MATHEMATICAL MODELS AND ELECTRICAL EQUIVALENT SCHEMES OF THE INDUCTION MOTOR

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The paper presents the mathematical models, the equivalent electrical schemes with the rotor winding referred to the stator one, the simplified mathematical models with their limits of application and physical interpretations concerning the performances obtained in the vector control of the induction motor. The general mathematical models can be simplified for slow transient processes, in this case the results obtained can be correlated with elements of the steady state, and the physical interpretations will be more adequate. It is to remark that the transformer e.m.f. can be neglected only in the synchronous reference system.

1. INTRODUCTION

Three-phase asynchronous machine mathematical models represent variants of the equations system characterizing the asynchronous machine operation in any state.

The voltage equations are established by basing on the analysis of the machine own and mutual inductances. Only the fundamental harmonic of the air gap magnetic field spatial distribution is taken into account; and at the beginning the magnetic circuit saturation is neglected. The complete mathematical model contains both the voltage equations and the moving equation.

By using the spatial phasors in the mathematical model of the three-phase asynchronous machine permits to obtain a simple model, physically rigorous interpretations and to identify new solutions for the electrical drives. The spatial phasors are correlated to the reference systems and are defined in the own reference system of stator or rotor. Finally, the mathematical model is written in a general reference system (for the steady state the synchronous reference system is considered).

General mathematical models for slow transient processes can be simplified, in this case the calculations volume is reduced and the results obtained can be have

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the same circuit elements (resistances and inductances) as the steady-state mathematical model, and the physical interpretations will be more adequate.

2. REFERENCE SYSTEMS AND MATHEMATICAL MODELS

In the asynchronous machine three reference systems are used:

– the general reference system noted K, with Ω_K as angular speed;
– the rotor fixed reference system noted by FR, with Ω as angular speed;
– the stator fixed reference system, noted as FS.

The speeds mentioned are considered in regard with the FS reference system (Fig. 1). If $\Omega_k = \Omega_1$ (synchronous speed), then the reference system K becomes the synchronous reference system noted by K_0. For the currents the positive conventional senses are to be considered. For $\lambda = 0$, the reference K corresponds to FS reference; for $\theta = 0$, the reference K corresponds to FR reference.

Asynchronous motor equations are written by means of the spatial phasors, which permits an easy passage from a reference system to another. Both the equations between the magnetic fluxes and the currents and the motion equation are added to the voltage equations. Then the complete equations system will be:

$$\begin{aligned}
&u_{sK} = R_s i_{sK} + \mathrm{j}\omega_K \psi_{sK} + \frac{\mathrm{d}\psi_{sK}}{\mathrm{d}t};\\
&\psi_{sK} = L_s i_{sK} + L_m i_{rK} = L_{s\sigma} i_{sK} + w_{es}\psi_{\mu K};\\
&-u_{rK} = R_r i_{rK} + \mathrm{j}(\omega_K - \omega)\psi_{rK} + \frac{\mathrm{d}\psi_{rK}}{\mathrm{d}t};\\
&\psi_{rK} = L_r i_{rK} + L_m i_{sK} = L_{r\sigma} i_{rK} + w_{er}\psi_{\mu K};\\
&\psi_{\mu K} = \frac{L_m}{w_{er}}\left(i_{sK} + i'_{rK}\right) = \frac{L_m}{w_{er}} i_{\mu K};\\
&i_{\mu K} = i_{sK} + i'_{rK};\, i'_{rK} = \frac{w_{er}}{w_{es}} i_{rK};\quad k = \frac{w_{er}}{w_{es}};\\
&J\frac{\mathrm{d}\Omega}{\mathrm{d}t} = M - M_2;\quad M = \frac{3}{2}P\left(\psi_{sd} i_{sq} - \psi_{sq} i_{sd}\right).
\end{aligned} \tag{1}$$

Here M_2 is the useful torque transmitted to the installation, moved by the motor. The torque M_2 is opposing to the movement that means it is a resistant torque and P is the number of pair poles*.

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* This notation is necessary, because later will be used p for the operational variable.

