STABILITY ANALYSIS OF A SYNCHRONOUS MOTOR FED BY A CURRENT SOURCE INVERTER

VASILE MANOLIU

Key words: Synchronous motor, Current inverter, Stability.

The operation stability of electrical drives with synchronous motors can be investigated by using models of small disturbance. The paper presents an analysis of operating stability for a synchronous motor fed by static frequency converter with current source inverter. On the basis of functional equations written in $d$-$q$ system, based on the model of small variations, the operation stability are studied using the Routh-Hurwitz criterion. For a given angle of commutation, the synchronous motor is evaluated in different cases, with and without load, with and without damper windings. An instability is observed, when running the synchronous motor without damper windings, but also when assessing the leading angle variation relative to torque load variation.

1. INTRODUCTION

The electrical drive systems with synchronous motors with field winding fed by current source inverter are used in specific applications, such as: training of compressors and centrifugal pumps, high speed trains, etc. In general, the analysis is carried out using mathematical models of small disturbances, and one of the criteria most often used is the Routh-Hurwitz criterion.

In [1] is presented a steady-state analysis of stability for a synchronous motor fed by a current source inverter. The control variable was the field angle between the the rotor axis and stator axis magnetomotive voltages. Through a transfer function expressing the (low variation) dependence of the field angle $\beta$ on the (small signal) variation of the torque, the stability analysis based on the Routh-Hurwitz criterion was possible.

As is known, the operation of the current inverter which feeds the stator windings of a synchronous motor with field winding is characterized by six pairs of sequences (one of conduction and one of switching) at each rotation. The electromagnetic torque, produced by the interaction between the magnetic fields generated by the field winding and the stator windings, is conditioned by an
adequate (relative) positioning of the two fields (expressed as the leading angle of commutation $x$).

Fig. 1 shows the equivalent circuit current inverter synchronous motor with field winding; note the presence of the short circuited damper windings (denoted by $D$ and $Q$).

Fig. 1 – The equivalent circuit current inverter – synchronous motor.

In [2] the static stability of the steady-state operation of the conventional synchronous motor is analyzed by the relation between the small signal variations of electromagnetic torque and the internal angle.

For a constant margin angle control of commutatorless motor, in [3] the stability is investigated using the Routh-Hurwitz criterion; the study is focused on the operating points in the current-speed plane.

Taking into account the practical aspect, the present analysis aims at obtaining transfer functions in which small signal variations of the equivalent currents, leading angle, electromagnetic torque $T_e$ and load torque $T_l$ are used.

The objective of the study is to verify that the synchronous machine, equipped with damper windings, fed by an inverter, can support low disturbances in load. The stability assessment of the drive system in the absence of the damper windings was also considered.

2. METHODS

2.1. EQUATIONS AND TRANSFER FUNCTIONS

In this analysis, the synchronous motor considered has salient poles and damper windings (one damper winding on each axis). Using Park’s transform, a constant fundamental harmonic of the stator current $I_s^{(1)}$ at a leading angle of commutation $x$, can be represented by direct and quadrature axis currents as (Fig. 2):
Fig. 2 – The simplified phasor diagram of the overexcited synchronous motor.

The control strategy, must uphold certain criteria, defining the two distinct stages in the operation (conduction and commutation), leading to a command in which the regulated quantities are interrelated.

A synchronous motor can be subject to a diversity of electrical or mechanical disturbances, leading to a (short) transitory process which can end in an abnormal operating state characterized by the lack of rigorous correspondence between the rotor speed and the converter frequency.

If the disturbance is of a very low value, the non-linear differential equations which describe the transitory process, can be linearized. In this context, it will be assumed that all operational variables make small deviations around the values corresponding to a given steady-state operation.

