MULTILEVEL CHAOS-BASED DS-CDMA SYSTEM WITH IMPROVED PERFORMANCE

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This paper investigates the chaos-based asynchronous direct sequence code-division multiple access (DS-CDMA) system performance optimization in accordance with spreading sequences set. The multiple access interference (MAI) power is estimated for conventional matched-filter receiver scheme, under the standard gaussian approximation (SGA) hypothesis, and minimized with respect to the auto-correlation function of the spreading sequences. The well-known family of optimum auto-correlation chaotic spreading sequences generated by the \((n, t)\) – tailed shifts map is reconsidered. The present work intends to prove that not only binary quantized sequences are optimum, but also for any number of levels an optimum of the auto-correlation function exists.

1. INTRODUCTION

Spread-spectrum based code division multiple access (CDMA) has taken on a significant role in cellular and personal communications. The already implemented second and third generation, and the future fourth generation mobile communications systems are taking benefits of the CDMA signal-processing scheme. The growing demand for higher data rates is determining the continuous search for communications systems with better performances. CDMA has been found to be attractive because of such characteristics as potential capacity increases over competing multiple access methods, anti-multipath capabilities, soft capacity, narrow-bandwidth anti-jamming, and soft handoff.

The bit error rate (BER) performances of direct sequence CDMA (DS-CDMA) systems depend mainly on the correlation properties of the spreading...
sequences set [1–3]. The use of low cross-correlation sets of sequences increases the BER performances and the system capacity as well. Hence, it is imperative to design optimum spreading sequences sets that minimize the BER.

Classical sets of spreading sequences used in actual standards of DS-CDMA mobile communications systems are binary sequences generated by linear-feedback shift register (LFSR) schemes. Even for minimum cross-correlation sequences, forming Gold and Kasami sets, the set dimension and the period of the sequences are limited by the LFSR polynomial degree. Another drawback of these sequences is induced by the generator linearity, which increases the interception probability.

A new direct-sequence spreading method assumes the use of discrete-time non-linear dynamical systems trajectories. These chaotic sequences present noise-like features that make them good for spreading in DS-CDMA systems [3–5]. A single system, described by its discrete chaotic map, can generate a very large number of distinct chaotic sequences, each sequence being uniquely specified by its initial value. This dependency on the initial state and the non-linear character of the discrete map make the DS-CDMA system using these sequences more secure.

It is known that binary quantized sequences generated by piece-wise affine Markov (PWAM) \((n, t)\) – tailed shifts maps minimize the BER under the standard Gaussian approximation (SGA) assumption in the asynchronous DS-CDMA system [4–6]. However, this paper considers the more general case of multilevel quantized \((n, t)\) – tailed shifts sequences [4], [7–9]. In fact, for any number of quantizing levels, an optimum set of \((n, t)\) – tailed shifts chaotic sequences exists. A similar chaos-based sequence generation method for reducing multiple access interference (MAI) in direct sequence UWB (Ultra Wide-Band) wireless-sensor-networks (WSNs) is presented in [10].

This paper is organized as follows. Section 2 presents the design method for optimal sets of chaotic multilevel quantized \((n, t)\) – tailed shifts sequences based on their auto-correlation shaping, in order to minimize the MAI power under the SGA assumption. This analysis of optimum chaotic sequences is performed as compared to the case of random sequences. Section 3 presents some simulation results compared to the theoretical average BER values for both chaotic optimal and random sequences. Finally, some conclusions are drawn in the last section.

2. OPTIMUM MULTILEVEL TAILED SHIFTS SEQUENCES

2.1. SECOND ORDER MOMENT ESTIMATION

One of the well known family of PWAM maps that generate chaotic sequences is the \((n, t)\) – tailed shifts map, defined in [3–8].

