TRANSFORMER INRUSH CURRENT PREDETERMINATION FOR DISTORTED WAVEFORM VOLTAGE SUPPLY

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Key words: Transformer inrush current, Harmonic distortion, Magnetic core characterization, Time domain analysis.

The magnetization current surged by a no-loaded power transformer at the connection instant has extremely high values. These inrush currents generate a significant electrodynamic and thermal stresses on the transformer windings and could determine the faulty operation of the protecting equipment devices. Moreover, since the majority of power distribution systems are affected by harmonic pollution, the applied voltage at the energizing moment has a non-sinusoidal waveform. Thus, the paper presents a predetermination of both the inrush current peak value and its harmonic spectrum under a distorted supply voltage waveform. Starting from the transient state equations along with a particular description of transformer core $H-B$ nonlinear characteristic and core stacking factor value, the inrush current time evolution is accurately predicted.

1. INTRODUCTION

Nowadays, the voltage waveform measured at any node of an electric energy distribution network is almost in every case a non-sinusoidal function. The network voltage degradation is mainly generated by the multitude of nonlinear loads (both industrial and domestic), which surge currents with high level of harmonic distortion [1, 2]. These currents flow throughout the entire installation and due to the finite (nonzero) value of the upstream network impedance, pollutes also the supply voltage waveform. This harmonic affected voltage will be applied to any other device connected at this grid-point of the installation, as Fig. 1 illustrates. Moreover, during a highly energy demanding process (starting a motor, energizing a transformer or electric arc welding machine) the voltage waveform additionally suffers from severe harmonic pollution. Thus, the power transformers are being affected by harmonics presence [2–5], but they also generate poor power quality during their connection to the network [3, 6]. The energizing current drawn by a non-loaded power transformer, known as inrush current (IC), is characterized by a highly amplitude and an even harmonic content of its waveform spectrum [7].

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As a result of its great importance in power systems, the inrush current problem has been studied in numerous papers [8–14], but only recently the harmonic content of the applied voltage was also considered [15–19]. Complexity of the problem lies in the dynamic of waveform spectrum and the mutual influences between the active network devices during the transient process [19]. This paper is going to investigate the main characteristics of the IC (peak value, first cycle harmonic spectrum and duration) for a single-phase power transformer (used in low-voltage installations) under a real (distorted) supply voltage. Additionally, a case study quantitatively illustrates the theoretical approach. Due to the low rated power range of the investigated energized transformers, it is assumed that the network voltage waveform is preserved during the transient process.

2. INRUSH CURRENT EVALUATION

The main purpose of the single-phase power transformers used in low-voltage installations is to provide electric energy (at different parameters) for a large variety of loads. Additionally, in certain applications, they may also perform the electrical isolation between the power supply and the electric appliances. During their operating time, transformers experience numerous commutations processes. Consequently, the predetermination of the IC features is required in order to properly consider its instantaneous effects both over the installation and the transformer (abnormal operation of the protective devices, overloading the nearby equipments and other power quality issues).
For a better illustration of the IC computation suggested procedure, our study focuses on a particular class of power transformers, commonly found in low-voltage applications. More specifically, UI magnetic core shape devices (with a rated power up to 25 kVA) having the geometrical data indicated in Appendix 1.

The inrush current time evolution \( i(t) \) immediately after the transformer switching could be derive from the instantaneous electric circuit equations along with Ampère's circuit law applied on the magnetic core device. Thus, taking the primary winding with resistance \( R_1 \) and turns number \( N_1 \), and the mean magnetic path length \( l_m \) of a cross section core area \( A_c \), the inrush current \( i(t) \) and magnetic induction \( B(t) \) time dependency can be expressed:

\[
\begin{align*}
\frac{d}{dt} B(t) &= u(t) + \frac{1}{N_1 A_c} \frac{d}{dt} \int H(t) dl_m, \\
N_1 i(t) &= H(t) / l_m, \quad H = f(B(t)),
\end{align*}
\]

where \( u(t) \) is the supplied non-sinusoidal voltage and \( H(t) \) is the core magnetic field strength.

In order to solve the above equation system, a representation of the magnetic core nonlinear characteristic in terms of \( H-B \) relationship is imposed. Various approximation functions can be chosen. Since the accuracy description of the core heavy saturation area is important for the IC generation phenomenon, the Brauer expression was here selected [20]:

\[
H = mB + nB \exp(pB^2),
\]

where \( m, n \) and \( p \) constants are accordingly determined for any type of core lamination steel by a simple fitting procedure (constraining the expression to pass through at least three of the electrical steel given data points).