Using the index "0" for the steady state value, and expressing small perturbation by "Δ", considering small disturbances, the operational expressions of the variations of the direct and quadrature axis currents ($I_d$ and $I_q$) are:

$$\Delta I_d(s) = (-I_d^{(0)} \cdot \cos x) \cdot \Delta x(s)$$  \hspace{1cm} (3)

$$\Delta I_q(s) = (-I_q^{(0)} \cdot \sin x) \cdot \Delta x(s) \hspace{1cm} (4)$$

The expression of the electromagnetic torque in small signals variations and in per-unit values is, after a series of calculations [1]:

$$\Delta T_e(s) = [(I_d - I_q) \cdot i_{d0} - i_{mq} \cdot i_{q0}] \cdot \Delta I_d(s)$$

$$+ [(I_d - I_q) \cdot i_{d0} + I_{md} \cdot i_{f0}] \cdot \Delta I_q(s)$$

$$+ I_{md} \cdot i_{q0} \cdot \Delta f(s) + I_{md} \cdot i_{q0} \cdot \Delta I_d(s) - I_{mq} \cdot i_{d0} \cdot \Delta I_q(s).$$ \hspace{1cm} (5)
Thus, the variation of the torque can be expressed as:

\[ \Delta T_e(s) = c_1 \cdot \Delta i_d(s) + c_2 \cdot \Delta i_q(s) + c_3 \cdot \Delta i_f(s) + c_4 \cdot \Delta i_D(s) + c_5 \cdot \Delta i_Q(s), \]  

(6)

where:

\[
\begin{align*}
    c_1 &= (I_d - I_q) \cdot i_{q0} - l_{mq} \cdot i_{q0}; \\
    c_2 &= (I_d - I_q) \cdot i_{d0} + l_{md} \cdot i_{d0} + l_{md} \cdot i_{f0}; \\
    c_3 &= l_{md} \cdot i_{q0}; \\
    c_4 &= l_{md} \cdot i_{q0}; \\
    c_5 &= -l_{mq} \cdot i_{d0}.
\end{align*}
\]

(7)

The equations of the small perturbations voltages of the field, and damper windings, D and Q, are expressed as:

\[
0 = \Delta U_f(s) = (R_f + s \cdot L_f) \cdot \Delta I_f(s) + s \cdot L_{md} \cdot (\Delta I_d(s) + \Delta I_D(s)),
\]

(8)

\[
0 = \Delta U_D(s) = (R_D + s \cdot L_D) \cdot \Delta I_D(s) + s \cdot L_{md} \cdot (\Delta I_d(s) + \Delta I_f(s)),
\]

(9)

\[
0 = \Delta U_Q(s) = (R_Q + s \cdot L_Q) \cdot \Delta I_Q(s) + s \cdot L_{mq} \cdot \Delta I_q(s).
\]

(10)

As the damper windings on the \(d\)-axis and \(q\)-axis are short-circuited, \(\Delta U_D = 0\) and \(\Delta U_Q = 0\). Furthermore, as the voltage fed to the field winding is constant, \(\Delta U_f = 0\). From the above equations one can express the field and damper winding current variations, \(\Delta I_f\), \(\Delta I_D\) and \(\Delta I_Q\), in relation to the direct and quadrature axis currents, \(\Delta I_d\) and \(\Delta I_q\). After a series of calculations and expressing (8)–(10) in per-unit values, substituting the expressions for the current variations in (6) yields:

\[
\begin{align*}
    \Delta T_e(s) &= c_1 \cdot \Delta i_d(s) + c_2 \cdot \Delta i_q(s) + c_3 \left( \frac{a_1 \cdot s^2 + a_2 \cdot s}{b_1 \cdot s^2 + b_2 \cdot s + b_3} \right) \cdot \Delta i_d(s) \\
    &+ c_4 \left( \frac{d_1 \cdot s^2 + d_2 \cdot s}{b_1 \cdot s^2 + b_2 \cdot s + b_3} \right) \cdot \Delta i_q(s) + c_5 \left( \frac{e_1 \cdot s}{f_1 \cdot s + f_2} \right) \cdot \Delta i_q(s),
\end{align*}
\]