It is known that these maps are exact and have a uniform invariant probability density function [4]. Using the tensorial algebra as in [4] it can be demonstrated...
that for the \((n, t) -\) tailed shifts map \(M\)-levels uniformly quantized sequences, the second order moment has the following approximate expression, for any user \(k\) [7]:

\[
A_k(l) = \mathbb{E}[f(a_0^{(k)})f(a_l^{(k)})] = \begin{cases}
\frac{M + 1}{3(M - 1)} & \text{for } l = 0;
\frac{1 + g}{M} \cdot \text{ceil}\left(\frac{M}{1 + g}\right) + M + 1 - 2 \cdot \text{ceil}\left(\frac{M}{1 + g}\right) + g(M - 1)^2, & \text{for } l \neq 0,
\end{cases}
\]

(1)

where \(g = t/(n - t)\), and the function \(f(x)\) is the \(M\) levels uniform quantization function [7–9].

For a constant power level at the output of the matched-filter receiver in the DS-CDMA system it is necessary to normalize all the shifted values of the second order moment by its non-shifted value. Hence, the equation (1) is rewritten in the normalized form as:

\[
\tilde{A}_k(l) = \frac{A_k(l)}{\tilde{A}_k(0)} = \frac{\mathbb{E}\{f(a_0^{(k)})f(a_l^{(k)})\}}{\mathbb{E}\{f(a_0^{(k)})\}^2} = \begin{cases}
1, & \text{for } l = 0;
G(-g)^l, & \text{for } l \neq 0, \forall k,
\end{cases}
\]

(2)

where we denoted the following term in (2):

\[
G = G(M, g) = \frac{3 + g}{M} \cdot \text{ceil}\left(\frac{M}{1 + g}\right) + M + 1 - 2 \cdot \text{ceil}\left(\frac{M}{1 + g}\right) + g(M - 1)^2.
\]

(3)

It is interesting to underline the existence of two limit cases for the quantization: the binary quantization \((M = 2)\), and the infinite quantization \((M \to \infty)\). The last quantization case, which considers an infinite number of levels, is theoretically equivalent to the case of no quantization at all. For the binary quantized sequences, by making \(M = 2\) in (3), results that:

\[
G(2, g) = 1, \quad \forall g \in [0, 1],
\]

(4)

and the normalized second order moment in (2) has the following expression [4, 6, 7]:

\[
\tilde{A}_k(l) \approx (-g)^l, \quad \forall l, k.
\]

(5)

It is important to note that if the number of quantizing levels \(M\) is very large \((M \to \infty)\) at the limit, but practical values larger than 20 are enough as approximations), then the value of \(G\) in (3) is:
and the normalized second order moment in (2) may be rewritten as follows:

\[
\overline{A}_k(l) = \begin{cases} 
1, & \text{for } l = 0 \\
\frac{3}{(1 + g)^2} (-g)^l, & \text{for } l \neq 0, \forall k.
\end{cases}
\]

2.2. MAI VARIANCE ESTIMATION FOR ASYNCHRONOUS DS-CDMA SYSTEM

In the asynchronous DS-CDMA system the overall (non-faded) mean interference (MAI – Multiple Access Interference) power for any user \(i\), \(\sigma_A^2(i)\) can be computed as \([1, 3, 7, 8]\):

\[
E_{\{a_i\}}[\sigma_A^2(i)] \approx \frac{PT^2(K - 1)}{12N^3}.
\]

\[
\cdot \sum_{l=1}^{N-1} \frac{N}{k+l} \left[ C_{k,l}^2(l) \right] + \frac{k+l}{k+l} \left[ C_{k,l}(l)C_{k,l}(l + 1) \right],
\]

where \(N\) is the sequence period (equal to the spreading factor), \(T\) is the data symbol duration, \(P\) is the common received power, \(K\) is the number of all the users in the system, \(C_{k,l}(l)\) is the discrete aperiodic cross-correlation for the sequences \(a_j^{(k)}\) and \(a_j^{(l)}\), and the average is estimated over the whole spreading set \([1, 3, 7, 8]\).