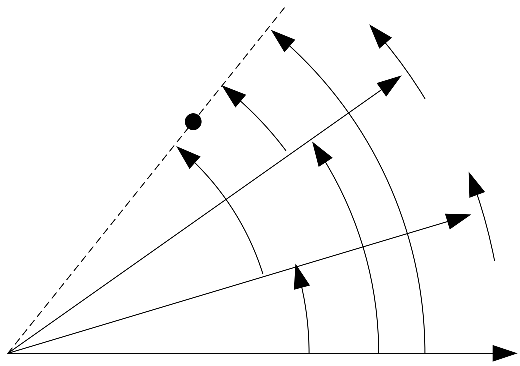

Fig. 1 – The reference systems.

The equations system (1) is the general mathematical model written in the reference system *K*. The equations system (1) being written in a general reference system *K*, it can be transcribed in any reference by simple particularizations. If $\Omega_K = \Omega_1$, the synchronous reference system can be obtained. The synchronous reference has the angular speed equal to the synchronous speed Ω_1 determined by the stator currents angular frequency ($\Omega_1 = \omega_1/P$), and in the equations system (1) Ω_K will be replaced by Ω_1. For $\Omega_K = 0$, the stator fixed reference system (FS) is obtained. It is to remark in this case the motion e.m.f. in the stator circuit is zero, because the stator winding is fixed in regard with the reference FS.

For $\Omega_K = \Omega$, the rotor fixed reference system (FR) is obtained. By replacing this magnitude in the system (1) the equations system in fixed rotor reference system (FR) is obtained; it is to remark in this case the motion e.m.f. in the rotor circuit is zero, because the rotor is fixed in regard with reference FR.

Using the factor *k*, the magnetic fluxes (spatial phasors) ψ_{sK} and ψ_{Rk} have been expressed as functions of the leakage magnetic flux in the winding considered and the main magnetic flux ψ_μ; consequently, the spatial phasors ψ_{sK} and ψ_{rK} have precise physical significances. The use of another referring factor as *k* does not lead to similar interpretations (leakage field and air-gap field).

The equations system (1) can be written in per units by choosing some basic magnitudes correlated with the rated magnitudes (*e.g.* $\sqrt{2}\,U_n$ for the voltages and $\sqrt{2}\,I_n$ for the currents, a.s.o.). After some calculations, the following equations are obtained:

$$
\begin{aligned}
&u_{sK}^{*} = r_s i_{sK}^{*} + \mathrm{j}\nu_K \psi_{sK}^{*} + \frac{\mathrm{d}\psi_{sK}^{*}}{\mathrm{d}\tau};\\
&\psi_{sK}^{*} = x_{s\sigma} i_{sK}^{*} + x_{\mu} i_{\mu K}^{*} = x_s i_{sK}^{*} + x_m i_{rK}^{'*};\\
&-u_{rK}^{'*} = r_r^{'} i_{rK}^{'*} + \mathrm{j}(\nu_K - \nu)\psi_{rK}^{'*} + \frac{\mathrm{d}\psi_{rK}^{'*}}{\mathrm{d}\tau};\\
&\psi_{rK}^{'*} = x_{r\sigma}^{'} i_{rK}^{'*} + x_{\mu} i_{\mu K}^{*} = x_r^{'} i_{rK}^{'*} + x_m i_{sK}^{*};\\
&i_{\mu K}^{*} = i_{sK}^{*} + i_{rK}^{'*};\quad H\frac{\mathrm{d}\nu}{\mathrm{d}\tau} = m - m_2;\\
&\nu = \frac{\omega}{\omega_b};\quad \nu_K = \frac{\omega_K}{\omega_b};\\
&m = \frac{M}{M_b} = \psi_{sd}^{*} i_{sq}^{*} - \psi_{sq}^{*} i_{sd}^{*};\\
&m_2 = \frac{M_2}{M_b};\quad H = \frac{J\omega_b^2}{pM_b}.
\end{aligned}
\tag{2}
$$

In the vector control theory, the spatial phasors ψ_{sK} and ψ_{rK} written in other forms present interest, for example the diagram with pure resistive rotor circuit. The mathematical model (2) written in per units, organized in such a way to obtain easily the equivalent diagrams will be envisaged. A factor a taking particularly the magnitude of $a = w_{es}/w_{er} = k$ is defined. In the expression of the equivalent diagrams the magnetic fluxes expressions are essential:

$$
\begin{aligned}
&\psi_{sK} = x_s i_{sK} + x_m i_{rK} = (x_s - a x_m) i_{sK} + a x_m\left(i_{sK} + \frac{i_{rK}}{a}\right) = x_{sa} i_{sK} + a x_m i_{aK};\\
&a\psi_{rK} = a x_r i_{rK} + a x_m i_{rK} = a^2\left(x_r - \frac{x_m}{a}\right)\frac{i_{sK}}{a} + a x_m\left(i_{sK} + \frac{i_{rK}}{a}\right) = x_{ra}^{'}\frac{i_{sK}}{a} + a x_m i_{aK}.
\end{aligned}
\tag{3}
$$

