POSITION CONTROL OF SWITCHED RELUCTANCE MOTOR USING AN ADAPTIVE BACKSTEPPING CONTROLLER

AHMED TAHOUR[1], ABDEL GHANI AISSAOUI[2], AHMED CHAWKI MEGHERBI[3]

Key words: Switched reluctance motor, Backstepping design, Nonlinear control, Position control, Adaptive backstepping.

In this paper the position control of a switched reluctance motor using backstepping design with adaptive action is proposed. First, a backstepping design for position control of SRM is proposed. Finally, an adaptive backstepping controller is investigated for a class of nonlinear system to tackle the nonlinear control problems with modelling uncertainties, plant parameters variations and external disturbances. The proposed scheme gives fast dynamic response with no overshoot and zero steady-state error. To show the validity and the effectiveness of the control method, simulations are performed for the position control of a switched reluctance motor. The simulation results show that the controller designed is more effective than the conventional backstepping controller in enhancing the robustness of control systems with high accuracy.

1. INTRODUCTION

Switched reluctance motors (SRMs) can be applied in many industrial applications due to their cost advantages and ruggedness. The switched reluctance motor is simple to construct. It is not only features a salient pole stator with concentrated coils, which allows earlier winding and shorter end turns than other types of motors, but also features a salient pole rotor, which has no conductors or magnets and is thus the simplest of all electric machine rotors. Simplicity makes the SRM inexpensive and reliable, and together with its high speed capacity and high torque to inertia ratio, makes it a superior choice in different applications [1, 2]. Due to new developments in nonlinear control theory, several nonlinear control techniques have been introduced in the last two decades. One of the nonlinear control methods that have been applied to switched reluctance motor control is the backstepping design [3, 4]. The backstepping is a systematic and recursive design methodology for nonlinear feedback control. This approach is based upon a systematic procedure for the design of feedback control strategies

[1] University of Mascara 29000, Algeria; E-mail: tah_ahmed13@yahoo.fr
[2] Laboratoire IRECOM, Département d'électrotechnique, Faculté d'ingenieur, Université de Sidi Bel Abbes, Sidi Bel Abbes 22000, Algerie
[3] University of Biskra 07000, Algeria

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suitable for the design of a large class of feedback linearisable nonlinear systems exhibiting constant uncertainty, and guarantees global regulation and tracking for the class of nonlinear systems transformable into the parametric-strict feedback form. It offers a choice of design tools to accommodate uncertainties and nonlinearities and can avoid wasteful cancellations. The idea of backstepping design is to select recursively some appropriate functions of state variables as pseudo-control inputs for lower dimension subsystems of the overall system. When the procedure terminates, a feedback design for the true control input results which achieves the original design objective by virtue of a final Lyapunov function, which is formed by summing up the Lyapunov functions associated with each individual design stage [5, 6]. The organization of this paper is as follows: in section 2, the diagram of the controller for SRM is presented; in section 3, the proposed adaptive backstepping, and used to control the position of the switched reluctance motor. Simulation results are given to show the effectiveness of this controller. Conclusions are summarized in the last section.

2. **SRM MODEL**

2.1. DESCRIPTION OF THE SYSTEM

In a switched reluctance machine, only the stator presents windings, while the rotor is made of steel laminations without conductors or permanent magnets. This very simple structure reduces greatly its cost. Motivated by this mechanical simplicity together with the recent advances in the power electronics components, much research has being developed in the last decade [1, 2]. A cross-sectional view is presented in Fig. 1.

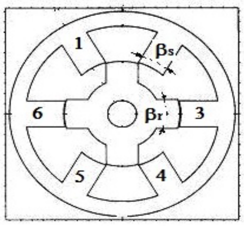


Fig. 1 – Switched reluctance motor.

2.2. MOTOR EQUATION

The switched reluctance motor has a simple construction, but the solution of its mathematical models is relatively difficult due to its dominant non-linear behaviour. The flux linkage is a function of two variables, the current I and the rotor position (angle θ). The mathematical model from the equivalent circuit is [1, 2, 11] :

$$v_j = Ri_j + \frac{\mathrm{d}\Psi_j(\mathbf{i},\theta)}{\mathrm{d}t}, \quad j = 1, 2, 3, \tag{1}$$

