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METHOD FOR ANALYZING THREE-PHASE NETWORKS WITH NONLINEAR RESISTIVE ELEMENTS

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Key words: Three-phase nonlinear networks, Fix point method for nonlinear controlled sources, Direct, inverse and homopolar succession network decomposition.

In this paper there is presented a procedure in which the nonlinear resistive elements are replaced by voltage or current generators whose sources are corrected, in an iterative manner, function of the voltage or of the current corresponding to the terminals. The resistances of the generators do not modify along the iterations and they can be chosen such that the three-phase network to be balanced, from its passive elements point of view. The harmonic analysis of the network is performed, by decomposing it into a direct, inverse and homopolar succession. By applying the proposed procedure, it results a spectacular reduction of the circuit's complexity. After determining the voltages and the currents at the terminals of the generators, their values, in time domain, are being computed and the sources are corrected. Then, the complex images are obtained, and the next iteration is applied. It results immediately the content of the harmonics and therefore the power quality.

1. INTRODUCTION

A three-phase unbalanced network is an electric circuit of big complexity, whose solving requires a big computation effort. This is the reason for which there have been developed methods suitable for such networks. For the case of balanced networks and symmetrical sources, the delta receivers are replaced by the Wye equivalent and the computation is performed only for one phase, the quantities corresponding the other two phases being obtained by multiplying with $\varepsilon = \exp(-j2\pi/3)$.

If the sources are not symmetrical, then they are decomposed in direct (positive), inverse (negative) and homopolar (zero) (DIH) components, and the computation is done on components. For the local unbalanced case (network with defects), the defect can be separated from the rest of the balanced network (equivalent generators) and the equations corresponding to the defect are written for DIH and then, the obtained circuit is solved. If the network contains nonlinear elements (see Fig. 1), the procedures described above are no longer valid. In this paper we propose a new method, which allows the analysis of an unbalanced three-phase network, which contains nonlinear resistive elements, by using the decomposition in DIH networks.

2. THREE-PHASE NETWORK SYMMETRIZATION

Hantila proposed in [1, 2, 4] an iterative method for electromagnetic field computation in non-linear media. The real medium, with the nonlinear characteristic $H = f(B)$, is replaced by the linear medium having a constant magnetic permeability but, with magnetic polarization I, which is updated (corrected) function of the magnetic induction B or of the magnetic field strength H: $B=\mu H+I$. Due to the important advantages given by this method, as compared to other method reported in literature and used for field analysis in nonlinear media (for example: Newton-Raphson, static permeability), this new procedure has been adopted by many specialists and it is called polarization fixed point method.

Also, in [1], the method proposed for electromagnetic field have been extended to circuits with nonlinear resistive circuits [2, 3]. The correspondent in field analysis is that of variable general electromagnetic field in electrokinetic

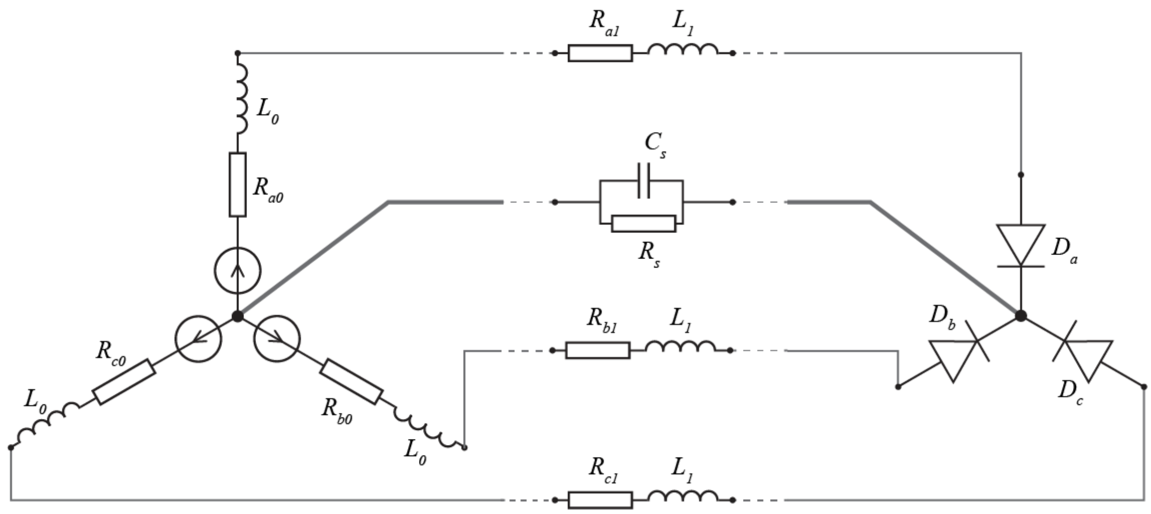

Fig. 1 – Example of three-phase network. Phase and line resistors can be nonlinear. Inductivities of the phases and of the line are equal.

