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THERMAL TREATMENT OF FERROMAGNETIC BARS

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This paper examines heating by eddy currents of a ferromagnetic plate that moves at an imposed speed. The periodic electromagnetic field problem in nonlinear media is solved by using the iterative method of polarization, of the Fourier series decomposition and, at each iteration, the harmonics of the electromagnetic field are obtained by solving the eddy currents integral equation. The *B-H* characteristic of the ferromagnetic medium depends on the temperature. The procedure has remarkable advantages over commercial programes. The numerical results are presented.

1. INTRODUCTION

For efficient mechanical machining of ferromagnetic bars, they are heated up and, most commonly, eddy currents are used for this purpose. The bar moves through the inside of a solenoid powered with a sinusoidal current. The analysis of this heating process is particularly complicated, as it involves the solving of coupled electromagnetic and thermal field problems, the bar being in motion. The physical properties of the medium depend on the temperature: the nonlinear *B–H* constitutive relation (Fig. 1), electrical resistivity, thermal conductivity, volumetric thermal capacity (Table 1). The electromagnetic field is the source of heat for the thermal problem, and the boundary conditions change in time.

Commercial programes (for example, FLUX) can be adopted to solve this problem. All commercial programes have some important disadvantages. They do not use an efficient and accurate enough procedure to treat nonlinearity in solving the periodic regime. The iterative method of static permeability is most frequently adopted, using different correction criteria [1]. In this way, we can use the complex images of the electromagnetic field quantities. The procedure does not take into account field harmonics, which are important when nonlinearity is significant. In addition, the method is not always convergent. To obtain better accuracy, the transient response method can be adopted, solving the field problem in the time domain for several periods. But the time required to obtain the asymptotic solution can be very long. We can approximate the electromagnetic field quantities with a finite number of harmonics. Then, in this form, they are introduced into the equations of the field (Harmonic balance method [2]). Unfortunately, due to the nonlinear B–H relation, the harmonics are coupled and a huge nonlinear system results. A particularly effective method for solving the periodic electromagnetic field regime in nonlinear media was proposed in [3]. The nonlinearity of B–H is treated by the polarization fixed point method (PFPM) [4], in which the nonlinear medium is replaced with a linear calculation medium with magnetic polarization nonlinearly corrected depending on the magnetic induction B. Polarization is decomposed into Fourier series and, for each harmonic, complex images can be used. In [2] it is recommended to start computations only with the fundamental, using the overrelaxation method described in [5]. Then, after reaching the solution on the fundamental, the higher harmonics are successively added, obtaining a superior accuracy of the result. The polarization method allows the choice of the free-space magnetic permeability for the calculation medium. Hence the advantage of using the integral equation of the eddy currents for solving each field harmonic. The procedure was applied for solving the heating problems of a non-moving ferromagnetic bar [6] and for the determination of the solidification surface at the change of the liquid-solid phase [7].

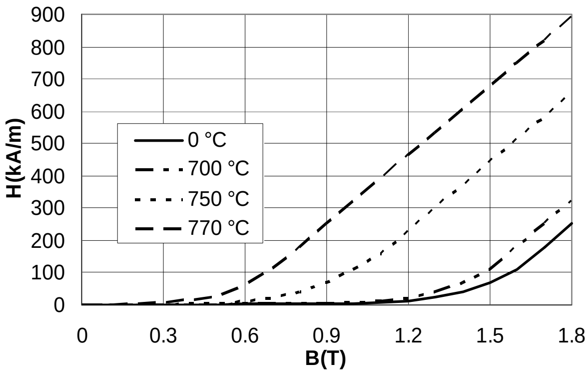

Fig. 1 – *H–B* characteristics for different temperatures. Values corresponding to other temperatures are obtained by linear interpolation.

We believe that the last method presented above has obvious advantages over other methods and we will adopt it in the analysis of the heating of the ferromagnetic bars in motion. The adopted model is parallel-plan.