The following notations have been used:

$$
\begin{aligned}
&x_{sa} = x_s - a x_m;\quad x_{ra}^{'} = a^2\left(x_r - \frac{x_m}{a}\right);\\
&i_{aK} = i_{sK} + \frac{i_{rK}}{a};\\
&a = k \Rightarrow x_{sa} = x_{s\sigma};\quad x_{ra}^{'} = x_{r\sigma}^{'};\quad i_{aK} = i_{\mu K}.
\end{aligned}
\tag{4}
$$

For $a = \underline{k}$, the reactances have known physical significances. The reactances x_{sa} and x'_{ra} become stator leakage reactance x_{s0} and respectively rotor leakage reactance x'_{r0}; the reactance ax_m becomes the magnetizing reactance x_μ and the current i_{aK} becomes the magnetizing current i_μ.

For $a \neq k$, there are not any physical significances similar to those for $a = k$. As consequence the voltage equations in the reference K can be written as follows:

$$
\begin{aligned}
&u_{sK} = r_s i_{sK} + x_{sa}\frac{\mathrm{d}i_{sK}}{\mathrm{d}t} + ax_m\frac{\mathrm{d}i_{aK}}{\mathrm{d}t} - e_{sma},\\
&e_{sma} = -\mathrm{j}\nu_K \psi_{sK};\\
&-au_{rK} = -u'_{rK} = r'_r i'_{rK} + x'_{ra}\frac{\mathrm{d}i'_{rK}}{\mathrm{d}t} + ax_m\frac{\mathrm{d}i_{aK}}{\mathrm{d}t} - e'_{rma},\\
&e'_{rma} = -\mathrm{j}(\nu_K - \nu)\ \ \psi'_{rK},
\end{aligned} \tag{5}
$$

where e_{sm} is the motion e.m.f. induced in the stator winding, and e'_{rm} is the motion e.m.f. induced in the rotor winding; it is obvious, these e.m.f. are defined in the reference K.

In the general reference system, the asynchronous machine equations in operational form are written as:

$$
\begin{aligned}
&u_{sK}(p) = R_s i_{sK}(p) + pL_s^* i_{sK}(p) + pL_{\mu a} i_{\mu a}(p) + \mathrm{j}\omega_K L_{\mu a} i_{\mu a}(p) - p\psi_{sK}(0) -\\
&-a\ \ u_{rK}(p) = -u'_{rK}(p) = R'_r i'_{rK}(p) + pL_{\mu a} i_{\mu a}(p) + \mathrm{j}(\omega_K - \omega) L_{\mu a} i_{\mu a}(p) - a\ \ p\psi_{rK}(0),\\
&\psi_{sK}(p) = L_s^* i_{sK}(p) + L_{\mu a} i_{\mu a}(p),\\
&a\psi_{rK} = \psi'_{rK}(p) = L_{\mu a} i_{\mu a}(p),\ a = \frac{L_m}{L_r},\\
&e_{sK}(0) = -p\psi_{sK}(0),\ e'_{rK}(0) = a\ \ p\psi_{rK}(0).
\end{aligned} \tag{6}
$$

3. **THE EQUIVALENT SCHEMES**

On the basis of the equations (5) the equivalent diagram in the Fig. 2 corresponding to general reference K is built up; the diagram is similar in the synchronous reference K_0. In this reference frame the motion e.m.f. induced in the stator and that induced in the rotor are different from zero, that means $e_{sm} \neq 0$ and $e_{rm} \neq 0$. As known, for an observer in the reference K, both stator winding and rotor winding are moving.

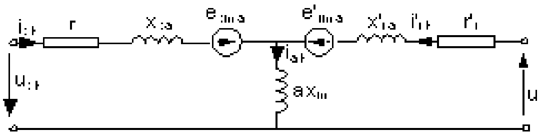

Fig. 2 – Equivalent diagram of a three-phased induction machine in reference *K*.

