ESTIMATION OF THE INFLUENCE TERMS INVOLVED IN STATIC DIAMAGNETIC LEVITATION

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Key words: Diamagnetic levitation, Stability area, Influence terms.

The stabilization of the static magnetic levitation can be achieved by using pieces of strong diamagnetic materials, which placed nearby the suspended magnet stabilize its intrinsically unstable equilibrium. This paper aims a better evaluation of the influence term added by the presence of the diamagnetic materials in order to realize a more accurate prediction of the floater stability area. The analyze mainly uses mechanical restrictions and stationary magnetic field equations.

1. INTRODUCTION

A classical result of the Maxwell’s equations, known as the theorem of Earnshaw [1], does not allow the stable levitation of a permanent magnet to occur in a static or stationary field. In spite of this, the introduction of a piece of diamagnetic material in the proximity of the suspended magnet, can escape the configuration from theorem incidence stabilizing the equilibrium due to the special features of the diamagnetic material [2, 3]. This paper suggests an improved estimation of the influence term [4], brought by the presence of the diamagnetic material by taking into account its finite geometrical data. That allows a more accurate estimation for the width of the stability area of the levitated magnet. In order to obtain realizable analytical results, the whole study is done over a vertical cylindrical symmetric levitation setting.

2. EQUILIBRIUM AND STABILITY

The stable static levitation of a permanent magnet asks for a minimum of the total energy along with the satisfied equilibrium condition [2]. If an $M$ dipole
magnet is imbedded in a field $B$, and $M$ and $B$ are parallel, then the potential energy $U$ of the system can be written as:

$$U = -MB + mgz = -MB + mgz,$$

where $mgz$ is the gravitational potential energy.

Assuming a circularly symmetric field $B(r, z)$, the equilibrium points will be on $z$-axis of the symmetry. Then the condition that $(z_0, 0)$ to be an equilibrium point becomes:

$$F = -\nabla U\bigg|_{z=z_0} \Rightarrow \nabla B\bigg|_{z=z_0} = -\frac{mg}{M}.$$

The stability conditions in $(z_0, 0)$ requires a minimum value of energy, which means positive curvature of energy function in every direction:

$$\left.\frac{\partial^2 U}{\partial z^2}\right|_{r=0, z=z_0} > 0; \text{ vertical stability}; \quad \left.\frac{\partial^2 U}{\partial r^2}\right|_{r=0, z=z_0} > 0; \text{ radial stability}. \quad (3)$$

To complete the problem, we express the magnitude of the magnetic field $B$ in terms of its $z$-component $B_z(r, z)$ only. Using the local magnetic laws: $\nabla B = 0$ and $\nabla \times B = 0$, the following extension of $B(r, z)$ around $(z_0, 0)$ derive [5]:

$$B(r, z) = B_0 + B'_0(z - z_0) + \frac{1}{2} B''_0(z - z_0)^2 + \frac{1}{4} \left( \frac{B'_0}{2B_0} - B'_0 \right) r^2 + \cdots. \quad (4)$$

Here $B_0 = B_z(0, z)|_{z=z_0}$; $B'_0 = \frac{\partial B_z}{\partial z}|_{z=z_0}$; $B''_0 = \frac{\partial^2 B_z}{\partial z^2}|_{z=z_0}$.

As we already explained in the previous section, a piece of diamagnetic material placed beneath the levitated magnet stabilizes the equilibrium of the floater. In this case, a new term $Cz^2$ is added to (1), which represents the diamagnetic material contribution to the potential energy. According to (4) and considering the diamagnetic material presence, the whole potential energy $U$ is reconsidered:

$$U = -M \left[ B_0 + \left(\frac{B'_0 - \frac{mg}{M}}{B_0}\right)(z - z_0) + \frac{1}{2} B''_0(z - z_0)^2 + \frac{1}{4} \left( \frac{B'_0}{2B_0} - B'_0 \right) r^2 + \cdots \right] + Cz^2(z - z_0)^2. \quad (5)$$

From the equilibrium condition required by (2), we obtain the magnetic field gradient $B'$ equal to $-mg/M$. This means that the quantity in the first curly
branches of (5) approaches zero. Vertical and horizontal stability conditions given by (3) can be now rewritten:

\[
\begin{align*}
\frac{\partial^2 U}{\partial z^2} & \bigg|_{z=\alpha_0, r=0} > 0 \Rightarrow D_v = C_z - \frac{MB_0^*}{2} > 0; \\
\frac{\partial^2 U}{\partial r^2} & \bigg|_{z=\alpha_0, r=0} > 0 \Rightarrow D_h = \frac{M}{4} \left( B_0^* - \frac{B_0^2}{2B_0} \right) > 0.
\end{align*}
\]

The above conditions, called discriminants of stability \(D_v\) and \(D_h\), must be simultaneously satisfied in order to achieve the stable static levitation state of the floater.

