ESTIMATION OF THE NUMERICAL MODEL PARAMETERS FOR THE SYNCHRONOUS MACHINE USING TESTS AT STANDSTILL

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The paper presents two methods, dc decay test and standstill frequency response (SSFR) test, used to estimate the parameters of the synchronous machine mathematical model. The synchronous and differential magnetization inductances, the resistances and leakage inductances of equivalent rotor circuits for starting and damping squirrel cages, as well as the incremental magnetization inductances are determined by experimental tests. Magnetic saturation is taken into account through synchronous and differential magnetization inductances variation against total magnetization current and the skin effect is considered by using rotor short-circuited windings with constant parameters. The d-q rotor coordinate system is used, mathematical computations are done using Matlab-Simulink professional program, while for experimental tests high performance measuring apparatus were used.

1. INTRODUCTION

In general, the analysis of the synchronous machine operation is performed by adopting mathematical models generated from electric circuit equations and the general motion equation. Taking into account all constructive aspects, as well as the variation of the material properties against temperature and mechanical stress, complex and impractical mathematical models are commonly generated. That is why simplifications are made according to targeted objectives. Therefore, in many cases, mathematical models based on the orthogonal coordinate system and constant parameters are used instead of more complex models with variable parameters and phase coordinates. However, for an accurate analysis of the synchronous machine operation, two main phenomena occurring in most operation situations – magnetic saturation and skin effect – must be considered.

Saturation can be modeled by variable magnetization inductances, while skin effect is taken into account by considering the resistance and leakage inductance of the starting and dumping cage as variable with frequency [1]. This frequency dependency prevents modeling the machine by a simple circuit; instead, parallel multiple circuits with constant parameters are preferred. For an accurate determination of the parameters, numerical and experimental methods are commonly used [2–5]. Numerical methods used for electromagnetic field computation lead also to accurate results, but require the exact knowledge of material properties and the machine geometry must be precisely defined [6]. Experimental tests can be conducted for machine real operation conditions or by using special methods that provide some practical advantages. Common experimental methods are frequently used, but they are difficult to implement for high power machines, and imply high energy consumption. There are, however, special methods trying to overcome these shortcomings and to provide proper results by adopting easier to implement and less costly procedures [7–13]. The paper presents two such special methods, dc decay test and standstill frequency response test, used for establishing the parameters of the synchronous mathematical model considering magnetic saturation and skin effect.

2. SYNCHRONOUS MACHINE MATHEMATICAL MODEL

If the rotor coordinate system (with orthogonal d and q axes) is taken as reference, then the equation system corresponding to the mathematical model of the synchronous generator can be written as follows [1]:

\[
\begin{align*}
\dot{i}_d \cdot R_s + u_d &= -\frac{d\varphi_d}{dt} + \omega_r \cdot \varphi_q, \\
\dot{i}_q \cdot R_s + u_q &= -\frac{d\varphi_q}{dt} - \omega_r \cdot \varphi_d, \\
\dot{i}_e \cdot R_e - u_e &= -\frac{d\varphi_e}{dt}, \\
R_D \cdot \dot{\varphi}_D &= -\frac{d\varphi_D}{dt}, \\
R_Q \cdot \dot{\varphi}_Q &= -\frac{d\varphi_Q}{dt},
\end{align*}
\]

where: \(u_d, u_q, u_e\) are stator winding voltages; \(i_d, i_q, i_e\) are stator and rotor currents; \(\varphi_d, \varphi_q, \varphi_e, \varphi_D, \varphi_Q\) are stator and rotor magnetic fluxes; \(R_s\) is the stator winding resistance; \(R_e\) is the field coil resistance; \(R_D\) and \(R_Q\) are the starting and damping cage equivalent resistances; \(\omega_0\) is the rotor angular velocity.