(11)

where:

\[
\begin{align*}
    a_1 &= l_{md}^2 - l_{md} \cdot l_D, \\
    a_2 &= -l_{md} \cdot r_D, \\
    b_1 &= l_D \cdot l_f - l_{mD}^2, \\
    b_2 &= l_D \cdot r_f + l_f \cdot r_D, \\
    b_3 &= r_D \cdot r_f, \\
    d_1 &= l_{md}^2 - l_{md} \cdot l_f, \\
    d_2 &= -r_f \cdot l_{md}, \\
    e_1 &= -l_{mq}, \\
    f_1 &= r_Q, \\
    f_2 &= l_Q.
\end{align*}
\]

(12)

(13)

(14)

(15)

Equation (11) can be expressed as:

\[
\Delta T_e(s) = \frac{(g_1 \cdot s^3 + g_2 \cdot s^2 + g_3 \cdot s + g_4) \cdot \Delta i_d(s) + (h_1 \cdot s^3 + h_2 \cdot s^2 + h_3 \cdot s + h_4) \cdot \Delta i_q(s)}{l_1 \cdot s^3 + l_2 \cdot s^2 + l_3 \cdot s + l_4}
\]

(16)

where \(g_1, g_2, g_3, g_4, h_1, h_2, h_3, h_4, l_1, l_2, l_3\) and \(l_4\) are functions of the machine parameters.
Substituting the expressions for the current variations $\Delta i_d$ and $\Delta i_q$ in the torque variation relation, and expressing the currents in per-unit values, a first transfer function can be expressed as:

$$H_1(s) = \frac{\Delta T_e(s)}{\Delta x(s)} = \frac{p_1 \cdot s^3 + p_2 \cdot s^2 + p_3 \cdot s + p_4}{l_1 \cdot s^3 + l_2 \cdot s^2 + l_3 \cdot s + l_4}, \quad (17)$$

where:

$$p_1 = -h_1 \cdot i_s \sin x_0 - g_1 \cdot i_s \cos x_0; \quad p_2 = -h_2 \cdot i_s \sin x_0 - g_2 \cdot i_s \cos x_0;$$
$$p_3 = -h_3 \cdot i_s \sin x_0 - g_3 \cdot i_s \cos x_0; \quad p_4 = -h_4 \cdot i_s \sin x_0 - g_4 \cdot i_s \cos x_0. \quad (18)$$

This expression allows the analysis of stability regarding the relation between the small variation of the leading angle and the variation of electromagnetic torque generated by the synchronous machine.

The torque dynamic equation is:

$$T_e - T_l = J_\Sigma \frac{d\Omega}{dt}, \quad (19)$$

where: $J_\Sigma =$ polar moment of inertia of motor and load [kgm$^2$], $\Omega =$ rotor angular speed [rad/s], $t =$ time [s]

Starting from the equation of motion, as a result of the small sudden change of the load torque, $\Delta T_l$, the rotor will lag the unchanging resultant stator field by the geometrical angle $\Delta \theta$ [2].

The relations between the position angle (the angle between the axis of the excitation field and the fixed reference axis) and the leading angle of commutation were obtained from the static torque variation analysis, using FLUX 2D [4]. The correspondence between conduction sequences and the angle of the position $\theta$ is shown in Table 1.

Consequently, the small variations of the internal angle of the motor (as described in [2]) and of the leading angle of commutation have the same sign and the same value.