The general factor a allows to modify the equivalent diagram structure. For $a = k$ equivalent diagram with classical parameters is obtained: the leakage reactance and the magnetizing reactance. For the case $a = x_m/x_r$ the equivalent diagram with resistive rotor circuit is obtained and the following relationships result:

$$\begin{aligned} & a = \frac{x_m}{x_r} = \frac{L_m}{L_r} \Rightarrow x'_{ra} = 0,\ \psi'_{rK} = a \quad x_m \quad i_{aK}, \\ & x_{sa} = x_s^{\bullet} = x_{s\sigma} + \frac{1}{1/x'_{r\sigma} + 1/x_{\mu}}, \\ & e'_{rma} = -\mathrm{j}(\nu_K - \nu) \quad a \quad x_m \quad i_{aK}. \end{aligned} \tag{7}$$

The rotor circuit reactance x_{ra} is null, and the stator circuit reactance x_{sa} is equal to the stator transient reactance x^*_s (the reactance is concentrated in the stator circuit). The equivalent diagram is shown in Fig. 3. This equivalent diagram has real importance in the vector control because the stator current has its active component equal to the rotor current, and the reactive component equal to the magnetizing current.

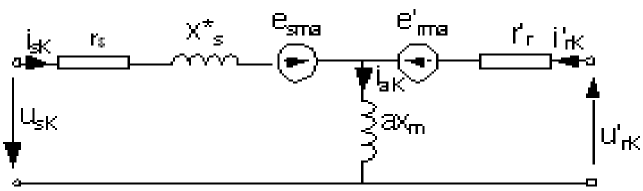

Fig. 3 – Equivalent diagram with resistive rotor circuit.

By means the stator current components and on the basis of the relationships (8) the electromagnetic torque (the active component) and the magnetic flux (reactive component) are controlled:

$$i_{sK} = i'_{rK} + \mathrm{j} i_{aK}\ ; \quad m = k_m i'_{rK}\ ; \quad \psi_{aK} = k_{\psi} i_{aK}\,. \tag{8}$$

On the basis of the equations (6) the general equivalent diagram with the resistive circuit rotor of Fig. 4 can be obtained and particularized for each of the reference-used systems.

On this scheme were noted:

ω_k – the angular speed, in electrical degrees, for the reference system;

$s = (\omega_k - \omega) / \omega_k$ the slip in regard with the reference angular speed;

$L_{\mu a} = aL_m$ – the magnetizing inductance;

L_s^* – the stator transient inductance.

In this scheme were neglected the iron losses.

For $\omega_k = \omega_1$ the synchronous reference equivalent scheme is obtained (ω_1 being angular speed of the turning magnetic field). For $\omega_k = 0$ the equivalent scheme in stator reference system is obtained. For $\omega_k = \omega$ ($s = 0$), the equivalent scheme in rotor reference system is obtained. For $p = 0$, the equivalent circuit corresponding to steady state is obtained.

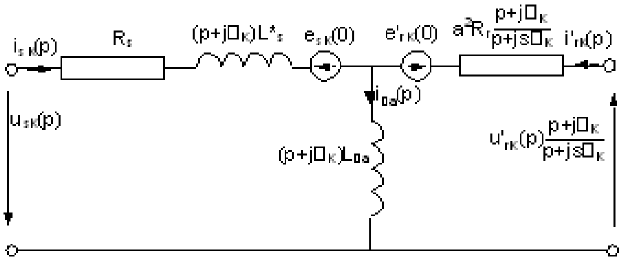

Fig. 4 – General diagram with resistive rotor circuit.

4. SIMPLIFIED MATHEMATICAL MODELS OF THE INDUCTION MACHINE

It is possible to obtain some simplified mathematical models having an advantageous application when the speed is not variable in large limits. Simplifications are important in the cases where the speed can be considered approximately constant. In the case of the start, at the first moments, the transformer e.m.f cannot be neglected; consequently the simplified models can not be applied.

