ANALYTICAL FLUX LINKAGE MODEL OF SWITCHED RELUCTANCE MOTOR

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Key words: Switched reluctance motor, Analytical model.

A simple flux linkage analytical model of the switched reluctance motor (SRM) that fully considers the saturation effect, is proposed in the paper. The model is obtained by using three flux linkage-current characteristics calculated via a two dimensions finite element method (2D-FEM). The model is compared with another one based on a Fourier series approximation of the flux linkage-current characteristics that employs the same three 2D-FEM calculated characteristics. The proposed model accuracy is evinced by comparing the analytical calculated values with the 2D-FEM computed ones.

1. INTRODUCTION

Switched reluctance motor (SRM) is a viable alternative to conventional motors, like induction or synchronous, due to its simple and robust construction, wide speed range capability and reduced cost [1, 2]. The SRM drive performances strongly depend on its design features and control, therefore the motor's mathematical model is very important. A key factor for all developed SRM models is the phase flux linkage calculation, which poses significant challenge, since both the stator and rotor have salient poles and the iron core saturation has a significant influence on the motor's operation. The SRM's phase flux linkage calculation is done analytically or via a numerical method, usually finite element method (FEM).

Many valuable works were published in the last years in this domain, as [3, 4] which introduce analytical models to calculate the flux linkage, [5, 6] which develop models based on magnetic equivalent circuits, or [7, 8] where the analytical model is created using FEM analysis results.

In [9–11] SRM's analytical models are developed based on 2D-FEM calculations, while in [12, 13] specific nonlinear models are introduced. In [14, 15]

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the 2D-FEM analysis is employed to obtain analytical models of SRM and respectively linear transverse flux reluctance motor (LTFRM).

In this paper a model proposed in [14, 15] is brought to a higher level of generality in the case of SRM. This model is compared with one derived from the proposals made in [8, 9] where the SRM's phase flux linkage is approximated by Fourier series with limited number of terms.

The basics of the newly proposed SRM model, containing a saturation function and a referred inductance function are presented in Section 2 while in Section 3 the model based on a phase flux linkage approximation using a limited number of Fourier series terms is introduced. Section 4 is dedicated to the SRM's electromagnetic torque calculation for both models and Section 5 contains the calculated values for a sample SRM. The conclusions are presented in Section 6.

2. **SATURATED SRM MODEL BASICS**

The SRM stator pole flux linkage depends on the rotor position and core saturation. The pole flux linkage has a maximum value in aligned position when the stator pole axis coincides to a rotor pole axis, and a minimum value in unaligned position when the stator pole axis coincides to a rotor slot axis. Typical SRM pole flux linkage versus phase current characteristics are shown in Fig. 1, where the actual and the ideal (unsaturated) flux linkage characteristics are given.

In Fig. 1 the following notations are made: λ_{0al} – aligned unsaturated flux linkage ($\lambda_{0al} = L_{0al} \cdot i$), λ_{al} – aligned saturated flux linkage, λ_{0un} – unaligned unsaturated flux linkage ($\lambda_{0un} = L_{0un} \cdot i$), λ_{un} – unaligned saturated flux linkage, L_{0al} – aligned unsaturated phase main inductance, L_{0un} – unaligned unsaturated phase main inductance.

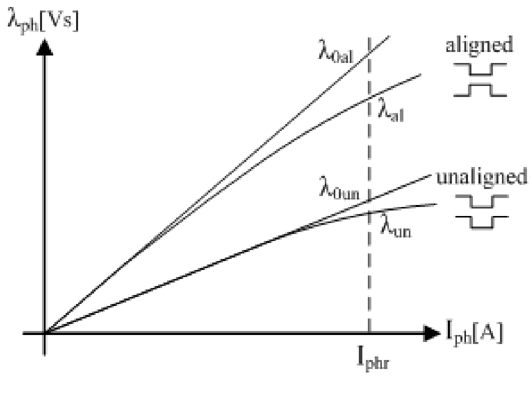

Fig. 1 – Aligned and unaligned flux linkage versus phase current.