It is known that when perfectly random sequences (white noise-like sequences) are employed, the average MAI variance in (8) may be written as \([1]\):

\[
E_{\{a_i\}}[\sigma_A^2, \text{white}(i)] = \frac{PT^2(K - 1)}{6N}.
\]

According to \([3], [6], \text{and } [8]\) the lower bound of average BER for all the users can be attained under the SGA (Standard Gaussian Approximation) assumption, if using spreading sequences that have the auto-correlation ensemble (2nd order moment) expression which minimizes the mean MAI power in (8), as:
\[ A_k(l) = \frac{\Delta}{\|a_k\|} \mathbb{E}[a_j^{(k)} a_{j+l}^{(k)}] = (-1)^l \frac{r^{l-N} - r^{N-l}}{r^N - r^{-N}}, \quad l = 0, 1, 2, \ldots, N - 1, \quad \forall k, \] (10)

where \( r = 2 - \sqrt{3} \). Note that when \( l \ll N, A_k(l) \approx (-r)^l \), which decays exponentially with alternate sign. By introducing relation (10) into (8), the minimum average interference power is obtained for user \( i \) [6, 8]:

\[
\mathbb{E}[\sigma^2_{A, \text{optimum}(i)}] \approx \frac{PT^2(K - 1)\sqrt{3}}{12N}. \tag{11}
\]

Comparing the optimum case with the case when white sequences are employed for spreading, the first one offers an increase in the system user capacity by more than 15% as compared with the white spreading case, for the same mean MAI variance.

Then, considering the normalized second order moment in (2) into the mean MAI variance in (8), the latter can be demonstrated to have the following approximate expression [8]:

\[
\mathbb{E}[\sigma^2_{A, \text{optimum}(i)}] \approx \frac{PT^2(K - 1)}{6N} \left[ 1 - Gg + 2G^2g^2 + G^2g^3(2g - 1) \right], \tag{12}
\]

that can be rewritten for the binary \((n, t)\) – \textit{tailed shifts} sequences, with the second order moment defined in (5), as:

\[
\mathbb{E}[\sigma^2_{A, (n, t)\rightarrow\text{TS}, 2\text{levels}(i)}] \approx \frac{PT^2(K - 1)}{6N} \left( \frac{1 - g + g^2}{1 - g^2} \right), \tag{13}
\]

where the minimum mean MAI value is reached for \( g_2 = g_{M=2} = r = 2 - \sqrt{3} \approx 0.26795 \), as in equation (10) which defines the optimum case.

Using the same procedure for infinite number of levels uniformly quantized \((n, t)\) – \textit{tailed shifts} sequences with the second order moment defined in (7), the mean MAI variance over the spreading set can be estimated as:

\[
\mathbb{E}[\sigma^2_{A, (n, t)\rightarrow\infty\text{levels}(i)}(g)] \approx \frac{PT^2(K - 1)}{6N} \left[ \frac{1 + g + 17g^2 - 9g^2 + g^4 - g^5 - g^6}{1 + 4g + 5g^2 - 5g^4 - 4g^5 - 6g^6} \right], \tag{14}
\]
where the minimum mean MAI variance value in (11) is reached for not 
\[ g_{\infty} = g_{M \to \infty} = 0.10952. \]

It was demonstrated in [9] that for a system over a Rician selective fading channel the BER can be minimized using the same optimization procedure.

The mean MAI variance per user is depicted in Fig. 1 as a function of the parameter \( g \), considering three values for the number of quantizing levels \( M \). Hence, the two limit cases were considered, for binary and infinitely quantized \((n, t) - \text{tailed shifts}\) sequences (equations (13) and (14)), and the quaternary quantization case (equation (12), with \( M = 4 \)).