Then we can write:

$$v_j = Ri_j + \frac{\mathrm{d}\Psi_j(\mathbf{i},\theta)}{\mathrm{d}i}\frac{\mathrm{d}\mathbf{i}}{\mathrm{d}t} + \frac{\mathrm{d}\Psi_j(\mathbf{i},\theta)}{\mathrm{d}\theta}\omega, \quad j = 1, 2, 3, \tag{2}$$

in which: $\omega = \frac{\mathrm{d}\theta}{\mathrm{d}t}$. The motion equation is:

$$J\frac{\mathrm{d}\omega}{\mathrm{d}t} = T_e - T_l - f\omega. \tag{3}$$

The average torque can be written as the superposition of the torque of the individual motor phases:

$$T_e = \sum_{phase=1}^{n} T_{phase}. \tag{4}$$

where V – the terminal voltage, I – the phase current, R – the phase winding resistance, Ψ – the flux linked by the winding, J – the moment of inertia, f – the friction, $L(\theta,i)$ – the instantaneous inductance and T_e is the total torque.

The schematic diagram of the position control system under study is shown in Fig. 2 [1, 2, 11].

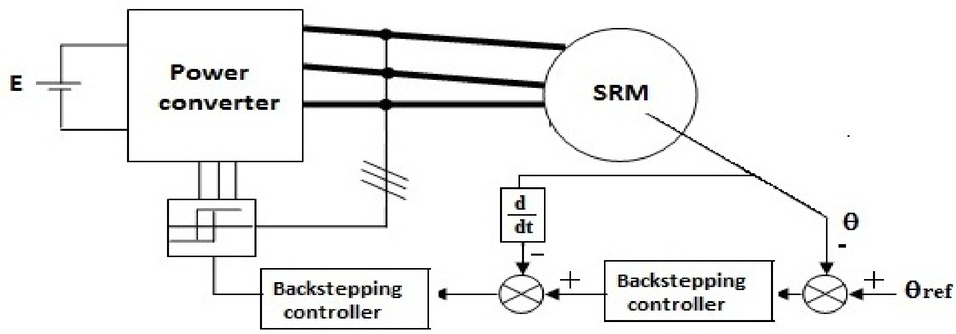


Fig. 2 – Control of SRM.

3. SRM BACKSTEPPING POSITION CONTROLLER

3.1. BACKSTEPPING PRINCIPLE

The system input and output are respectively u and y, while the reference trajectory is denoted y_r [3, 8, 9, 10].

Step 1: The first error variable is defined as

$$\varepsilon_1 = y - y_r = x_1 - y_r \,. \tag{5}$$

The first candidate Lyapunov function is chosen as:

$$V_1 = \frac{1}{2}\varepsilon_1^2 \tag{6}$$

and its derivative is

$$\dot{V}_1 = \varepsilon_1\dot{\varepsilon}_1 = \varepsilon_1(x_2 - \dot{y}_r)\,. \tag{7}$$

To render the later negative, x_2 is taken as the first virtual control. Its desired value

$$\alpha_1 = (x_2)_d = -k_1\varepsilon_1 + \dot{y}_r \qquad k_1 > 0\,, \tag{8}$$

where k_1 is a positive design parameter. With the above choice, (8) becomes definite negative.

Step 2: The new error variable is

$$\varepsilon_1 = x_2 - \alpha_1 = x_2 + k_1\varepsilon_1 - \dot{y}_r \,. \tag{9}$$

An augmented candidate function Lyapunov is introduced:

$$V_2 = \frac{1}{2}\varepsilon_1^2 + \frac{1}{2}\varepsilon_2^2 \,. \tag{10}$$

Since

$$\dot{\varepsilon}_1 = x_2 - \dot{y}_r = \varepsilon_2 + k_1\varepsilon_1\,, \tag{11}$$

the derivative of (10) is given by:

$$\dot{V}_2 = -k_1\varepsilon_1^2 + \varepsilon_2(\varepsilon_1 + \dot{x}_2 - \dot{\alpha}_1) = -k_1\varepsilon_1^2 + \varepsilon_2[(1-k_1^2)\varepsilon_1 + k_1\varepsilon_2 + x_3 - \ddot{y}_r]\,. \tag{12}$$

Choosing x_3 as the second virtual control, and selecting its value to render $\dot{V}_2$ definite negative, gives:

$$\alpha_2 = (x_3)_d = -[(1-k_1^2)\varepsilon_1 - (k_1 + k_2)\varepsilon_2 + \ddot{y}_r] \qquad k_2 > 0\,. \tag{13}$$