Under the assumption of distinct main field path from the leakage field path and neglecting mutual leakage inductances, the magnetic fluxes can be written as [1]:

\[
\begin{align*}
\varphi_d &= L_{sd} \cdot i_d + \varphi_{dp}, \\
\varphi_q &= L_{sq} \cdot i_q + \varphi_{qp}, \\
\varphi_D &= L_{Dd} \cdot i_D + \varphi_{dp}, \\
\varphi_Q &= L_{Qq} \cdot i_Q + \varphi_{qp}, \\
\varphi_e &= L_{es} \cdot i_e + \varphi_{dp},
\end{align*}
\]

where \(\varphi_{dp}, \varphi_{qp}\) are main magnetization fluxes; \(L_{sd}, L_{sq}, L_{Dd}, L_{Qq}, L_{es}\) represent leakage inductances corresponding to the field coil and to the rotor circuit respectively.

The conversion from quantities describing the real machine to those defining the mathematical model is achieved using Park transformation [1].

In order to model magnetic saturation, magnetic characteristics of the machine on d-axis and q-axis are defined by variations of the main magnetic fluxes against the total magnetization current: \(\Psi_{dq}(i)\) and \(\Psi_{pq}(i)\). Taking into account these dependences, the main magnetic flux components result [1]:

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\[ \Psi_{d}(i_t) = \frac{\Psi_{dp}(i_t)}{i_t}, \quad L_d(i_t) = \frac{\Psi_{dp}(i_t)}{i_t}, \]
\[ \Psi_{q}(i_t) = \frac{\Psi_{qp}(i_t)}{i_t}, \quad L_q(i_t) = \frac{\Psi_{qp}(i_t)}{i_t}. \]  

Equations (3) and (4) in eq. (3) are the synchronous magnetization inductances, which depend on the total magnetization current and are determined by:
\[ i_t = \sqrt{i_d^2 + i_q^2} \cdot i_d = i_d + i_e + i_{D}, \quad i_{D} = i_q + i_{Q}. \]

For a more accurate reflection of the electromagnetic phenomena under time-variable operating conditions, differential magnetization inductances are also defined:
\[ L_{d}(i_t) = \frac{d\Psi_{dp}(i_t)}{di_t}, \quad L_{q}(i_t) = \frac{d\Psi_{qp}(i_t)}{di_t}. \]  

The skin effect which is predominantly noticed in the starting and damping rotor cage can be taken into account by introducing multiple circuits with constant parameters, connected in parallel, which will best approximate the parameters’ dependency with frequency (Fig. 1).

Two rotor circuits are used to model the skin effect for \( d \)-axis, and only one circuit for \( q \)-axis, as skin effect influence is reduced in the \( q \)-axis.

![Fig. 1 – Synchronous machine equivalent circuits for SSFR test [1]: a) for \( d \)-axis; b) for \( q \)-axis.](image)

![Fig. 2 – Positioning the rotor and winding connections: a) rotor positioned with \( d \)-axis along direction of phase U; b) windings connections for \( d \)-axis measurements; c) windings connections for \( q \)-axis.](image)

### 3. STANDSTILL METHODS USED TO ESTIMATE THE PARAMETERS FOR THE SYNCHRONOUS MACHINE MATHEMATICAL MODEL

#### 3.1. POSITION OF THE ROTOR AND WINDINGS CONNECTION

The rotor is placed with its \( d \)-axis aligned with the phase U. In the meantime, phases V and W are connected in series and fed by an ac low voltage source. The field winding is short-circuited with an ammeter while the rotor is turned slowly following the meter indication. When the ammeter shows zero, the rotor position is achieved (Fig. 2a).