A first simplified model consists to neglect the transitive state in the stator windings, that means one neglects the transformer e.m.f.:

$$\frac{\mathrm{d}\Psi_{sd}}{\mathrm{d}t} = 0,\ \frac{\mathrm{d}\Psi_{sq}}{\mathrm{d}t} = 0 \Rightarrow$$
$$u_{sd} = R_s i_{sd} - \frac{\omega_k}{\omega_b}\varepsilon_{sq},$$
$$u_{sq} = R_s i_{sq} + \frac{\omega_k}{\omega_b}\varepsilon_{sd}, \tag{9}$$
$$-u'_{rd} = R'_r i'_{rd} - \frac{\omega_k - \omega}{\omega_b}\varepsilon'_{rq} + \frac{p}{\omega_b}\varepsilon'_{rd},$$
$$-u'_{rq} = R'_r i'_{rq} + \frac{\omega_k - \omega}{\omega_b}\varepsilon'_{rd} + \frac{p}{\omega_b}\varepsilon'_{rq}.$$

In these equations the magnetic fluxes are represented by magnitudes having dimension of the e.m.f., named conventional e.m.f.. These have the form:

$$
\begin{aligned}
&\varepsilon_{sd} = \omega_b \Psi_{sd} = X_{s\sigma} i_{sd} + X_{\mu} i_{\mu d} = X_s i_{sd} + X_{\mu} i'_{rd},\\
&\varepsilon_{sq} = \omega_b \Psi_{sq} = X_{s\sigma} i_{sq} + X_{\mu} i_{\mu q} = X_s i_{sq} + X_{\mu} i'_{rq},\\
&\varepsilon'_{rd} = \omega_b \Psi'_{rd} = X'_{r\sigma} i'_{rd} + X_{\mu} i_{\mu d} = X'_r i'_{rd} + X_{\mu} i_{sd},\\
&\varepsilon'_{rq} = \omega_b \Psi'_{rq} = X'_{r\sigma} i'_{rq} + X_{\mu} i_{\mu q} = X'_r i'_{rq} + X_{\mu} i_{sq},\\
&i_{\mu d} = i_{sd} + i'_{rd},\ i_{\mu q} = i_{sq} + i'_{rq}.
\end{aligned}
\tag{10}
$$

Using the relations (10), the matrix form of system (9) is obtained:

$$
\begin{bmatrix} u_{sd} \\ u_{sq} \\ -u'_{rd} \\ -u'_{rq} \end{bmatrix} =
\begin{bmatrix}
R_s & -\dfrac{\omega_K}{\omega_b} X_s & 0 & -\dfrac{\omega_K}{\omega_b} X_{\mu} \\
\dfrac{\omega_K}{\omega_b} X_s & R_s & \dfrac{\omega_K}{\omega_b} X_{\mu} & 0 \\
\dfrac{p}{\omega_b} X_{\mu} & -\dfrac{\omega_K - \omega}{\omega_b} X_{\mu} & R'_r + \dfrac{p}{\omega_b} X'_r & -\dfrac{\omega_K - \omega}{\omega_b} X_{\mu} \\
\dfrac{\omega_K - \omega}{\omega_b} X_{\mu} & \dfrac{p}{\omega_b} X_{\mu} & \dfrac{\omega_K - \omega}{\omega_b} X'_r & R'_r + \dfrac{p}{\omega_b} X'_r
\end{bmatrix} \cdot
\begin{bmatrix} i_{sd} \\ i_{sq} \\ i'_{rd} \\ i'_{rq} \end{bmatrix}.
\tag{11}
$$

To the matrix equation the move equation is added. If in the relationships (11) the currents function of conventional e.m.f. is replaced, a new matrix equation will be obtained:

$$
\begin{bmatrix} u_{sd} \\ u_{sq} \\ -u'_{rd} \\ -u'_{rq} \end{bmatrix} =
\begin{bmatrix}
R_s & -\dfrac{\omega_K}{\omega_b} X_s & 0 & -\dfrac{\omega_K}{\omega_b} X_{\mu} \\
\dfrac{\omega_K}{\omega_b} X_s & R_s & \dfrac{\omega_K}{\omega_b} X_{\mu} & 0 \\
\dfrac{p}{\omega_b} X_{\mu} & -\dfrac{\omega_K - \omega}{\omega_b} X_{\mu} & R'_r + \dfrac{p}{\omega_b} X'_r & -\dfrac{\omega_K - \omega}{\omega_b} X_{\mu} \\
\dfrac{\omega_K - \omega}{\omega_b} X_{\mu} & \dfrac{p}{\omega_b} X_{\mu} & \dfrac{\omega_K - \omega}{\omega_b} X'_r & R'_r + \dfrac{p}{\omega_b} X'_r
\end{bmatrix} \cdot
\begin{bmatrix} i_{sd} \\ i_{sq} \\ i'_{rd} \\ i'_{rq} \end{bmatrix}.
\tag{12}
$$

It is to remark the derivatives of the stator magnetic flux ψ_{sd} and ψ_{sq} do not appear and the rotor magnetic fluxes ψ'_{rd} and ψ'_{rq} can be envisaged as state variables.