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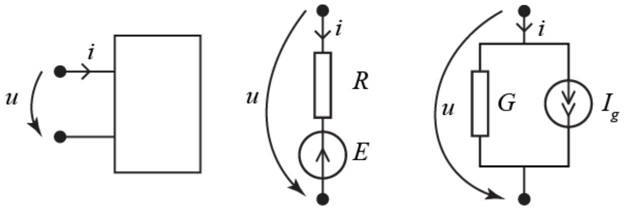

(a) Nonlinear element (b) Voltage equivalent generator (c) Current equivalent generator (G=1/R, I_g=E/R)

Fig. 2 – Replacing the nonlinear element by equivalent generators.

nonlinear media. For simplicity, in the case of circuits, one takes into consideration the one-port elements. The nonlinear circuit element is replaced by the equivalent generators. The description is given in Fig. 2.

2.1. SOURCES CORRECTION USING THE CURRENT FROM THE TERMINALS

The *u-i* relation

$$u = f(i) \tag{1}$$

is replaced by

$$i = Gu + I_g , \tag{2}$$

where the current source is corrected in an iterative manner, function of the current from the terminals, i

$$I_g = i - Gf(i) = p(i) . \tag{3}$$

We assume the function f is uniformly monotone

$$\left(f(i') - f(i'')\right)(i' - i'') \geq \lambda (i' - i'')^2 \quad \forall i', i'' \tag{4}$$

and Lipschitzian

$$\left| f(i') - f(i'') \right| \leq \Lambda \left| i' - i'' \right| \ \forall i', i'' . \tag{5}$$

Let us consider

$$R_{\max} = \sup_{\substack{i', i'' \in (0, \infty) \\ i' \neq i''}} \frac{f(i') - f(i'')}{i' - i''} ,$$

$$R_{\min} = \inf_{\substack{i', i'' \in (0, \infty) \\ i' \neq i''}} \frac{f(i') - f(i'')}{i' - i''} . \tag{6}$$

If we choose

$$G \in \left(0, \frac{2}{R_{\max}} \right) , \tag{7}$$

then the function p, defined in relation (3), is a contraction [4]

$$\left| p(i') - p(i'') \right| \leq \theta \left| i' - i'' \right| , \ \forall i', i'' \tag{8}$$

with the contraction factor

$$\theta = \max\left((1 - GR_{\min}), (GR_{\max} - 1) \right) < 1 . \tag{9}$$

For

$$R_{opt} = \frac{1}{G_{opt}} = \frac{R_{\min} + R_{\max}}{2} \tag{10}$$

there is obtained the smallest value for the contraction factor

$$\theta_{opt} = \frac{R_{\max} - R_{\min}}{R_{\max} + R_{\min}} . \tag{11}$$

2.2. SOURCES CORRECTION USING THE VOLTAGE FROM THE TERMINALS

If the function f is uniformly monotone and Lipschitzian, then the function is invertible and its inverse $g = f^{-1}$ is also uniformly monotone and Lipschitzian. By replacing the following equation

$$i = g(u) \tag{12}$$

with

$$u = Ri + E , \tag{13}$$

where the voltage source is corrected in an iterative manner function of the voltage u from the terminals

$$E = u - Rg(u) = q(u) . \tag{14}$$

If we choose

$$R \in \left(0, \frac{2}{G_{\max}} \right) \tag{15}$$

then the function q defined in relation (14) is a contraction.

This procedure is the “dual” to the one described in Subsection 2.1. From equation (7) and (15) it results that, instead of nonlinear elements, we can use a generator having the same values for the inner resistances. Therefore, we can replace a nonlinear (and/or unbalanced) network with a balanced three-phase network. The sources which appear in the equivalent generators are non-symmetrical three-phase sources, which can be immediately decomposed in DIH components. So, each network can be solved separately.