3. DIAMAGNETIC INFLUENCE TERM FOR STABLE VERTICAL CONFIGURATION

If we consider now the case when the magnetic field curvature is positive and large enough to determine a horizontal stability \((D_h > 0)\), adding a diamagnetic material nearby the floater the vertical stabilized configuration can be obtained [3].

![Fig. 1](image1.png)  
*Fig. 1 – A compact setting for the vertically stabilized levitation system.*

![Fig. 2](image2.png)  
*Fig. 2 – The levitated magnet nearby the diamagnetic material.*
A simple vertical configuration that uses a common cylindrical coil as the magnetic field source is shown in Fig. 1, while Fig. 2 magnifies the floater beneath the diamagnetic plate. The piece of diamagnetic material can be made from graphite, bismuth or pyrolytic graphite (materials with great absolute values for magnetic susceptibility). For the floating magnet one uses rare-earth materials such as mixture of Nd₂Fe₁₄B, which could reach 1.5 T value for remnant flux density.

The stationary magnetic field provided by the coil and the physical properties of the levitated magnet must be adjusted that both the equilibrium condition (2) and the stability requirement (6) to be simultaneous satisfied.

According to (6) the vertical stability condition can be written as follows:

\[
\frac{2C_z}{M} > B_0 > \frac{B_0^2}{2B_0} = \frac{(mg)^2}{2B_0M^2}.
\]

From (7) one can notice that the inferior limit of the magnetic flux curvature in the levitation point is imposed by the \(C_z\) term, added by the diamagnetic material presence. Thus, it is very important to have a good estimation of the \(C_z\) term for a accurate prediction of the floater stability area.

The \(C_z\) term is proportional to the diamagnetic susceptibility (\(\chi\)) and magnetic momentum \(M\) of the floater and gets smaller if the gap \(d\) between the levitated magnet and diamagnetic body increases – Fig. 2. An analytical expression for \(C_z\), bellow denoted with \(C_z(d)\), is obtained in [4] by using the dipole approximation of the floater and the method of magnetic images in the diamagnetic piece, considering an infinite thickness of the diamagnetic plate:

\[
C_z(d) = \frac{3|\chi|\mu_0M^2}{\pi(2d + L)^3},
\]

where \(\mu_0\) represents vacuum permeability and \(L\) is the height of levitated magnet.

According to (7) and (8) the maximum value for the gap distance between the floater and the diamagnetic material: \(d_{\text{max}}\) (the stability area) can be analytically predicted:

\[
d_{\text{max}} \leq \left( \frac{3|\chi|\mu_0B_0M^4}{8\pi(mg)^2} \right)^{1/5} - \frac{L}{2}.
\]

Because (8) does not take into account the thickness \(T\) of the diamagnetic material, we further calculate its contribution by using a superposition method, allowed by the linear characteristic of the diamagnetic materials. This is done by finding the resulting effect of two separate cases having semi-infinite thick layers positioned at the top and at the bottom surfaces of the finite layer, respectively, and then by subtracting the second case effect from the first one, as shown in Fig. 3. The new diamagnetic influence term denoted \(C_z(d, T)\) becomes:
Influence terms involved in static diamagnetic levitation

\[
C_z(d, T) = \frac{3|x|\mu_0M^2}{\pi} \left[ \frac{1}{(2d + L)^5} - \frac{1}{(2d + L + 2T)^5} \right].
\] (10)

Considering the finite dimension of the diamagnetic plate, inequality (7) gives the maximum gap distance \(d_{\text{max}}\) from the expression:

\[
\left[ \frac{1}{(2d_{\text{max}} + L)^5} - \frac{1}{(2d_{\text{max}} + L + 2T)^5} \right] > \frac{\pi(mg)^2}{12|x|\mu_0B_0M^3},
\] (11)

which can be easily numerically solved.

![Fig. 3 – Superposition method for evaluation \(C(d, T)\).](image)

The contribution of finite thickness of diamagnetic material to the influence term is also suggested in [6], where the building of a novel diamagnetic motor is presented.

### 4. NUMERICAL APPROACH

For achieving a quantitative analyze over the above-proposed configuration (Fig. 1), we set up specific geometrical data and material properties to the model.