<table>
<thead>
<tr>
<th>Thyristors to be fired</th>
<th>Position angle $\theta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_1$ and $T_2$</td>
<td>$-\pi/2 - x_0$</td>
</tr>
<tr>
<td>$T_2$ and $T_3$</td>
<td>$-\pi/6 - x_0$</td>
</tr>
<tr>
<td>$T_3$ and $T_4$</td>
<td>$\pi/6 - x_0$</td>
</tr>
<tr>
<td>$T_4$ and $T_5$</td>
<td>$\pi/2 - x_0$</td>
</tr>
<tr>
<td>$T_5$ and $T_6$</td>
<td>$5\pi/6 - x_0$</td>
</tr>
<tr>
<td>$T_6$ and $T_1$</td>
<td>$7\pi/6 - x_0$</td>
</tr>
</tbody>
</table>
Therefore, the small change in speed $\Omega$ can be related to an inverse proportional variation of the leading angle. Thus, one can write:

$$\frac{d\Omega}{dt} = -\frac{1}{p} \cdot \frac{d^2 (\Delta \theta)}{dt^2} = -\frac{1}{p} \cdot \frac{d^2 (\Delta \alpha)}{dt^2}.$$  \hspace{1cm} (20)

By applying the model with small perturbations and the Laplace transform, after some calculations, the transfer function expressing the leading angle dependence on the load torque variation, becomes:

$$H_2(s) = \frac{\Delta \alpha(s)}{\Delta T_L(s)} = \frac{l_1 \cdot s^3 + l_2 \cdot s^2 + l_3 \cdot s + l_4}{k_1 \cdot s^4 + k_2 \cdot s^4 + k_3 \cdot s^3 + k_4 \cdot s^2 + k_5 \cdot s + k_6},$$  \hspace{1cm} (21)

where, for the number of pole pairs $p = 2$:

$$k_1 = J \cdot l_1; k_2 = J \cdot l_2; k_3 = J \cdot l_3 + p_1; k_4 = J \cdot l_4 + p_2; k_5 = p_3; k_6 = p_4;$$ \hspace{1cm} (22)

and $J = J_L/p$.

The stability analysis in steady-state for the transfer function (relation (21)) is achieved by calculating the roots of the characteristic equation (the denominator of the transfer function polynomial equalized to 0) and applying the Routh-Hurwitz criterion.

2.2. THE ROUTH-HURWITZ CRITERION

The Routh-Hurwitz criterion is a method for determining whether a linear system is stable or not, by examining the locations of the roots of the characteristic equation of the system.

Consider the characteristic equation associated to a transfer function (the denominator polynomial of the transfer function being set to zero):

$$a_n \cdot s^n + a_{n-1} \cdot s^{n-1} + ... + a_1 \cdot s + a_0 = 0.$$  \hspace{1cm} (23)

If all roots of the characteristic equation have negative real parts, the system is stable. If the above condition are fulfilled, then compute the Routh-Hurwitz array as follows:

$$\begin{array}{c|cccc}
S^n & a_n & a_{n-2} & a_{n-4} & \ldots \\
S^{n-1} & a_{n-1} & a_{n-3} & a_{n-5} & \ldots \\
S^{n-2} & b_1 & b_2 & b_3 & \ldots \\
S^{n-3} & c_1 & c_2 & c_3 & \ldots \\
\vdots & \vdots & \vdots & \vdots & \ddots \\
S & & & & \ddots \\
S^1 & & & & \ddots \\
S^0 & & & & \ddots \\
\end{array}$$
where:

\[
\begin{align*}
    b_1 &= -\frac{1}{a_{n-1}} \begin{bmatrix} a_n & a_{n-2} \\ a_{n-1} & a_{n-3} \end{bmatrix}, \\
    b_2 &= -\frac{1}{a_{n-1}} \begin{bmatrix} a_n & a_{n-4} \\ a_{n-1} & a_{n-5} \end{bmatrix}, \\
    c_1 &= -\frac{1}{b_1} \begin{bmatrix} a_{n-1} & a_{n-3} \\ b_1 & b_2 \end{bmatrix}, \\
    c_2 &= -\frac{1}{b_1} \begin{bmatrix} a_{n-1} & a_{n-5} \\ b_1 & b_3 \end{bmatrix}.
\end{align*}
\]

(24)

The necessary condition that all roots have negative real parts is that all the elements of the first column of the array have the same sign. The number of roots of the polynomial that are in the right half plane is equal to the number of sign changes in the first column.