Table 1

The temperature dependency of the medium physical properties

θ (°C)	500	1300	2000
ρ (Ωm)	1.2	1.6	2

θ (°C)	500	1300	1800
c_v (MJ·K·m^{-3})	3.8	3.5	3

θ (°C)	100	500	1000	1500
λ (W·K^{-1}·m^{-1})	40	35	30	25

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2. HARMONIC ANALYSIS

According to the polarization method [4], the nonlinear relation $\boldsymbol{H} = \mathrm{F}(\boldsymbol{B}, \theta)$ is replaced by $\boldsymbol{H} = \boldsymbol{B}/\mu_0 - \boldsymbol{M}$, where the magnetization $\boldsymbol{M}$ is nonlinearly corrected by the equation [4]

$$\boldsymbol{M} = (1/\mu)\boldsymbol{B} - \mathrm{F}(\boldsymbol{B},\theta) \equiv \mathrm{G}(\boldsymbol{B},\theta) \qquad (1)$$

The B–H characteristic of the temperature θ is obtain by interpolation of that given in Fig. 1[8]. We expand the magnetization $\boldsymbol{M}$ into Fourier series and retain a finite number of harmonics:

$$\boldsymbol{M}(t) \cong \sum_{n=1,3,\dots,2N-1} \left(\boldsymbol{M}'_n \sin(n\omega t) + \boldsymbol{M}''_n \cos(n\omega t)\right). \qquad (2)$$

It should be pointed out here that the approximation function $\boldsymbol{S}$ by truncated Fourier series is non-expansive [3]. For each magnetization harmonic, a sinusoidal regime problem is solved using complex images of field quantities. The eddy-current integral equation for each pulse harmonic of angular frequency $\omega_n \equiv (2n-1)\omega$ is:

$$\rho \underline{J}_n(\boldsymbol{r}) + \frac{\mu_0}{2\pi} \mathrm{j}\omega_n \int_{\Omega_f} \underline{J}_n(\boldsymbol{r}') \ln\frac{1}{R} \mathrm{d}s' =$$

$$-\frac{\mu_0}{2\pi} \mathrm{j}\omega_n \int_{\Omega_0} \underline{J}_{0_n}(\boldsymbol{r}') \ln\frac{1}{R} \mathrm{d}s'$$

$$-\frac{\mu_0}{2\pi} \mathrm{j}\omega_n \int_{\Omega_f} \boldsymbol{k}\cdot\left(\nabla' \times \underline{\boldsymbol{M}}_n(\boldsymbol{r}')\right) \ln\frac{1}{R} \mathrm{d}s', \qquad (3)$$

where $\boldsymbol{k}$ is the longitudinal unit vector, $\boldsymbol{r}$ and $\boldsymbol{r}$' are the position vectors of the observation and integration points, $R = |\boldsymbol{r} - \boldsymbol{r}'|$. After solving the integral equation (3), the harmonic n of the magnetic induction is determined with the equation:

$$\underline{\boldsymbol{B}}_n(\boldsymbol{r}) = \frac{\mu_0}{2\pi}\left[\boldsymbol{k} \times \int_{\Omega_f} \frac{\underline{J}_n(\boldsymbol{r}')\boldsymbol{R}}{R^2} \mathrm{d}s' + \boldsymbol{k} \times \int_{\Omega_0} \frac{\underline{J}_{0_n}(\boldsymbol{r}')\boldsymbol{R}}{R^2} \mathrm{d}s' + \int_{\Omega_f} \frac{\nabla' \times \boldsymbol{M}_n(\boldsymbol{r}')}{R^2} \times \boldsymbol{R}\,\mathrm{d}S'\right]. \qquad (4)$$

The function $\underline{\boldsymbol{M}} \xrightarrow{\ \boldsymbol{Z}\ } \underline{\boldsymbol{B}} = \boldsymbol{Z}(\underline{\boldsymbol{M}})$ is non-expansive. Magnetic induction time value is obtained from the Fourier series:

$$\boldsymbol{B}(t) = \sum_{n=1,3,\dots,2N-1} \left(\boldsymbol{B}'_n \sin(n\omega t) + \boldsymbol{B}''_n \cos(n\omega t)\right). \qquad (5)$$