Thus for the cylindrical symmetric coil we assume the length \(l = 150\) mm, the thickness bounded by radius \(a_1 = 20\) mm and \(a_2 = 50\) mm, \(N = 300\) windings flowed by a direct current \(I = 5\) A. The floater was chosen as a tiny NdFeB cylinder (diameter \(2R = 4\) mm and \(L = 4\) mm height) with magnetic momentum \(M = 0.047\) Am\(^2\) and mass \(m = 0.39\) g. A piece of pyrolitic graphite (\(\chi = 450 \times 10^{-6}\)), of thicknesses \(T\) placed initially at \(d = 1.2\) mm underneath the levitated magnet, stabilizes the equilibrium.
In order to show the difference between the two expressions of the influence term given by (8) and (10) respectively, Fig. 4 plots their values due to the gap distance alternation assuming a thickness $T = 2$ mm and the above numerical data.

![Fig. 4 – The two influence terms variation due to the gap distance $d$ alternation.](image1)

![Fig. 5 – The influence term $C_\varepsilon(d, T)$ variation due to the distance gap $d$ and thickness $T$ alternation.](image2)

The variation of the influence term $C_\varepsilon(d, T)$ as a function of both gap distance $d$ and the thickness $T$ according to (10) is shown in Fig. 5.

Our approach toward is to estimate for the studied configuration the equilibrium point and its stability area with and without the consideration of the finite thickness of diamagnetic material.

The magnetic field variation on the $z$-symmetry axis of the coil could be determinate from the finite solenoid equation [7]:

$$B(z) = \frac{\mu_0 MI}{2l} \left( \frac{l + z}{a_2 - a_1} \ln \frac{a_2 + \sqrt{a_2^2 + (l + z)^2}}{a_1 + \sqrt{a_1^2 + (l + z)^2}} - \frac{z}{a_2 - a_1} \ln \frac{a_2 + \sqrt{a_2^2 + z^2}}{a_1 + \sqrt{a_1^2 + z^2}} \right)$$  \quad (12)

Setting $d = 1.2$ mm and $T = 2$ mm, the equilibrium equation (2) has one solution, namely $z_0 = 27.56$ mm. In this point, both stability restrictions (6) are satisfied, obtaining for $C_\varepsilon(d) = 0.115$ and $C_\varepsilon(d, T) = 0.105$, respectively.

These two distinct numerical values of influence term lead to different lengths of stability area interval according to (9) and (11). Thus, when the thickness is assumed endless we obtain $d_{\text{max}} = 1.404$ mm and considering the finite dimension of the stabilizing material ($T = 2$ mm) the stability interval has an abatement to $d_{\text{max}}(T) = 1.336$ mm.

Fig. 6 and Fig. 7 show the variation of stability discriminants on $z$-axis, along with the equilibrium point prediction with or without the diamagnetic material thickness consideration.
The various numerical results regarding the stability area (maximal gap distance) for different thicknesses $T$ of diamagnetic material piece are detailed in Table 1.

**Table 1**

<table>
<thead>
<tr>
<th>Thickness $T$ (mm)</th>
<th>Maximal gap distance $d_{\text{max}}(T)$ [mm]</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\infty$</td>
<td>1.404</td>
</tr>
<tr>
<td>10</td>
<td>1.404</td>
</tr>
<tr>
<td>9</td>
<td>1.402</td>
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<tr>
<td>7</td>
<td>1.401</td>
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<tr>
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<td>1.397</td>
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<tr>
<td>3</td>
<td>1.370</td>
</tr>
<tr>
<td>1</td>
<td>1.201</td>
</tr>
</tbody>
</table>

As one can notice from Table 1, the fact of taking into account the diamagnetic material thickness led us to a decreasing stability area, phenomenon which becomes more significant when the diamagnetic plate thickness decreases.

**5. CONCLUSIONS**

The problem of stabilizing the static levitation of the permanent magnets using diamagnetic materials is treated. The analyze examines the influence of the diamagnetic material geometrical data to the floater stability area interval. The equilibrium and stability conditions are obtained predicting the location of suspension point and its fixity. For a specific levitation array a numerical simulation was done, presenting the quantitative aspects regarding the real finite dimensions of the diamagnetic stabilizer.
As a result of this study, we achieve the values for the gap spacing between the floater and the diamagnetic material, which diminishes when the thickness of diamagnetic material has comparable values to the floater height.

This maximum gap distance (the stability area) can be enlarged by using diamagnetic pieces with higher absolute value for magnetic susceptibility, strong magnetic field sources and floater magnets having the highest magnetization – mass ratio.

The applicability area of this permanent magnet levitation can be found in very high-sensitive gravity sensors or in frictionless suspension design. Due to its unique features, this kind of levitation conquers new application in micro-nano devices, where complex classical instruments are difficult or impossible to integrate.

Received on 15 January, 2007

REFERENCES