The unsaturated phase flux linkage in aligned position λ_{0al} and in an arbitrary rotor position λ_0 are:

$$\lambda_{0\mathrm{al}} = L_{0\mathrm{al}} \cdot i, \quad \lambda_0 = L_0 \cdot i, \tag{1}$$

where L_0 is the unsaturated value of the phase inductance in an arbitrary rotor position and i the phase current.

The saturated phase flux linkage in the same positions are:

$$\lambda_{\mathrm{al}} = \lambda_{0\mathrm{al}} / k_{sal}, \quad \lambda = \lambda_0 / k_s, \tag{2}$$

k_{sal}, k_s being the saturation factor function of phase current in the aligned, respectively arbitrary rotor position.

Finally the flux linkage analytical expression is:

$$\lambda = \lambda_{al} \cdot l_{0r} / k_{sr}. \tag{3}$$

The refered inductance l_{0r} and saturation k_{sr} functions:

$$l_{0r} = L_0 / L_{0al}, \ \ k_{sr} = k_s / k_{sal} \tag{4}$$

are estimated in the paper by simple analytic functions, each containing a cosinusoidal term in θ, the electrical angle between two teeth on stator and rotor:

$$l_{0r}(\theta) = a_1 + b_1 \cos\theta, \tag{5}$$

$$k_{sr}(i,\theta) = a_s(i) + b_s(i)\cos\theta. \tag{6}$$

Three values of unsaturated phase inductance are necessary to obtain the referred inductance function $l_{0r}(\theta)$, aligned L_{0al}, averaged L_{0av} and unaligned L_{0un}. The averaged position of the rotor is situated at midway between aligned and unaligned position. Three points of $l_{0r}(\theta)$ characteristics are known then:

$$l_{0un} = L_{0un} / L_{0al}, l_{0av} = L_{0av} / L_{0al}, l_{0al} = 1. \tag{7}$$

The $l_{0r}(\theta)$ characteristic coefficients a_l and b_l can be calculated *via* a curve fitting procedure.

The saturation function k_{sr} is calculated through a similar procedure.

First, a saturation function depending on the rotor position is calculated for each considered flux linkage characteristic, at constant current. If i_1 is a value of the phase current, then the corresponding saturation function k_{sr1} comes as:

$$k_{sr1}(\theta) = k_{s1} / k_{sal1} = a_{s1} + b_{s1}\cos\theta. \tag{8}$$

The coefficients a_{s1}, b_{s1} result through a curve fitting procedure by considering three points of the saturation function k_{sr1},

$$k_{sr1un} = k_{s1un} / k_{s1al}; \quad k_{sr1av} = k_{s1av} / k_{s1al}; \quad k_{sr1al} = 1; \tag{9}$$

A set of saturation functions for different phase currents, i_1, i_2, …, are calculated:

$$\begin{aligned} k_{sr1} &= a_{s1} + b_{s1}\cos\theta; \\ k_{sr2} &= a_{s2} + b_{s2}\cos\theta; \\ &\ldots\ldots. \end{aligned} \tag{10}$$

Finally the saturation function $k_{sr}(i,\theta)$ (6) results, its coefficients $a_s(i)$ and $b_s(i)$, polynomial estimations, being obtained from already existing values a_{s1}, a_{s1}…and b_{s1}, b_{s2}…respectively, by using a curve fitting procedure.

3. SRM MODEL BASED ON FOURIER SERIES

In many references, for instance [8, 9], the SRM's phase flux linkage is approximated by a Fourier series with limited number of terms.