Step *n*+1: Defining [12, 13, 14]

$$\varepsilon_{n+1} = x_{n+1} - \alpha_n \,, \tag{14}$$

$$V_{n+1} = \frac{1}{2}\sum_{j=1}^{n+1} \varepsilon_j^2 \,. \tag{15}$$

It gives:

$$\begin{aligned} \dot{\varepsilon}_n &= \varepsilon_{n+1} - k_n \varepsilon_n - \varepsilon_{n-1} \,, \\ \dot{V}_{n+1} &= -\sum_{j=1}^{n} k_j \varepsilon_j^2 + \varepsilon_{n+1}(\varepsilon_n + \dot{x}_{n+1} - \dot{\alpha}_n). \end{aligned} \tag{16}$$

which leads to

$$\dot{\varepsilon}_{n+1} = -k_{n+1}\varepsilon_{n+1} - \varepsilon_n \tag{17}$$

and

$$\alpha_{n+1} = (\dot{x}_{n+1})_d = -k_{n+1}\varepsilon_{n+1} - \varepsilon_n + \dot{\alpha}_n \qquad k_{n+1} > 0 \,. \tag{18}$$

3.2. BACKSTEPPING CONTROLLER

Step1: Define the position tracking error as:

$$\varepsilon_1 = \theta_{ref} - \theta \,, \tag{19}$$

where θ_{ref} is the desired reference trajectory of the rotor angle. The position error dynamics is then:

$$\dot{\varepsilon}_1 = \dot{\theta}_{ref} - \dot{\theta} = \dot{\theta}_{ref} - \omega_r \,. \tag{20}$$

The stabilizing function is determined by differentiating the Lyapunov function $V_1 = \frac{1}{2}\varepsilon_1^2$ to get:

$$\dot{V}_1 = \varepsilon_1\dot{\varepsilon}_1 = \varepsilon_1(\dot{\theta}_{ref} - \omega_r) \,. \tag{21}$$

We now choose the first stabilizing function as:

$$\omega_r = k_1\varepsilon_1 + \dot{\theta}_{ref} \,. \tag{22}$$

Equation (22) indicates the desired velocity for position tracking. The next step is to design a speed controller so that the rotor speed will follow.

Substituting (22) back into equation (21) would yield: $\dot{V}_1 = -k_1\varepsilon_1^2$, where $k_1 > 0$ is design constant. Thus the virtual control is asymptotically stable.

Step 2: Now we define the speed tracking error as:

$$\varepsilon_2 = \omega_{ref} - \omega = k_1\varepsilon_1 + \dot{\theta}_{ref} - \omega . \tag{23}$$

From equation (23) the position error dynamics can be written as: $\varepsilon_1 = -k_1\varepsilon_1 + \varepsilon_2$. The speed error dynamics is defined as:

$$\dot{\varepsilon}_2 = \dot{\omega}_{ref} - \dot{\omega} = -k_1^2\varepsilon_1 + k_2\varepsilon_2 + \ddot{\theta}_{ref} - (\frac{1}{J})(T_e - f\omega - T_l) . \tag{24}$$

Now define a new Lyapunov function as:

$$V_2 = \frac{1}{2}\varepsilon_1^2 + \frac{1}{2}\varepsilon_2^2 . \tag{25}$$

Differentiate to get:

$$\dot{V}_2 = \varepsilon_1\dot{\varepsilon}_1 + \varepsilon_2\dot{\varepsilon}_2 = -k_1\varepsilon_1^2 + \varepsilon_2[(1-k_1^2)\varepsilon_1 + k_1\varepsilon_2 + \ddot{\theta}_{ref} - (\frac{1}{J})(T_e - f\omega - T_l)] . \tag{26}$$

We define the reference currents as:

$$I_r = \frac{J}{k_t}\ddot{\theta}_{ref} + \frac{f}{k_t}\omega + \frac{T_l}{k_t} + \frac{J}{k_t}(1-k_1^2)\varepsilon_1 + \frac{J}{k_t}(k_1 + k_2)\varepsilon_2, \qquad k_1,\ k_2 > 0 . \tag{27}$$

Substituting (27) back into equation (26) would yield: $\dot{V}_2 = -k_1\varepsilon_1^2 - k_2\varepsilon_2^2$ where k_1, $k_2 > 0$ are design constants. Thus the virtual control is asymptotically stable.