The equivalent diagram for this model (Fig. 4) is obtained if $p = 0$, in the expression $p + j\omega_K$, is considered.

A second simplified mathematical model consists in neglecting the stator as well the rotor transformer e.m.f.:

$$\frac{\mathrm{d}\Psi_{sK}}{\mathrm{d}t} = 0, \ \frac{\mathrm{d}\Psi'_{rK}}{\mathrm{d}t} = 0. \tag{13}$$

Such a mathematical model can be used for slow transient processes presenting interest when the mechanical time constant is greater than the electric circuits time constants. In the synchronous reference frame K_0 and the conditions (13) the mathematical model becomes:

$$\begin{aligned}
&u_{sK} = R_s i_{sK} + \mathrm{j}\,\omega_1 \Psi_{sK}; \quad \omega_K = \omega_1;\\
&\Psi_{sK} = L_s i_{sK} + L_m i_{rK} = L_{s\sigma} i_{sK} + w_{es}\Psi_{\mu K} = L_{s\sigma} i_{sK} + L_\mu i_{\mu K};\\
&-u'_{rK} = R'_r i'_{rK} + \mathrm{j}(\omega_1 - \omega)\Psi'_{rK};\\
&\Psi'_{rK} = L'_r i'_{rK} + \frac{w_{es}}{w_{er}} L_m i_{sK} = L'_{r\sigma} i'_{rK} + L_\mu i_{\mu K} = L'_{r\sigma} i'_{rK} + L_\mu i_{\mu K};\\
&w_{es}\Psi_{\mu K} = \frac{w_{es}}{w_{er}} L_m i_{\mu K} = L_\mu i_{\mu K};\\
&\frac{J}{p}\cdot\frac{\mathrm{d}\omega}{\mathrm{d}t} = M - M_2;\\
&M = \frac{3}{2} p\left(\Psi_{sd} i_{sq} - \Psi_{sq} i_{sd}\right).
\end{aligned} \tag{14}$$

The voltage equations system (14) coincides formally with the equation system in steady state. To remark that the transformer e.m.f. may be neglected only in the synchronous reference frame K_0, where the induced motion e.m.f. in the stator winding and in the rotor are different from zero and are leading in comparison with the transformer e.m.f.-s. If in the stator fixed reference FS or in the rotor fixed reference FR the transformer e.m.f. are neglected, we will obtain:

$$\begin{aligned}
&FS: \quad \frac{\mathrm{d}\,\Psi_s}{\mathrm{d}\,t} = 0 \quad \Rightarrow \quad u_s = R_s i_s;\\
&FR: \quad \frac{\mathrm{d}\,\Psi'_r}{\mathrm{d}\,t} = 0 \quad \Rightarrow \quad -u'_r = R'_r i'_r.
\end{aligned} \tag{15}$$

The relationships (15) are absurd, that means in the respective reference frame the transformer e.m.f. can not be neglected. It is possible to neglect the transformer e.m.f. only in the reference frames where the motion e.m.f. have a significant magnitude compared with transformer e.m.f.-s. In the synchronous reference frame K_0 the stator winding has as angular speed Ω_1 and the rotor winding has as angular speed $s\Omega_1$, $s \neq 0$ and consequently relationships (13) are possible.

On the basis of the relationships (14) it results the equivalent diagram, which is similar to the diagram of steady state, in Fig. 4, $p = 0$ all over. The fact explains the extension of the steady state relationships towards certain transient states when the hypothesis (13) can be valuable.

4. MATHEMATICAL MODELS APPLIED IN THE AIR-GAP FLUX ORIENTED CONTROL

In the air-gap flux oriented control electrical drive it is required the motor to operate at ψ_μ constant air-gap flux and a permanent co-linear with synchronous reference axis envisaged as real axis in the complex plane. The mathematical form will be the following:

$$\begin{aligned} \Psi_{\mu\, d} &= L_\mu i_{\mu\, d} = \Psi_\mu = \text{ct.}, \\ \Psi_{\mu\, q} &= L_\mu i_{\mu\, q} = 0 \Rightarrow i_{\mu q} = 0. \end{aligned} \tag{16}$$

The relationships (16) have an important physical significance in the explanation of the results obtained by the vector control. In the vector control, the windings in the axis d and those in the axis q are decoupled, that means the magnitudes on the axis d can be controlled without influencing the magnitudes on the axis q, and reciprocally. In the synchronous reference K_0 (Fig. 5) the three-phase stator winding is equivalent to two orthogonal windings S_d with magnetic axis d and S_q with magnetic axis q; the winding S_d is crossed by the current i_{sd} and the winding S_q is flown by the current i_{sq}.