In order to conduct measurements on \( d \)-axis, the windings are connected according to Fig. 2b. In this case, the stator winding currents meet the following relation: \( i_e = i_{w} = i_d/2 \). Applying Park transformation, the quantities of the orthogonal model are determined:
\[ u_{d} = u_{vw} \cdot \sqrt{2}/3, \quad i_{d} = i_{u} \cdot \sqrt{3}/2, \quad u_{q} = 0, \quad i_{q} = 0. \]

For the \( q \)-axis experimental tests, only phases V and W are connected in series (Fig. 2c), which means: \( i_e = -i_{w} \) and \( i_{w} = 0 \). Using Park transformation, the quantities of the orthogonal model are:
\[ u_{q} = u_{vw} / \sqrt{2}, \quad i_{q} = i_{v} \cdot \sqrt{2}, \quad u_{d} = 0, \quad i_{d} = 0. \]

#### 3.2. DC DECAY TESTS

The dc decay tests imply feeding the stator and field windings with dc voltage; next, one winding circuit is short-circuited, while recording the current and voltage decay [1, 7–9, 13].

For dc decay tests in \( d \)-axis, the stator and field windings are fed with dc currents (\( i_{w0} \) and \( i_{e0} \)). Then, the stator winding is short-circuited, while maintaining the dc supply for the field circuit; the decay current characteristic is recorded. Short-circuit condition applied to the first equation of (1) for the rotor at standstill, leads to:

\[ \psi_{dp} = \frac{\psi_{dp}(i_t)}{i_t}, \quad \psi_{qp} = \frac{\psi_{qp}(i_t)}{i_t}. \]
Integrating this equation and taking into account relations (2–4) and (6), after some computation, the following relations result [9, 13]:

\[
\Psi_{dp} = \Psi_{dp}^\text{final} + \sqrt{3/2 \cdot R_s \cdot i} \int_0^{t \rightarrow \infty} \frac{u_{avw} \text{ d}t}{\sqrt{3/2 \cdot i_{u0}}},
\]

\[i_t = \sqrt{3/2 \cdot i_{u0} + i_{d0}}.\]

The final value of the main magnetic flux is obtained only from the field circuit and it can be determined by short-circuiting the field coil and recording the voltage decay at the stator terminals. With the equation corresponding to the \(d\)-axis of the orthogonal mathematical model (1) for the rotor at standstill and equation (6), it results:

\[
\Psi_{dp}^\text{final} = \sqrt{2/3} \cdot \int_0^{t \rightarrow \infty} u_{avw} \text{ d}t.
\]

Synchronous and differential inductances are determined for the magnetization current \((i_t)\) using equations (3), (5), (9) and (10). The procedure is repeated for different values of stator and field currents obtaining \(L_d(i_t)\) and \(L_{dq}(i_t)\). Conducting the dc decay tests in \(q\)-axis implies a similar procedure: while the field and stator windings are fed by dc currents \((i_{d0}\) and \(i_{q0}\)) the stator winding is short-circuited. Short-circuit condition is applied to the second equation of (1) for the rotor at standstill; after some computation [9, 13] the following equations result:

\[
\Psi_{eq} = \sqrt{2} (R_s \int_0^{t \rightarrow \infty} i_t \text{ d}t - L_{qf} i_{q0} \cdot \sqrt{1/2 \cdot (i_{e0} / i_{q0})^2 + 1})
\]

\[i_t = \sqrt{i_{e0}^2 + 2 \cdot i_{q0}^2}.
\]

Using equations (3), (5) and (11) for different values of the stator and field currents, \(L_q(i_t)\) and \(L_{q(f)}(i_t)\) are obtained.

### 3.3. STANDSTILL FREQUENCY RESPONSE TESTS

Under the standstill frequency response (SSFR) tests, the stator circuit is fed from an ac voltage supply with variable frequency the voltage and current waveforms are recorded [1, 10–13]. Since the skin effect occurs mainly in rotor starting and dumping cage, the field current will be neglected. Therefore, if the electrical diagram corresponding to the \(d\)-axis (Fig. 1a) is considered, an equivalent operational inductance \(L_d(j\omega)\) can be defined satisfying the following relation:

\[
U_d - R_s \cdot i_d = L_d(j\omega) \cdot (j\omega L_d^\text{eff}).
\]