If the system equations (single input and single output) are written in state-space matricial form:

\[
\begin{cases}
    \dot{X} = A \cdot X + B \cdot U \\
    Y = C \cdot X + D \cdot U.
\end{cases}
\]

(25)

The poles of the transfer function \(Y(s)/U(s)\) are obtained solving the equation: \(\text{det}(s \cdot I - A) = 0\), where \(I\) is the unity matrix, and \(s\) is the operational variable.

2.3. THE STUDY OF THE SEQUENCE OF COMMUTATION

The commutation sequence is influenced by the following parameters: \(\Phi_D\), \(\Phi_Q\), \(i_k\), \(i_o\), \(i_f\). Based on the parameters of the synchronous machine and the constant values of the field and d.c. link currents, one can calculate the variation in time of the commutation current, \(i_k\) [4]. The commutation current \(i_k\) equation is presented below:

\[
\frac{di_k}{dt} = \omega_n \left( -\frac{r_k}{l_k} \cdot i_k + \frac{u_k}{l_k} \right),
\]

(26)

where:

\[
\begin{align*}
    u_k &= u_q \sin \theta + u_d \cos \theta + \left( r_c + 2v \cdot (l_q - l_d) \sin(2\theta + 2\pi/3) \right) \cdot i_l, \\
    r_k &= 2r_c - 2v \cdot (l_q - l_d) \sin 2\theta; \quad l_k = l_d + l_q + (l_q - l_d) \cos 2\theta.
\end{align*}
\]

(27)

\(i_0\) – the d.c. link current, \(\omega_n\) – the natural pulsation and \(v\) – the speed in per-unit value.

In (27) the subtransient voltage equations expressed in the coordinate system \(d-q\), have the form:

\[
\begin{align*}
    u_d^n &= -v \cdot (l_q - l_d) \cdot (\sqrt{6}/\pi) i_0 \cos x_0; \quad u_q^n = v \cdot \left( (l_d - l_q) \cdot (\sqrt{6}/\pi)i_0 \sin x_0 + l_{md} \cdot i_f \right).
\end{align*}
\]

For the equations in matrixial form, the damper windings fluxes and the commutation, the d.c. link and field currents can be considered as state variables.
For the case described above, as the elements of the state matrix $A$ contain the trigonometric functions of the position angle $\theta$, the system is time-varying. Using Matlab-Simulink programming, reasonable accuracy is achievable by splitting the range of the commutation interval in small increments, over which the time-varying system can be approximately treated as a time-invariant system [5].

In this case, one can approximate $\sin \theta \approx \theta$; $\cos \theta \approx 1$.

If, in the system described above, supposing the damper and field windings fluxes, as well as the d.c. link current as constant quantities, considering the field circuit voltage, $u_f$, as control variable, one can rewrite a simplified form of state-space matriceal equation, containing the currents $i_k$, $i_0$ and $i_f$ as state variables. The state-space equations system can be rewritten in the matriceal form:

$$\ddot{X} = A \cdot X + B \cdot U,$$

where $X = [i_k \ i_0 \ i_f]^T$.

The commutation is over when $i_k$ reaches the value of dc link current.

3. RESULTS

3.1. THE STABILITY OF THE SYNCHRONOUS MACHINE WITH DAMPER WINDINGS

The synchronous machine (of the experimental bench [4]) parameters (per-unit values): $r_s = 0.07$; $l_f = 1.397$; $l_d = 1.102$; $l_q = 0.758$; $l_{md} = 1.005$; $l_{mq} = 0.661$; $l_D = 1.069$; $l_Q = 0.812$; $l_d = 0.379$; $l''_d = 0.166$; $l''_q = 0.22$; $l_{md} = 0.051$; $l_{mq} = 0.151$; $r_D = 0.022$; $r_Q = 0.087$; $r_f = 0.02$; $l_{sf} = 0.39$.