3. NUMERICAL SOLUTION OF THE EDDY CURRENT EQUATION

For the numerical solution of the equation (3), we divide the domain Ω_f occupied by the ferromagnetic bar into I subdomains ω_i, and the domain Ω_0 of the imposed currents, in Q subdomains ω_{0q}, on each subdomain considering that the values of $\underline{J}_n$, J_{0n}, and $\boldsymbol{M}_n$ are constant. By integrating the equation (3) on each subdomain in Ω_f, we obtain the system of algebraic equations:

$$\rho_m S_m \underline{J}_m + \mathrm{j}\omega_n \sum_{i=1}^{I} \beta_{mi} \underline{J}_i = -\mathrm{j}\omega_n \sum_{q=1}^{Q} \beta_{0mq} \underline{J}_{0q}$$

$$+ \mathrm{j}\omega_n \sum_{i=1}^{I} \boldsymbol{\gamma}_{mi} \cdot \underline{\boldsymbol{M}}_i, \; m = 1,2,\dots,I, \qquad (6)$$

where

– ρ_m, S_m and J_m are the resistivity, surface and current density for the domain ω_m;

– J_{0q} is the current density imposed in the subdomain ω_{0q};

– $\boldsymbol{M}_i$ is the magnetization in the subdomain ω_i;

$$\beta_{mi} = \frac{\mu_0}{2\pi} \int_{\omega_m} \int_{\omega_i} \ln\frac{1}{R} \mathrm{d}s_i' \mathrm{d}s_m =$$

$$\frac{\mu_0}{8\pi} \oint_{\partial\omega_m} \oint_{\partial\omega_i} R^2 \ln R \,\mathrm{d}\boldsymbol{l}_m \cdot \mathrm{d}\boldsymbol{l}_i' \qquad (7)$$

$$\beta_{0mq} = \frac{\mu_0}{2\pi} \int_{\omega_m} \int_{\omega_{0q}} \ln\frac{1}{R} \mathrm{d}s_q' \mathrm{d}s_m =$$

$$\frac{\mu_0}{8\pi} \oint_{\partial\omega_m} \oint_{\partial\omega_{0q}} R^2 \ln R \,\mathrm{d}\boldsymbol{l}_m \cdot \mathrm{d}\boldsymbol{l}_q' \qquad (8)$$

$$\boldsymbol{\gamma}_{mi} = -\frac{\mu_0}{2\pi} \int_{\omega_m} \oint_{\partial\omega_i} \ln\frac{1}{R} \mathrm{d}\boldsymbol{l}_i' \mathrm{d}s_m =$$

$$\frac{\mu_0}{8\pi} \oint_{\partial\omega_m} \oint_{\partial\omega_i} (2\ln R - 1)(\boldsymbol{R}\cdot\boldsymbol{n}_m) \mathrm{d}l_m \mathrm{d}\boldsymbol{l}_i'. \qquad (9)$$

In comparison with [6], the value of β_{0mq} is time depending because of bar moving, The most convenient alternative is to choose polygonal subdomains, and, in this case, the above integrals can be obtained analytically. By separating the complex quantities of the system (6) into real and imaginary components we can write it by using matrices.

$$\begin{pmatrix} \beta & \delta/n \\ -\delta/n & \beta \end{pmatrix} \begin{pmatrix} J'_n \\ J''_n \end{pmatrix} = -\begin{pmatrix} A'_{0n} \\ A''_{0n} \end{pmatrix} + \begin{pmatrix} A'_{Mn} \\ A''_{Mn} \end{pmatrix} \qquad (10)$$

where

– β is the matrix with the elements β_{mi};

– δ is the diagonal matrix with the elements

$$\delta_{mm} = \rho_m S_m / \omega;$$

– J'_n and J''_n, A'_{0n} and A''_{0n}, A'_{Mn} and A''_{Mn} are the column matrices of the real and imaginary components of the current density J_n, of the potential vector A_{0n} components, of the imposed current density and the potential vector A_{Mn} due to magnetization:

$$\underline{A}_{0n} = \sum_{q=1}^{Q} \beta_{0mq} \underline{J}_{0n,q}, \quad \underline{A}_{Mn} = \sum_{i=1}^{I} \boldsymbol{\gamma}_{mi} \cdot \underline{\boldsymbol{M}}_{n,i}. \qquad (11)$$

$\underline{A}_{0n}$ remains unchanged during the iterations of the polarization method, while $\underline{A}_{Mn}$ is corrected for each iteration. In the example of computation, smaller rectangular subdomains were chosen with smaller width in the edge area in order to adapt to the electromagnetic field skin effect.