For simplicity, we chose one-port resistive circuit elements. We proceed in a similar manner for the multi-port case, the coupling being describes by ports’ voltage or current controlled sources (eventually nonlinear) [5, 6].

3. **PERIODIC SOLUTION OF THE LINEAR CIRCUIT**

Replacing the nonlinear resistors with voltage (or current) dependent sources, we obtain a linear circuit with “linear” relationship (2) or (13) for resistors [5, 6]. Without affecting the generality, we can admit that we have only two voltage sources E', E''. Let u_β', u_β'' and i_β', i_β'' be the branch voltages and currents, respectively, due to the sources E', E'' and $\Delta u_\beta = u_\beta' - u_\beta''$, $\Delta i_\beta = i_\beta' - i_\beta''$, $\Delta E = E' - E''$. If we take into account the relationships between the currents and voltages from the currents corresponding to the branches (for the equivalent generators case, relation (13)), then the Tellegen’s theorem gives

$$\begin{aligned} \left\langle \Delta u_\beta, \Delta i_\beta \right\rangle_{R^b} &= \left\langle \Delta u_\rho, G\Delta u_\rho - G\Delta E \right\rangle_{R^r} \\ &+ \left\langle \Delta u_\gamma, C\frac{\mathrm{d}\Delta u_\gamma}{\mathrm{d}t} \right\rangle_{R^c} + \left\langle L\frac{\mathrm{d}\Delta i_\eta}{\mathrm{d}t}, \Delta i_\eta \right\rangle_{R^l} \\ &= 0 , \end{aligned} \tag{16}$$

where

- u_ρ, u_γ are the vectors of the resistor and capacitor voltages;
- i_η is the vector of the inductor currents;
- *r, c, l, b* are the numbers of the resistors, capacitors, inductors and of all branches respectively;
- C and L are positively defined matrices of the capacitances and inductances respectively, and $G = R^{-1}$ is the symmetric and positively defined matrix of all conductances.

The vector ΔE has nonzero entries only for nonlinear resistor ports. Therefore, for the periodic solution we have

$$\int_0^T \left\langle \Delta u_\rho, G\Delta u_\rho \right\rangle_{R^r} \mathrm{d}t \le \int_0^T \left\langle \Delta u_\rho, G\Delta E \right\rangle_{R^r} \mathrm{d}t \qquad (17)$$

or, in Hilbert space of the periodic functions with weight G:

$$\left\| \Delta u_\rho \right\|_{R^r}^2 \le \left\| \Delta u_\rho \right\|_{R^r} \left\| \Delta E \right\|_{R^r} . \qquad (18)$$

It results that

$$\left\| \Delta u_\rho \right\|_{R^r} \le \left\| \Delta E \right\|_{R^r} . \qquad (19)$$

It follows that the function $E \xrightarrow{W} u$, giving the periodic solution of the linear circuit, is non-expansive in the Hilbert space of the periodic functions.

From relation (14), it results that

$$\left\| \Delta E_\rho \right\|_{R^r} \le \theta \left\| \Delta u \right\|_{R^r} \qquad (20)$$

because q is a contraction.

4. PERIODIC SOLUTION OF THE NONLINEAR CIRCUIT

The proposed method consists of solving the linear network in periodic steady state, and then the sources are corrected by using the relationships given in (14) or (3). The scheme of the procedure is the following

$$\ldots E^{(n)} \xrightarrow{W} u^{(n)} \xrightarrow{(14)} E^{(n+1)} \ldots . \qquad (20)$$

When the error: $\left\| E^{(n+1)} - E^{(n)} \right\|_{R^r}$ is sufficiently small, the iterations stop.