Finally, the nonlinear differential algebraic equations system described by (1) is numerically solved using Rosenbrock method [21]. The magnetic core flux density before the energizing process \( B_{0} \) represents the necessary initial value parameter, which is always taken as the remnant magnetization. Thus, the worst case scenario is considered.

Many transformers’ protecting equipment devices discriminate the IC from different internal fault currents (e.g., short circuits) by investigating the transient current harmonic spectrum of the first cycle (during a period time of the fundamental voltage supply waveform \( T = 2\pi/\omega \)). This principle is also known as the second harmonic restraint procedure [7, 22, 23]. In that respect, a numerical harmonic analyze over the first cycle of IC waveform, which relies on the Fourier series [9, 11, 24, 25], is also accomplished. Hence, the spectrum of IC first cycle is accordingly evaluated [1–3] along with each harmonic percentage level \( (H_h) \) with respect to the fundamental and its total harmonic distortion \( (THD) \):
In order to quantitatively describe the level of supply voltage harmonic pollution, the total harmonic distortion \( THDU \) indicator was also adopted (derived from voltage signal Fourier series decomposition):

\[
THD_U = \frac{\sqrt{\sum_{k=2}^{n} U_k^2}}{U_1} \cdot 100, \quad u(t) = \sum_{k=1}^{n} U_k \sqrt{2} \sin(kt\omega),
\]

with \( k \) the harmonic order of effective value \( U_k \), and angular frequency \( k\omega \).

The IC evaluation described above can be easily implemented in any general purpose computation environment, once all the required data (for both transformer and applied voltage) are available. The applied distorted voltage is to be investigated before starting the energizing process. That can be achieved by using a high performance measuring and monitoring device, such a modern power quality analyzer, which is able to provide a complete harmonics signal profile up to the 51st harmonic. This valuable information may lead to accurate IC predetermination parameters. In the following section a case study is going to illustrate the flexibility of the suggested IC computation procedure. Supplementary, the harmonic voltage spectrum influence of the IC characteristics is revealed by performing specific quantitative parametric estimations.

3. NUMERICAL APPROACH – CASE STUDIES

A low-voltage single phase power transformer of 2 kVA rated power, whose complete data are indicated in Appendix 1, is to be energized at a certain grid point of an electric installation. At this specific network node numerous nonlinear loads are already on duty (adjustable speed drives, switching power supplies, discharged lamps and other) as depicted in Fig. 1. The supply voltage characteristics were investigated with a class A power quality analyzer [26].
Fig. 2 – The grid node voltage waveforms of the three-phase supply voltages.

Fig. 3 – The grid node voltage harmonic content of the three-phase supply voltages.

Hence, the captured images of voltages waveform and its harmonic content (with percentage contribution of each component related to the fundamental) are presented in Fig. 2 and Fig. 3, respectively. One can notice that the total harmonic distortion of the three voltages waveforms reaches $THD_U = 6.4\%$. The transformer core magnetic material is a non-oriented grain silicon steel M 27 and corresponds to a stacking factor of $k_{ST} = 0.94$. The magnetic material characterization is described by the magnetization curve as a finite number of measured pair of points.

Fig. 4 – Transformer core magnetization curve (marker) and the adopted Brauer functions (curve).

Fig. 5 – Transformer inrush current computed variation during a five period time.
Figure 4 represents this magnetization curve along with corresponding Brauer approximation of the $B$-$H$ relationship. Applying the IC procedure evaluation presented in the previous section, the transformer IC variation immediately after the transformer switching process is obtained. Figure 5 depicts IC time evolution during 100 ms. The amplitude reaches 513.78 A at 7.62 ms after the commutation occurred. One can notice that it decays very rapidly (only after a few cycles) and ultimately its steady state value, also called magnetization current, is practically around one percent from the transformer rated current (for this particular device only 0.11 A). The harmonic investigation performed over the first cycle of IC waveform, indicates a significant percentage level with respect to the fundamental of zero (DC) and second harmonic. Correspondingly, Fig 6 presents the spectrum histogram of IC first cycle.