The static stability was investigated for a leading angle $\alpha_0 \approx 60^\circ$.

The Routh table for the transfer function $H_1(s)$ shows, on the first column, the values: 0.422; 0.0879; 0.0047; 0.000038 indicating the stability. When changing the leading angle, the results keep their convergence.

The static stability of the steady-state operation expressed in [2] by the relation $\left(\frac{dT_e}{d\alpha_0}\right)_{\alpha_0=0} > 0$ gives the idea to verify the stability of the characteristic equation resulted from the denominator of the transfer function $H_1(s)$. It may be noted that the term internal angle [2] corresponds, in a small variations analysis, to the term leading angle referred in the present paper.

For the transfer function $H_2(s)$ the values of the first column show a change of sign, which indicate instability.

3.2. THE STABILITY OF THE SYNCHRONOUS MACHINE WITHOUT DAMPER WINDINGS AT NO-LOAD

At no-load $\alpha_0 \approx 90^\circ$, the transfer function of the electrical drive system, for the motor without damper windings is modified as [1]:
\[ H_s (s) = \frac{\Delta \tau (s)}{\Delta T_s (s)} = \frac{1}{J \cdot s^2 + i_{sd} \cdot i_{f} + (i_{md} - i_{mq}) \cdot i_{d}^2}. \] (29)

The roots of the characteristic equation are pure imaginary and complex conjugate; such roots lead to a marginally stable or undamped system which does not clearly indicate the steady state stability [1].

If the damper windings are considered at no-load operation \((x = 90^\circ)\), the values on the first column of the Routh table do not change sign; therefore, the situation concerning the stability remains unchanged.

Generally it is observed that a synchronous motor without damper windings does not satisfy the steady state stability criteria.

3.3. THE STABILITY FOR THE COMMUTATION INTERVAL

For a rotor speed (in per-unit) \( \nu = 1 \) and a leading angle \( x_0 = \pi/4 \), the following coefficients matrix of the state variable are obtained:

\[ a_{11} = -97.1; \quad a_{12} = 632.6; \quad a_{13} = 1014.8; \]
\[ a_{21} = a_{22} = a_{23} = a_{31} = a_{32} = 0; \quad a_{33} = -4.5, \]

so that the determinant of the matrix \((s \cdot I - A)\) has the expression:

\[ s^3 + 101.6 \cdot s^2 + 436.95 \cdot s = 0. \]

When considering the current commutation in the simplifying assumptions, because the free term is zero, upon completing the Routh table we obtain a marginally stable system behavior.

In this case, in the absence of the damper windings, the stability analysis leads to a marginally stable or undamped system behavior.

4. CONCLUSIONS

Static stability, as defined in [2], considers disturbances of very small values, being characterized by the condition: \( d\tau / d\theta \) \( \theta = 0 > 0 \). So, regardless of the type of power source, the electromagnetic torque variation in relation to the internal angle must be positive. When the principle of self-control is used, by associating the synchronous motor with a current inverter and a position transducer, it's more natural to express the steady-state stability in relation to the leading angle of commutation.

Practically, the stability is ensured for any value of \( x_0 \) between 10° and 80°. However, when investigating the variation of the leading angle of commutation in relation to the variation of load torque, the stability criterion evaluated by the Routh-Hurwitz criterion could not be confirmed.
Even so, there are at least two arguments that still explain the operational stability in case of the drive system with a self-controlled synchronous motor:

– the d.c. link current is not (rigorously) constant
– the control schema includes the $i_0$ current controller as well as two function generators, one of which implements the relationship between the stator current and the leading angle of commutation.

In [3] it is stated, as well, that the constant margin-angle control scheme has intrinsic instability, especially in heavy load and high-speed operation. Usually, the operating instability in case of a synchronous motor fed by a current source inverter can be associated with the situation when the leading angle becomes lower than the load angle.

The absence of damper windings is inconvenient, leading to unacceptable levels of commutation overlap angle, but also to unstable functional conditions.

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