This equivalent operational inductance can be determined by acquiring \(u_{avw}\) voltage and \(i_u\) current waveforms at different frequencies and using equations (6). The equivalent operational inductance can be expressed as follows:

\[
L_d(j\omega) = (L_{sat} + L_d^\text{eff}(j\omega)) \cdot \left(\frac{1 + j\omega T_d}{1 + j\omega T_d^2}\right) = \left(\frac{1 + j\omega T_d}{1 + j\omega T_d^2}\right) \cdot \left(\frac{1 + j\omega T_d}{1 + j\omega T_d^2}\right)^2.
\]

where the time constants \(T_{d1}, T_{d2}, T_{d3}, T_{d4}\) are functions of the equivalent circuit parameters.

The equivalent operational inductance given by (13) must be estimated in order that dependency (12) is met with minimal errors for a domain of frequency as large as possible. The estimation can be achieved using an output-error model estimator defined in Matlab-Simulink professional program, which allows finding a transfer function based on a given input quantity for which an output quantity with minimum error is obtained. Thus, if equation (12) is considered within magnitude domain, together with equations (6), the input and output functions necessary for the estimation bloc are determined [12, 13]:

\[
\text{In}_d = (3/2) \cdot i_u + \omega
\]

\[
\text{Out}_d = \left[\text{Re}(U_{avw}) - \frac{3}{2} \cdot \text{Re}(L_d \cdot R_s)\right]^2 + \left[\text{Im}(U_{avw}) - \frac{3}{2} \cdot \text{Im}(L_d \cdot R_s)\right]^2.
\]

On the other hand, if the previously mentioned time constants are determined, the following system of equations could be generated:

\[
\begin{align*}
T_{d1}^2 + T_{d2}^2 - T_{d3}^2 - T_{d4}^2 &= \\
&= \frac{(R_D^2 L_{D2} + R_{D2} L_{D2a} \cdot L_{D2} + L_{D2}^2) + L_{D2}^2 L_{D2a} \cdot L_{D2} + L_{D2}^2}{L_{D2} L_{D2a} + L_{D2}^2 L_{D2a}} \cdot \frac{L_{D2} L_{D2a} + L_{D2}^2 L_{D2a}}{L_{D2} L_{D2a} + L_{D2}^2 L_{D2a}}.
\end{align*}
\]

where \(L_{D1}, L_{D2}, L_{D2a}, L_{D2a}\) are the resistances and leakage inductances of multiple circuits for simulating the skin effect; \(L_d^\text{eff}(j\omega)\) is the incremental magnetization inductance defined for low magnetic saturation; \(L_a\) is the stator leakage inductance.

The solution of this equations system determines the parameters of the equivalent circuit.

Conducting the SSFR test for the \(q\)-axis implies a similar procedure. The equivalent circuit, however, has only one circuit for simulating the skin effect, while the equivalent operational inductance has the following form:

\[
L_q(j\omega) = (L_{sat} + L_q^\text{eff}(j\omega)) \cdot \left(\frac{1 + j\omega T_q}{1 + j\omega T_q^2}\right).
\]
Using equations (7), the input and output functions result as follows [12, 13]:

\[
\begin{align*}
\text{In}_q &= 2 \cdot I_q \cdot \omega, \\
\text{Out}_q &= \sqrt{[\Re(U_{vw}) - 2 \cdot \Re(L_q) \cdot R_s]^2 + \left[5m(U_{vw}) - 2 \cdot 5m(L_q) \cdot R_q\right]^2}.
\end{align*}
\] (17)

Once the time constants are determined, solving the equivalent circuit generates the following relations:

\[
T_{q1} = \frac{L_{Q1} \sigma}{R_{Q1}} + \frac{1}{R_{Q1}} \cdot \frac{L_{\sigma} L_{q(diff)}}{(L_{\sigma} + L_{q(diff)})},
\]

\[
T_{q2} = \frac{L_{Q1} \sigma + L_{q(diff)}}{R_{Q1}},
\] (18)

where \(R_{Q1}, L_{Q1,\sigma}\) are the resistance and leakage inductance of circuit for simulating the skin effect; \(L_{q(diff)}\) is the incremental magnetization inductance defined for low magnetic saturation; \(L_{\sigma}\) is the stator leakage inductance.