3.3. ADAPTIVE BACKSTEPPING

Step 3: The goal now is to make I_r follow the reference trajectory I_{ref}. The final current error signals are defined as:

$$\varepsilon_3 = I_{ref} - I_r . \tag{28}$$

The speed error can be represented by:

$$\dot{\varepsilon}_2 = -\varepsilon_1 - k_1\varepsilon_2 + \frac{k_t}{J}\varepsilon_3 + \tilde{C} + \tilde{D}\omega , \tag{29}$$

$$\tilde{C} = \hat{C} - C \quad \text{and} \quad \tilde{D} = \hat{D} - D,$$
$$C = \frac{-T_l}{J}, \quad D = \frac{-f}{J}.$$

Step 4: The final Lyapunov function includes the current error and parameter estimation errors:

$$V_3 = \frac{1}{2}\varepsilon_1^2 + \frac{1}{2}\varepsilon_2^2 + \frac{1}{2}\varepsilon_3^2 + \frac{1}{\lambda_1}\tilde{C}^2 + \frac{1}{\lambda_2}\tilde{D}^2, \tag{30}$$

where λ_1, λ_2 are adaptive gains. Now differentiate and substitute all error dynamic equations to get:

$$\begin{aligned}\dot{V}_3 &= \varepsilon_1\dot{\varepsilon}_1 + \varepsilon_2\dot{\varepsilon}_2 + \frac{1}{\lambda_1}\tilde{C}\dot{\hat{C}} + \frac{1}{\lambda_2}\tilde{D}\dot{\hat{D}} \\ &= \varepsilon_1(-k_1\varepsilon_1 + \varepsilon_2) + \varepsilon_2(-\varepsilon_1 - k_2\varepsilon_2 + \frac{k_t}{J}\varepsilon_3 + \tilde{C} + \tilde{D}\omega) + \frac{1}{\lambda_1}\tilde{C}\dot{\hat{C}} + \frac{1}{\lambda_2}\tilde{D}\dot{\hat{D}}.\end{aligned} \tag{31}$$

The update laws are defined as:

$$\dot{\hat{C}} = -\lambda_1(\varepsilon_2 + \frac{J}{k_t}(k_1 + k_2)\varepsilon_3) \quad \text{and} \quad \dot{\hat{D}} = -\lambda_2(\varepsilon_2\omega + \frac{J}{k_t}(k_1 + k_2)\varepsilon_3\omega). \tag{32}$$

Substituting (33) back into equation (32) would yield:

$$\dot{V}_2 = -k_1\varepsilon_1^2 - k_2\varepsilon_2^2 - k_2\varepsilon_3^2 + \frac{k_t}{J}\varepsilon_2\varepsilon_3 < 0, \tag{33}$$

where $k_1, k_2, k_3 > 0$ are design constants. Thus the virtual control is asymptotically stable.

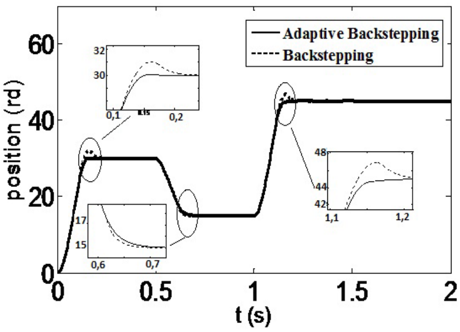
4. RESULT AND SIMULATION

In order to validate the control strategies as discussed above, digital studies were made the system described in Fig. 2. The position and the currents loops of the drive were also designed and simulated respectively with backstepping control and adaptive backstepping control. The simulation with starting mode without load is done, followed by changing of the reference:

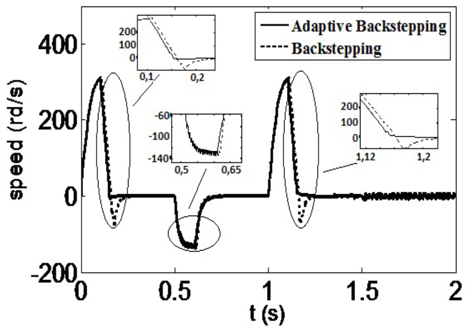
1. 0 to 0.5s $\theta_{ref} = 30$ rd.
2. 0.5 to 1s $\theta_{ref} = 15$ rd.

3. 1 to 2s $\theta_{ref} = 45$ rd .