![Fig. 1 – MAI variance per user as a function of \( g \), for several values of \( M \).](image)

It is obvious from Fig. 1 that for all three cases, the minimum mean MAI variance value in (11) is reached, but for different \( g \) values. Thus, a very important note is that the mean MAI variance in (12) is a convex function of \( g \), and reaches the minimum (optimum) value from (11) for any value of the number of levels \( M \). In fact, the optimum value of \( g \) is varying from \( g_2 \) to \( g_{\infty} \) when \( M \) varies from 2 to \( \infty \). This is illustrated in Fig. 2. As the Fig. 2 shows the optimum \( g \) rapidly converges to \( g_{\infty} \), when the number of quantization levels \( M \) is increasing. Values of \( M \) slightly larger than 20 are enough to approximate the optimum case for \( g_{\infty} \).
3. EXPERIMENTAL RESULTS

The transmission channel is assumed to be a non-selective fading Rician channel with two-sided power spectral density (PSD) $N_0/2$ additive Gaussian noise. The output signal of a Rician nonselective fading channel is the sum of a non-faded version of the input signal (specular component) and a non-delayed faded version of the input signal (scatter component). All communications links are assumed to fade independently. We also assume that all users have the same faded power ratio.

Considering the asynchronous DS-CDMA over the above presented channel, the average BER over the whole set of spreading sequences may be estimated under the SGA assumption as in [1–3], [8].

The asynchronous DS-CDMA system presented above using quantized $(n, t)$-tailed shifts sequences, and Gold (white, pseudo-noise (PN)) sequences generated by $m = 6$ degree primitive polynomials of period $N = 2^m - 1 = 63$ were considered for simulation. The two limits of the number of quantizing levels are assumed for the chaotic sequences: binary and infinite. The parameters values of the $(n, t)$-tailed shifts map were chosen to match the condition for optimizing the BER performances (the minimum MAI variance in (11), (13), and (14)):

$$g_2 = \frac{t_2}{n-t_2} = \frac{11}{39} \approx 0.282 \approx 0.268 \approx 2 - \sqrt{3} = r$$

$$g_\infty = \frac{t_\infty}{n-t_\infty} = \frac{5}{45} \approx 0.111 \approx 0.10952$$

for $n = 50$ \hspace{1cm} (15)
The estimated average BER was evaluated for $K = 10$ users and the energy per bit to noise DSP ratio $E_b/N_0$ taking values from 0 to 30 dB. The common faded power ratio is taking the value $\gamma^2 = 0.1$. The resulting simulated and theoretical BER curves as function of the ratio $E_b/N_0$ are depicted in Fig. 3. The asynchronous DS-CDMA system capacity is also an important parameter to measure. The average BER was estimated considering several values for the number of users $K \in \{12 \ldots 30\}$ having the same value of the energy-per-bit to noise DSP ratio $E_b/N_0 = 24$ dB. The resulting simulated and theoretical BER curves as function of the number of users $K$ are depicted in Fig. 4.

The simulation results show that optimum multilevel (for any $M$) quantized $(n, t)$ – tailed shifts sequences are better than Gold sequences in terms of allowable number of users (Fig. 4) by more than 15%. This result is consistent with the analytical result presented in Section II.B. However, there are some differences between the simulation and analytical results given the fact that Gold sequences are not perfectly white, quantized $(n, t)$ – tailed shifts sequences are in fact pseudo-optimal, and the SGA approximation is not quite valid for a small number of users.
4. CONCLUSIONS

A family of optimal spreading sequences for asynchronous DS-CDMA system for the SGA approximation hypothesis was considered for minimising the average BER. The quantized sequences generation method for \((n, t)\) - tailed shifts map and their correlation properties were also presented. The BER performance of the asynchronous DS-CDMA system was estimated assuming a frequency non-selective fading channel with AWGN noise. Hence, the use of these optimal sequences for the asynchronous DS-CDMA system offers a capacity increase of about 15% than when white sequences or Gold codes are used.

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