The equivalent circuit parameters are then determined from the above equations.

4. EXPERIMENTAL TESTS. RESULTS

In order to conduct the experimental tests under standstill condition and to determine the mathematical model parameters for the synchronous machine, the electric circuit shown in Fig. 3 is used. An oscilloscope Tektronix TDS 2000B and a grid analyzer Chauvin Arnoux CA 8334 were used for measurements.

Experimental tests were conducted on a synchronous machine with salient poles and following rated data: \(S_n = 2500\) VA; \(U_n = 208/120\) V; \(I_n = 6.95/12\) A; \(n_n = 1500\) rpm; \(U_{en} = 70\) V; \(I_{en} = 2.93\) A; \(\cos \phi_n = 0.8\).

The stator winding resistance measured in dc is \(R_s = 1\) Ω and the leakage inductance is approximated with the homopolar inductance \(L_{en} = 0.0044\) H.

For dc decay test the power supply was carried out using the shunt dc generator that allows short circuits at the terminals. For SSFR test we used a synchronous generator driven at different speeds to obtain different frequency values.

In Fig. 4 the decay of the stator current and voltage at the stator circuit terminals are shown for the dc decay test in \(d\)-axis, at different stator and field current values.

Applying the procedure described in paragraph 3.2, the synchronous and differential magnetization inductances in the \(d\) and \(q\) axes are determined for different values of the total magnetization current (Fig. 5).
The SSFR test was conducted according to the procedure described under Chapter 3.3, within the frequency range 10÷70 Hz and the following results are obtained.

For d-axis:
\[ T_{d1} = 0.003887 \text{ s}; \quad T_{d2} = 2.26 \times 10^{-6} \text{ s}; \quad T_{d3} = 0.006527 \text{ s}; \quad T_{d4} = 2.2 \times 10^{-6} \text{ s}; \quad R_{D1} = 2.327 \Omega; \quad L_{D1d} = 0.00608 \text{ H}; \quad R_{D2} = 3202.44 \Omega; \quad L_{D2d} = 0.00377 \text{ H}; \quad L_{d(df)}^{d} = 0.00917 \text{ H}. \]

For q-axis:
\[ T_{q1} = 0.007889 \text{ s}; \quad T_{q2} = 0.00813 \text{ s}; \quad R_{Q1} = 24.34 \Omega; \quad L_{Q1d} = 0.1891 \text{ H}; \quad L_{q(df)}^{q} = 0.0088 \text{ H}. \]

5. CONCLUSIONS

Taking into account the described experimental methods and the results obtained, the following conclusions can be drawn.

The methods used to determine the mathematical model parameters of a synchronous machine have some advantages: easy way of implementation, not affecting the driving system of the machine and low energy consumption. The synchronous and differential magnetization inductances have been determined using the dc decay test, as functions of total magnetization current. Analyzing the plots in Fig. 5 one could notice significant variations of inductances corresponding to \(d\)-axis and small variations of the inductances in \(q\)-axis, magnetic saturation occurring mainly in \(d\)-axis. The resistances and leakage inductances of the rotor multiple circuits used to simulate the skin effect were determined from the SSFR test. From measured data one could observe the reduced skin effect influence in the \(q\)-axis. Also, the second circuit used to simulate the skin effect in the \(d\)-axis has little influence therefore, one circuit would be sufficient in this case to model the skin effect. This can be explained by machine construction with salient poles and large polar pitch, and by the fact that the machine power is reduced.

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