The numerical calculation of the magnetic induction for each harmonic is done with

$$\underline{\boldsymbol{B}}_n = -\frac{\mu_0}{2\pi} \boldsymbol{k} \times \sum_{i=1}^{I} \underline{J}_{n,i} \oint_{\partial\omega_i} \ln R \,\mathrm{d}\boldsymbol{l}_i'$$

$$-\frac{\mu_0}{2\pi} \boldsymbol{k} \times \sum_{q=1}^{Q} \underline{J}_{0n,q} \oint_{\partial\omega_{0q}} \ln R \,\mathrm{d}\boldsymbol{l}_q'$$

$$-\frac{\mu_0}{2\pi}\sum_{i=1}^{I}\oint_{\partial\omega_i}\frac{\boldsymbol{R}}{R^2}(\underline{\boldsymbol{M}}_{n,i}\cdot \mathrm{d}\boldsymbol{l}_i'). \quad (12)$$

For magnetization correction, it is necessary to determine the average magnetic induction value on each subdomain ω_m:

$$\underline{\boldsymbol{B}}_{n,m}=-\frac{1}{S_m}\left(\sum_{i=1}^{I}\gamma_{mi}\underline{J}_{n,i}+\sum_{i=1}^{I}\overline{\overline{\zeta}}_{mi}\underline{\boldsymbol{M}}_{n,i}\right)+\underline{\boldsymbol{B}}_{0n,m} \quad (13)$$

where $\underline{\boldsymbol{B}}_{0n,m}$ is produced by the imposed current density and in comparison with [6], is time depending.

$$\underline{\boldsymbol{B}}_{0n,m}=-\frac{1}{S_m}\sum_{q=1}^{Q}\gamma_{mq}\underline{J}_{0n,q} \quad (14)$$

and

$$\overline{\overline{\zeta}}_{mi}=\frac{\mu_0}{2\pi}\oint_{\partial\omega_m}\oint_{\partial\omega_i}\ln R(\mathrm{d}\boldsymbol{l}_m;\mathrm{d}\boldsymbol{l}_i'), \quad (15)$$

where ($\mathrm{d}\boldsymbol{l}_m$; $\mathrm{d}\boldsymbol{l}_i'$) is the dyadic product between the two vectors. Note that the mediation function on AV subdomains is non-expansive. On each subdomain, the magnetization correction is done by the G function, also resulting that it is constant on the subdomains. The G function is contraction.

Consequently, the procedure for solving the eddy current problem by the polarization method and the eddy current equation follows the scheme

$$\underline{\boldsymbol{M}}^{(0)}\xrightarrow{\boldsymbol{Z}}\underline{\boldsymbol{B}}^{(1)}\xrightarrow{\boldsymbol{AV}}\underline{\tilde{\boldsymbol{B}}}^{(1)}\xrightarrow{Fourier\ Sum}\boldsymbol{B}^{(1)}(t)$$
$$\xrightarrow{\boldsymbol{G}}\boldsymbol{M}^{(1)}(t)\xrightarrow{\boldsymbol{S}}\underline{\boldsymbol{M}}^{(1)}....$$

The above chain involves the composition of the functions $\boldsymbol{Z}\circ\boldsymbol{AV}\circ\boldsymbol{G}\circ\boldsymbol{S}$ that are non-expansive or contractions. The result is a contraction and the numerical procedure is convergent (Picard-Banach).