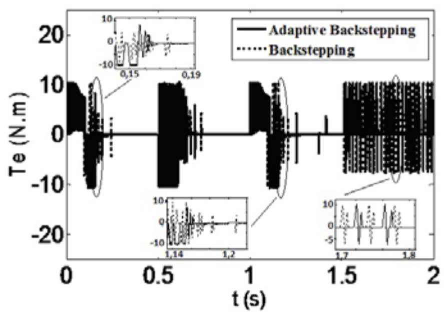
The simulation is realized using the SIMULINK software in MATLAB environment. Figure 3 shows the performance of adaptive backstepping controller. The actual position converges to the reference position in a short time with no overshot and no steady state error. Figures 3a, 3b, 3c, 3.d and 3e shows the corresponding position, speed, torque, current and voltage. At 1.5s, the load torque was applied. There is no noticeable change in the position results.



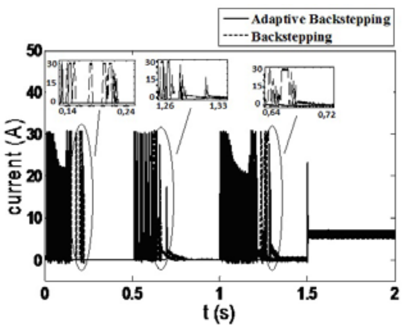
a) Position



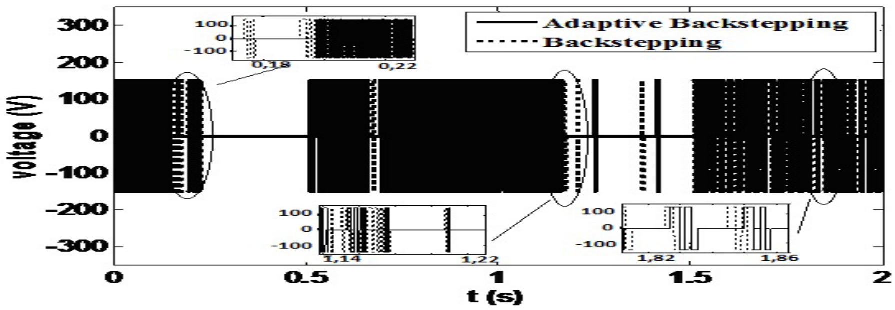
b) Speed



c) Torque



d) Current



e) Voltage

Fig. 3 – Simulation results of position control.

5. ROBUSTNESS

In order to test the robustness of the proposed control, we have studied the speed performances. Two cases are considered the stator resistance variation and inertia variation. Fig. 4 shows the tests of the robustness:

a) the robustness tests concerning the variation of the resistances,

b) the robustness tests in relation to inertia variations.

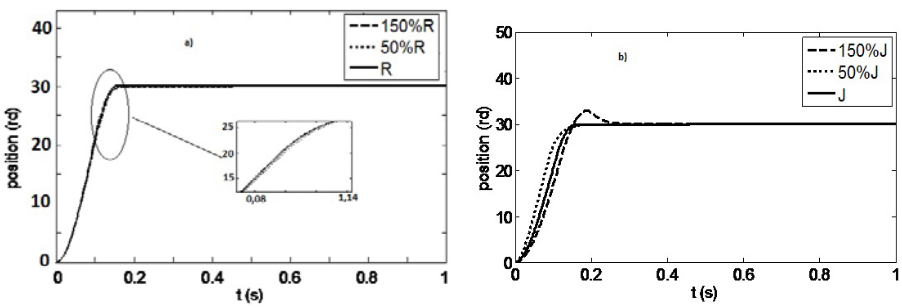


Fig. 4 – Test of robustness.

Figure 4b shows the parameter variation does not allocate performances of proposed control. The position response is insensitive to parameter variations of the machine, without overshoot and without static error. The other performances are maintained.

6. **CONCLUSION**

An adaptive backstepping based nonlinear control technique for SRM drive has been developed for both position controls. The control laws were derived based on the motor model incorporating various system uncertainties. Global stability of the developed nonlinear controller has been verified analytically using Lyapunov's stability theory. As show by the simulation results, position tracking were achieved with no steady state error or overshoot. The performance of these controllers was found be superior to the drive.

APPENDIX

Switched reluctance Motors Parameters

Number of phase 3, Number of stator poles 6, Pole arc 30°, Number of rotor poles 4, Pole arc 30°, Maximum inductance 60 mH, Minimum inductance 8 mH, Resistance 1.3 Ω, Moment of inertia 0.0013kgm^2, Friction 0.0183Nm/s, Inverter voltage 150 V.

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