Aiming to also estimate the harmonic voltage distortion level influence over the IC characteristics, various quantitative parametric evaluations have been considered. Thus, Fig. 7 represents the IC amplitude as a function of the voltage total harmonic distortion ($THD_U$).

The amplitude variation of zero ($H_0$) and second ($H_2$) harmonic component of the first cycle IC waveform, defined according to the last expression of (3), is depicted in Fig. 8. The total harmonic distortion ($THD_I$) of the IC first cycle alteration due to the voltage distortion is presented in Fig. 9.

![Fig. 6 – IC first cycle computed harmonic spectrum histogram.](image)

![Fig. 7 – Computed IC amplitude variation with the voltage total harmonic distortion.](image)
4. CONCLUSIONS

An analytical procedure that predicts the IC parameters of a single phase power transformer under a distorted voltage condition is presented. The evaluation method is based on the solution of the nonlinear differential algebraic equations system derived from the transformer’s circuit model and its magnetic core characterization. Since nowadays the voltage harmonic content can be accurately measured, the voltage distortion is accordingly modeled by Fourier series. The variations of the IC features corresponding to the level of voltage harmonic pollution ($THDU$) are investigated. Thus, the IC amplitude and its first cycle waveform zero harmonic component are increasing along with the voltage degradation, while IC distortion ($THDI$) and the second harmonic amplitude diminish. These parameters are particularly important because they generate the thermal and electrodynamical stresses experienced by transformer during the commutation process (IC amplitude) and also lead to the proper selection of the protection equipment or method (e.g. second harmonic attendance discrimination procedure between IC and short circuit). The level of voltage distortion which could be measured at different grid nodes of common low-voltage electric installations ($THDU < 20\%$), do not generate critical values for the transformer IC parameters. Nevertheless, the constant demand for energy efficiency, operating flexibility and high performance levels for the electric appliances require the large scale usage of power electronic equipment. Consequently, the voltage distortion is expected to constantly increase.
The proposed IC computing procedure could be further developed in terms of its preciseness. For instance, since the $THD_U$ is a non-unique quantitative parameter of voltage harmonic pollution (the same value can result from different voltage waveforms), other (qualitative) distortion indicators are to be found in order to better describe the non-sinusoidal influence over the IC features. Additionally, a better core magnetic material description (e.g. taking into account the laminations hysteresis loop instead of their magnetization curve) could also lead to a more accurate IC parameters prediction. In order to suppress the voltage distortion influence over the IC parameters, the classic harmonic mitigation techniques are also to be applied here, such as: increase the short circuit power, installing harmonic filters (active or passive), segregate the nonlinear loads, power equipment oversize.

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APPENDIX 1

The core geometry of the tested low-voltage transformer is presented in Fig. 10 and its main technical data are shown in Table 1.

![Fig. 10 – Magnetic core shape of the tested LV transformer and its geometrical dimensions.](image-url)
Table 1

The investigated single phase power transformer data

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$S_n = 2 \text{kVA}$</td>
<td>rated power</td>
</tr>
<tr>
<td>$U_{1n} = 230 \text{V}$</td>
<td>primary rated voltage</td>
</tr>
<tr>
<td>$U_{2n} = 24 \text{kV}$</td>
<td>secondary rated voltage</td>
</tr>
<tr>
<td>$N_1 = 262$</td>
<td>number of turns in primary</td>
</tr>
<tr>
<td>$N_2 = 27$</td>
<td>number of turns in secondary</td>
</tr>
<tr>
<td>$R_1 = 0.5555 \Omega$</td>
<td>primary loss resistance</td>
</tr>
<tr>
<td>$R_2 = 0.0066 \Omega$</td>
<td>secondary loss resistance</td>
</tr>
<tr>
<td>$B_n = 1.25 \text{T}$</td>
<td>operating magnetic core induction</td>
</tr>
<tr>
<td>$B_r = 1.01 \text{T}$</td>
<td>core remnant magnetization</td>
</tr>
<tr>
<td>$A_c = ab = 0.00317 \text{m}^2$</td>
<td>magnetic core area</td>
</tr>
<tr>
<td>$I_n = 12a = 0.528 \text{m}$</td>
<td>magnetic path mean length</td>
</tr>
<tr>
<td>$a = 0.044 \text{m}$</td>
<td>core main reference dimension</td>
</tr>
<tr>
<td>$b = 0.077 \text{m}$</td>
<td>core thickness</td>
</tr>
</tbody>
</table>

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REFERENCES