The most usual approximation of the SRM's phase flux linkage is:

$$\lambda(i,\theta) = \lambda_0 + \lambda_1\cos(\theta) + \lambda_2\cos(2\theta). \tag{11}$$

The coefficients λ_0, λ_1 and λ_2 are derived as functions of the aligned λ_{al}, unaligned λ_{un} and averaged λ_{av} flux linkage-current characteristics.

$$\lambda_0 = 0.5\left[0.5\left(\lambda_{al} + \lambda_{un}\right) + \lambda_{av}\right], \tag{12}$$

$$\lambda_1 = 0.5\left(\lambda_{al} - \lambda_{un}\right), \tag{13}$$

$$\lambda_2 = 0.5\left[0.5\left(\lambda_{al} + \lambda_{un}\right) - \lambda_{av}\right]. \tag{14}$$

The nonlinear flux linkage-current characteristics should be estimated via a curve fitting procedure. A ratio of polynomials was employed [11], for aligned and averaged flux linkage λ_{al}, λ_{av}, its general form being:

$$\lambda = i / \left(ai^2 + bi + c\right). \tag{15}$$

Due to its simplicity and accuracy, this estimation was preferred to other possible estimations, like the one with arc tangential function, used in [8].

$$\lambda = \tan^{-1}(mi)/n, \tag{16}$$

where m and n should be obtained, as a, b and c *via* a curve fitting procedure.

The 2D-FEM analysis is preferred since the construction of the 2D structure is simpler and the results, except the leakage inductance of the end winding, which are not considered in 2D, are quite the same as the ones obtained via 3D-FEM analysis, mainly for the magnetizing flux, which produces the torque.

4. ELECTROMAGNETIC TORQUE

The SRM's electromagnetic torque is:

$$T = \int_0^i \frac{\partial \lambda(i,\theta)}{\partial \alpha} \mathrm{d}i, \;\; \theta = Q_R \cdot \alpha, \tag{17}$$

Q_R being the rotor number of poles and α is the rotor angular displacement.

In the case of the saturated SRM model the electromagnetic torque final equation is:

$$T = Q_R \sin\theta \int_0^i \frac{a_l b_s(i) - b_l a_s(i)}{\left[a_s(i) + b_s(i)\cos\theta\right]^2} \frac{i \cdot \mathrm{d}i}{a_{al} i^2 + b_{al} i + c_{al}}, \tag{18}$$

a_{al}, b_{al} and c_{al} being the coefficients of the polynomial estimation (15) applied for aligned flux linkage-current characteristic.

An analytic integration of (17) is possible, but a complicated sum of functions results. Therefore, an approximation based on a particular case of the Newton-Cotes method [16, 17] is recomended.

In the case of SRM's model based on the approximation of the flux linkage-current characteristics by a Fourier series with limited number of terms, the electromagnetic torque results:

$$T = -Q_R \sin\theta \int_0^i \lambda_1 \, \mathrm{d}i - 2Q_R \sin(2\theta) \int_0^i \lambda_2 \, \mathrm{d}i. \tag{19}$$

All resulting integrals can be solved analytically.

5. CALCULATED RESULTS

A sample 8/6 poles SRM with four stator phases was considered. The main data and dimension ratios for the sample motor are given in Table 1.

The signification of the notations from Table 1 are: t_{Sp} –stator pole pitch, g – air-gap length in aligned position, w_{Ss}, w_{Rs} – stator and rotor slot opening,

w_{Sp}, w_{Rp} – stator and rotor pole width, l_{st} – axial stack length, N_c – number of turns per pole coil, I_{phr} – rated phase current.

Table 1

Sample SRM's main data

	t_{Sp}/g	$w_{Ss}/2g$	w_{Ss}/w_{Rs}	w_{Sp}/w_{Rp}	g	I_{phr}	l_{st}	N_c
	-	-	-	-	mm	A	mm	-
S1	86.4	25.9	0.676	0.675	0.5	10	66	93