4. SOLUTION OF THE THERMAL DIFFUSION PROBLEM

The temperature distribution θ is obtained by solving the thermal diffusion equation

$$-\nabla\cdot(\lambda\nabla\theta)+c_v\frac{\partial\theta}{\partial t}=p \quad (16)$$

where λ is the thermal conductivity; cv is the volumetric thermal capacity of the material, both depending on the temperature and p are the specific losses obtained by solving the eddy current problem. The boundary condition is

$$\lambda\frac{\partial\theta}{\partial n}+\alpha(t)(\theta-\theta_e)=0, \quad (17)$$

where θe is the outside temperature; α is the thermal convection coefficient, which depends on time, due to the movement of the bar. The time discretization of the equation (16) is done by the trapezium method, and the spatial discretization is done by the finite element method. Thermal conductivity and heat capacity are iteratively corrected depending on the temperature.

5. STAGES OF THE COMPUTATION PROGRAME

1. We set the maximum number of *nit(n)* iterations and *er(n)* equations of the PFPM for the harmonics *n* to be successively taken into account.

2. We determine the matrices β, δ and $\overline{\overline{\zeta}}$ that appear in the equations (10) and (13).

3. At first, we take into account only the fundamental; the calculations stop at the iteration n_1, when the error is:

$$\varepsilon_r=\frac{\left\|\underline{\boldsymbol{M}}_1^{(n_1)}-\underline{\boldsymbol{M}}_1^{(n_1-1)}\right\|_1}{\left\|\underline{\boldsymbol{M}}_1^{(n_1)}\right\|_1}<er(1), \quad (18)$$

or when the number of *nit(1)* iterations is exceeded. With $\underline{\boldsymbol{M}}_1$ being the fundamental of magnetization, we have $\|\underline{\boldsymbol{M}}\|_1^2=\int_{\Omega_f}\underline{\boldsymbol{M}}_1\cdot\underline{\boldsymbol{M}}_1^*\mathrm{d}s$. Then, when the first harmonic (the 3rd) is also added, the equation (18) becomes

$$\varepsilon_r=\frac{\left\|\boldsymbol{M}_2^{(n_2)}-\boldsymbol{M}_2^{(n_2-1)}\right\|_2}{\left\|\boldsymbol{M}_2^{(n_2)}\right\|_2}<er(2), \quad (19)$$

where $\|\boldsymbol{M}\|_2^2=\int_{\Omega_f}\underline{\boldsymbol{M}}_1\cdot\underline{\boldsymbol{M}}_1^*\mathrm{d}s+\int_{\Omega_f}\underline{\boldsymbol{M}}_2\cdot\underline{\boldsymbol{M}}_2^*\mathrm{d}s$ and so on.

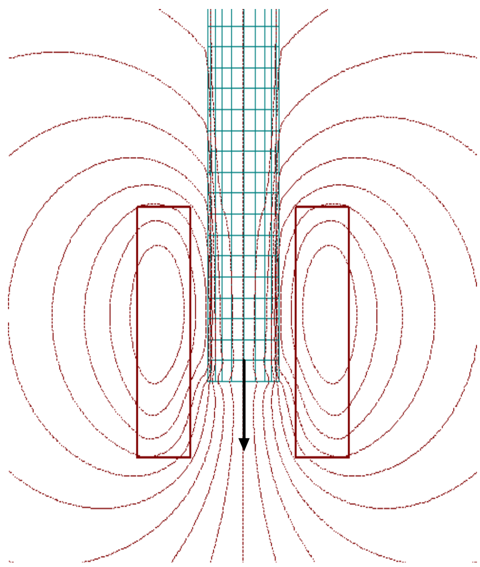
The material parameters correspond to the initial temperature.

4. With the current density values obtained at point 3, we determine the specific losses *p*. We choose the time step Δt. We determine the temperature field by solving the thermal diffusion equation

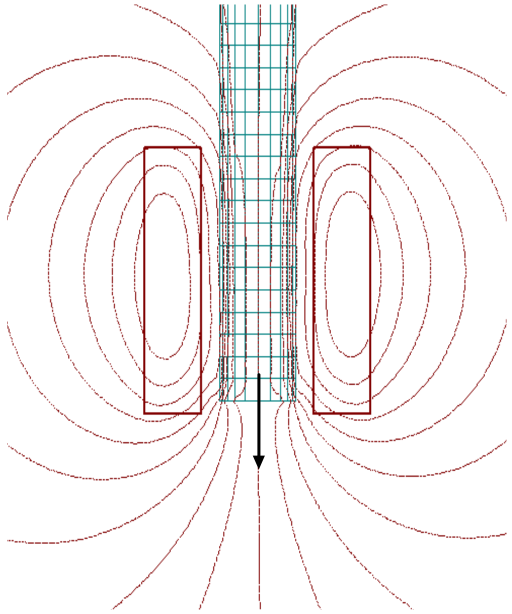
$$-\nabla\cdot(\lambda\nabla\theta)+c_v\frac{\partial\theta}{\partial t}=p. \quad (20)$$