4.1. THREE-PHASE NETWORK COMPUTATION

a) The nonlinear elements are replaced by generators having the same inner resistance. We proceed in the same manner for the unbalanced linear receivers' case.

b) Any source e_k is expanded in Fourier series

$$e_k(t) = \sum_n \left((e'_k)_n \sqrt{2} \sin(n\omega t) + (e''_k)_n \sqrt{2} \cos(n\omega t) \right). \quad (21)$$

For the numerical computation, we retain only a finite number N of harmonics, namely $e \cong e_{ap} \equiv Y(e)$, the approximation Y being non-expansive.

c) For each harmonic of e_k the corresponding complex value $\underline{e}_k$ is considered. Using the DIH decomposition, we obtain the sources $\underline{e}_{kd}$, $\underline{e}_{ki}$, $\underline{e}_{kh}$ for each harmonic. The DIH circuits are computed and we obtain the quantities which are used to correct the sources (in the DIH system). Let be $\underline{u}_{kd}$, $\underline{u}_{ki}$, $\underline{u}_{kh}$ these quantities.

d) Determine the complex values of the quantities corresponding to each phase. Let be $\underline{u}_{ka}$, $\underline{u}_{kb}$, $\underline{u}_{kc}$ these quantities.

e) Determine the values in time domain of the quantities used to correct the sources, using the inverse Fourier transform. We obtain: u_a, u_b, u_c.

f) Correct the values of the nonlinear controlled sources.

In the end of the iterations, using the values obtained at point d), we can obtain the power quality of the energy transmitted by the analyzed network.

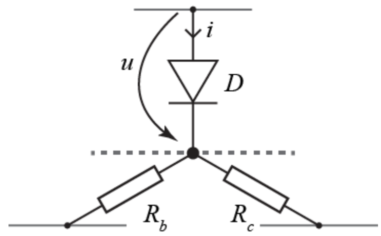

Fig. 3 – Receiver with nonlinear resistor.

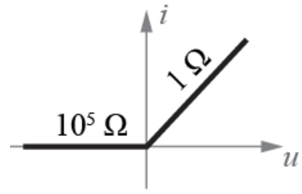

Fig. 4 – *u-i* characteristics of the diode.

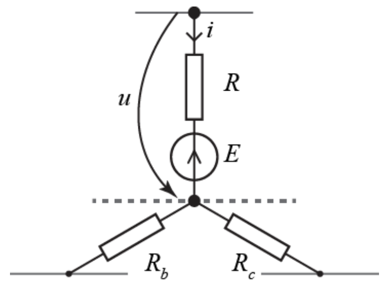

Fig. 5 – The equivalent balanced Wye connection.

5. EXAMPLE

Let's consider the Wye (Star) receiver given in Fig. 3, where the diode has the *u-i* characteristics described in Fig. 4.

For the nonlinear element we have: $R_{\max} = 10^5\ \Omega$ and $R_{\min} = 1\ \Omega$.

According to (7), if the correction of the source takes place function of the current from the diode, we must choose $G \in (0, 2/R_{\max})$ or $R > R_{\max}/2$.

If the correction of the source takes place function of the voltage from the terminals of the diode, according to (15), we must choose $R \in (0, 2/G_{\max})$, therefore $R < 2R_{\min}$, so $R < 2\Omega$.

If the other resistors on phases would have the values $R_a = R_b > 5 \cdot 10^4 \Omega$, one can choose the generator equivalent to the diode, with a resistance $R = R_a$, resulting the unbalanced Wye connection from Fig. 5, and the correction of the current source I_g is done function of the diode's current. We can proceed the same way if we place the voltage source $E = RI_g$. If the other resistors on phase would have the values $R_a = R_b < 2\Omega$, we can choose the generator equivalent to the diode, of resistance $R = R_a$, resulting also a balanced Wye connection described in Fig. 5, and the correction of the current source takes place function of the diode's voltage.

If the resistances R_a, R_b are not equal or they are placed in the interval $[R_{\min}, R_{\max}]$ then, in order to obtain a balanced Wye connection, these are also replaced by the equivalent generators. Considering that for the linear resistor's case $R_{\min} = R_{\max}$, one can choose any value for the inner resistance of the equivalent generators with linear resistors, in particular we can choose the value chosen for the one corresponding to the diode. The balanced Wye connection will have sources on each of its branches.

6. **CONCLUSIONS**

The presented method preserved the network's topology. There are modified only the resistive element of the network, allowing therefore the adoption of a balanced network. We obtain a remarkable advantage for network's computation: the decomposition into DIH sub-networks. The controlled sources which appear in the new network are subjected also to the DIH decomposition. For the nonlinear elements' case, the DIH circuits' analysis takes place on each harmonic. Then, we return to the values in time domain of the currents and of the voltages, to make the correction of the sources. Sometimes, one can select only the important harmonics. The iterative procedure remains convergent (the Fourier truncation is not expansive).

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