}$$

by substituting (4) into (A.9), we obtain the following expression:

$$F_\lambda(t) = \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} \frac{S^{2l+1}\left(1-q_d^4\right)^{2k}\left(1-q_c^4\right)^{2l}\left(1+q_d^4\right)\left(1+q_c^2\right)\Gamma(2k+2l+2)\,q_d^{4l+3}q_c^{4k+3}}{2^{2k+2l-2}\Gamma(k+1)k!\Gamma(l+1)l!}\cdot \int_0^t \frac{x^{2k}}{\left(x\left(1+q_d^2\right)^2 q_c^2 + S\left(1+q_c^2\right)^2 q_d^2\right)^{2k+2l+2}}dx \tag{A.10}$$

Previous integral can easily be solved, using the well-known definition of incomplete beta function:

$$\int_0^\lambda \frac{x^m}{\left(a+bx^n\right)^p} = \frac{a^{-p}}{n}\left(\frac{a}{b}\right)^{\frac{m+1}{n}} B_z\left(\frac{m+1}{n}, p-\frac{m+1}{n}\right); \qquad z = \frac{b\lambda^n}{a+b\lambda^n}, \quad a>0, \quad b>0, \quad n>0, \quad 0<\frac{m+1}{n}<p. \tag{A.11}$$

By using the famous relationship between incomplete beta and $_2F_1$ hypergeometric function:

$$B_z(a,b) = \frac{z^a}{a} F_1(a,1-b,1+a,z) \tag{A.12}$$

and after some straightforward manipulations we obtain CDF in the form of (5)

APPENDIX 3

For a noncoherent frequency shift keying (NCFSK) or a differentially coherent phase-shift keying (DPSK) system, the average bit error probability for a given SIR, λ, can be calculated from:

$$\bar{P}_e = \frac{1}{2}\int_0^\lambda \exp(-\alpha\lambda)p_\lambda(\lambda)\mathrm{d}\lambda\,, \tag{A.13}$$

with $\alpha = 1$ for binary PSK, and with $\alpha = 1/2$ for binary FSK. By substituting (4) into (A.13), we obtain the following expression:

$$\bar{P}_e = \sum_{k=0}^{\infty}\sum_{l=0}^{\infty} \frac{S^{2l+1}\left(1-q_d^4\right)^{2k}\left(1-q_c^4\right)^{2l}\left(1+q_d^4\right)\left(1+q_c^2\right)\Gamma(2k+2l+2)\,q_d^{4l+3}q_c^{4k+3}}{2^{2k+2l-1}\Gamma(k+1)k!\Gamma(l+1)l!} \int_0^{\infty}\frac{\lambda^{2k}}{\left(\lambda\left(1+q_d^2\right)^2 q_c^2+S\left(1+q_c^2\right)^2 q_d^2\right)^{2k+2l+2}}\exp(-\alpha\lambda)\,\mathrm{d}\lambda. \tag{A.14}$$

Previous integral can easily be solved, using the relation:

$$\int_0^{\infty} x^{q-1}\exp(-px)(1+ax)^{-v}\,\mathrm{d}x = a^{-q}\,\Gamma(q)\,\Psi\left(q,q+1-v,\frac{p}{a}\right), \tag{A.15}$$

where ψ (a, b, x) is the confluent hypergeometric function of the second kind [10]. After straightforward manipulations and integration (A.14) reduces into (7). Using relation (8) after applying same manipulations and integration coherent system average bit error probability can be presented in the form of (9).

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