If the temperature variation is too high, we reduce the time step, and if it is too low, we increase it. We correct the material parameters and follow the same calculations.

Observation. To determine the speed of the bar and to reduce the computation time, we calculate the time in which the bar is heated to θ_{fin} (for example, 1 000^{0}C), without moving. Then, admitting that the bar moves until its upper side leaves the inductor, the speed is determined.



a)

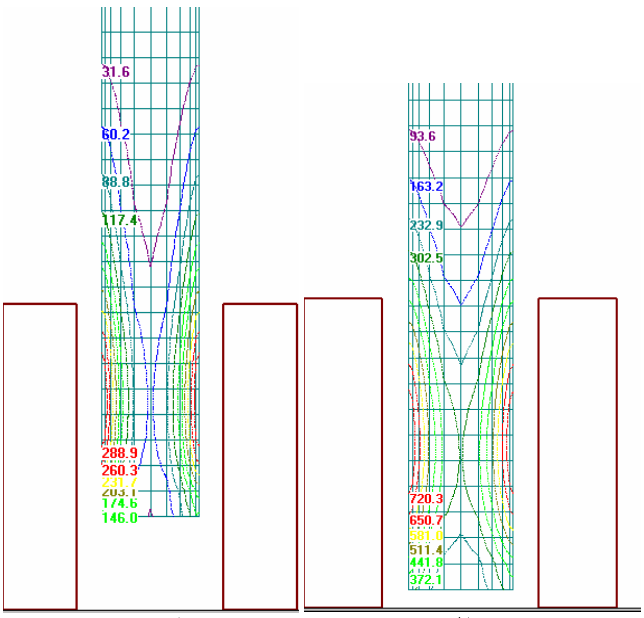


b)

Fig. 2 – Field lines of magnetic induction: a) $t = 2.33$ s, speed = 11.65 mm/s, $\theta_{max} = 312$°C, phase = 90°; b) $t = 5.41$ s, speed = 27 mm/s, $\theta_{max} = 791.3$°C, phase = 90°.

6. ILLUSTRATIVE EXAMPLE

The width of the bar is 20 mm, and the height is 100 mm. The current density in the coil is 8 A/mm^2, with the frequency of 5 kHz. The bar advances at 5 mm/s. For different positions of the bar, the field lines are drawn in (Fig. 2), and the isotherms in (Fig. 3).



a) b)

Fig. 3 – Isotherms: a) $t = 2.33$ s, speed = 11.65 mm/s, $\theta_{max} = 312$°C; b) $t = 5.27$ s, speed = 26.36 mm/s, $\theta_{max} = 775$°C.

7. CONCLUSIONS

The aim of this paper was to analyze the heating of the ferromagnetic bars by eddy currents. By analyzing the methods offered in specialized literature the higher efficiency of the procedure proposed in [3, 6] has been achieved. Although the *B–H* relation is nonlinear, the polarization method allows solving the periodic electromagnetic field problem on harmonics. The coupling of the harmonics is done only at the correction of the magnetization (1), while it is necessary to determine the magnetic induction in the time domain (5). The choice of the magnetic permeability of free-space for the calculation medium allows the use of the integral equation of the eddy currents for obtaining the solutions of each harmonic. Thus, the coefficients of the matrices of the systems necessary to determine the eddy currents (10) and the magnetic induction harmonics (13) are calculated once. Considering initially only the fundamental, from the harmonic spectrum we obtain a solution close to the exact one. The speed of convergence can be spectacularly increased by using over-relaxation.

Then the solution is refined by successively adding higher harmonics.

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