The curve fitting approximations of the aligned, averaged and unaligned flux linkage characteristics and the estimation of the referred inductance function are:

$$\lambda_{al} = i/\left(0.3386 \cdot i^2 - 2.6263 \cdot i + 45.55\right), \tag{20}$$

$$\lambda_{av} = i/\left(0.4198 \cdot i^2 - 3.53 \cdot i + 79.34\right), \tag{21}$$

$$\lambda_{un} = L_{0un} i\,; \quad L_{0un} = 2.953\,\text{mH} \tag{22}$$

$$l_{0r}(\theta) = 0.565 + 0.441\cos\theta\,. \tag{23}$$

The coefficients of the saturation function are:

$$a_s(i) = -\,0.0006387\, i^2 + 0.0437\, i - 0.2681, \tag{24}$$

$$b_s(i) = -\,0.001387\, i^2 + 0.003325\, i + 1.01. \tag{25}$$

In Fig. 2, a comparison between the flux linkage versus phase current characteristics, calculated via 2D-FEM respectively by using the saturated analytical model is given in the case of sample SRM. The same comparison in the case of the SRM model based on the approximation of the flux linkage-current characteristics by number of terms is given in Fig. 3.

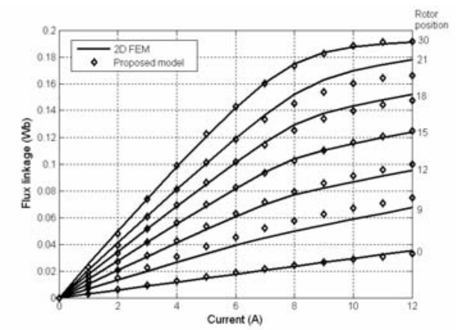

Fig. 2 – Phase flux linkage versus phase current at different rotor positions, proposed model.

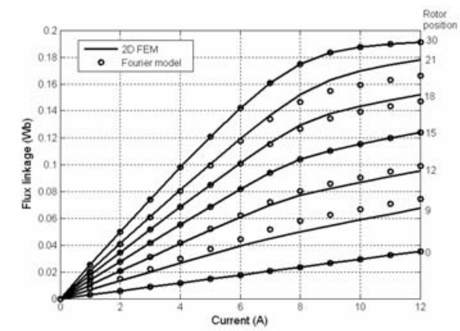

Fig. 3 – Phase flux linkage versus phase current at different rotor positions, Fourier model.

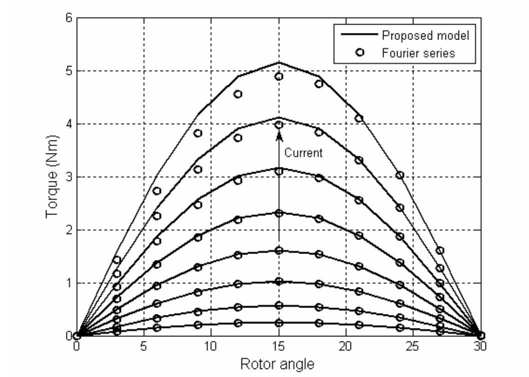

Fig. 4 – Proposed model vs. Fourier, $i = (2\ldots9)$ A.

The torque function of rotor position characteristics, at different currents, calculated via proposed model and respectively by using Fourier series with limited number of terms are given in Fig. 4. As expected, both models lead to almost sinusoidal variation fo the torque function of rotor position since both models contain sinusoidal functions slightly affected by the saturation effect.

6. **CONCLUSIONS**

A simple analytical model of SRM, developed by using 2D-FEM analysis results, is presented in the paper. The model contains three analytical functions for estimating the phase flux linkage versus current characteristics:

i) A function which approximates the phase flux linkage versus current characteristic calculated via 2D-FEM at aligned rotor position.

ii) A function for the variation of the unsaturated inductance ratio versus rotor position.

iii) A saturation factor function which depends on both phase current and rotor position and considers the influence of the variable saturation on the phase flux linkage.

The unsaturated inductance ratio function and the saturation factor function are obtained based on three phase flux linkage versus current characteristics, calculated via 2D-FEM for the aligned, averaged and unaligned rotor positions.

Another simple analytical model, based on a Fourier series approximation with limited number of terms of the flux linkage-current characteristics, is presented in the paper and the results obtained via this model are compared with the 2D-FEM computed ones, for a sample 8/6 SRM, as for proposed model too. The comparison shows that the accuracy of the proposed model is very good.

The final conclusion must state that the proposed analytical model is simple, and define clearly the influence of the rotor position and phase current on the SRM behaviour.

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