The three-phase rotor winding is equivalent to two orthogonal windings R_d with the magnetic axis d and R_q with the magnetic axis q. The winding R_d is flown by the current i_{rd} and the winding R_q is flown by the current i_{rq}. The resulting ampere-turns $\Theta_{\mu\mathrm{d}}$ of the windings S_d and R_d engender the magnetic flux ψ_{sd} oriented along the axis d, which the resulting ampere-turns $\Theta_{\mu\mathrm{q}}$ of the windings S_q and R_q is null. These elements are resulted from the relationships:

$$\begin{aligned} & S_d \cup R_d, \Rightarrow\ \psi_{\mu d} = L_\mu i_{\mu d} = \frac{w_s}{R_m}\left(w_s i_{sd} + w_r i_{rd}\right) = \frac{w_s}{R_m}\Theta_{\mu d}, \\ & S_q \cup R_q, \Rightarrow\ \psi_{sq} = L_{s\sigma} i_{sq} + L_\mu i_{\mu q}, \\ & i_{\mu q} = 0 \Leftrightarrow \Theta_{\mu q} = 0 \Rightarrow\ \psi_{sq} = L_{s\sigma} i_{sq}, \\ & i_\mu = i_{\mu d} + j i_{\mu q} = i_{\mu d}. \end{aligned} \tag{17}$$

Reluctance R_{m} corresponding to the inductance L_μ is considered.

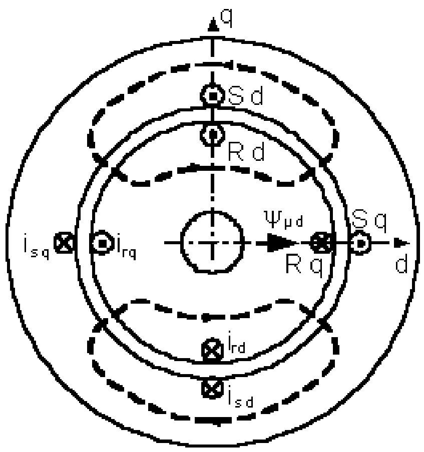

Fig. 5 – Orthogonal equivalent windings and the field lines for $\Psi_{\mu q}$=0.

The magnetizing current i_μ has only the component $i_{\mu d}$ different from zero; it engenders the magnetizing flux $\psi_{\mu d}$ in the axis *d*. The winding S_q has a behaviour similar to a compensating winding, cancelling the reaction magnetic field of the winding R_q as it happens in DC machine. It remarks the rotor winding R_q flown by the maximum current i_{rq} is in the zone where the magnetic field is maximum and it results a tangential rotor force to which corresponds a maximum electromagnetic torque. The explanations are identical for the other types of the flux-oriented control.

5. **CONCLUSIONS**

In this paper are analysed the mathematical models of the asynchronous machine and the equivalent diagram structures depending on the mode of refering the rotor winding to the statoe one.

It is to remark that the diagram structure of the induction motor is detrmined by the refering factor *a* of the rotor winding to the stator winding and of the reference system considered. In steady-state the structure of the equivalent diagrams is depending only on the factor *a* and not on the reference system.

The paper presents the simplified mathematical models in their applying limits. It is to remark that the transformer e.m.f. can be neglected only in the synchronous reference system. In this reference system, the greatest motion e.m.f.-s are bigger than the greatest transformer e.m.f.-s. To neglect the transformer e.m.f.-s is important because the time derivates are eliminated from the voltage equations, these equations becoming algebraic ones.

It is possible to find reference systems in which the transformer e.m.f.-s are prevalent and the motion e.m.f.-s can be neglected, but in this case important simplified mathematical models can not be obtained.

One of the advantages of the simplified mathematical model (the stator and rotor transformer e.m.f. considered zero) consists in the decoupling of the voltage equations system from the moving equation; in this case the voltage equations can be analysed separately from the moving equation.

The position of the equivalent windings along the longitudinal axis d and the transversal axis q in the synchronous reference K_0 gives physical explanations concerning the performances obtained in the vector control of the induction motor.

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