VARIABILITY MODELS FOR TRANSMISSION LINES

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The on-going reduction of feature size and the increase of variability are the main characteristics of today's integrated circuits. Certain variability effects previously considered negligible, now should be taken into account. Handling variability during the modelling procedure is an important topic and requires a great amount of research. The key solution of this problem is to build a parametric model of the device. This paper proposes new and efficient parametric models for interconnects modelled as transmission lines. The novelty consists of the parameterization of the longitudinal line parameters with respect to geometric transversal dimensions, subject to large or small variations. The models are based on the computation of first order sensitivities. The paper is focused on the simultaneously variation of multiple parameters.

1. INTRODUCTION

Manufacturing variability in the fabrication process of the ICs is gaining more attention as technology dimensions become smaller and the operation frequencies continue to go up. Such variations, which are hard to predict and to control, may have an important effect on the functionality of the design or on the accuracy of the resulting device. The parameter variability can no longer be disregarded during modeling, simulation and verification of the device. Much research is focused on interconnects as their performances impact has become important due to the fact that million closely spaced interconnections in one or more levels connect various components on the integrated circuit [1]. Parasitic capacitances, resistances and inductances of the interconnects have become major factors in the evolution of very high speed IC technology. The subject of this paper is how parametric models for interconnects can be extracted in a fast and robust manner. New numerical extracted models based on first order sensitivities for per unit length parameters with respect to a set of geometrical dimensions are considered. This represents one of the issues carried out with the European research project FP6/IST/Chameleon [2].

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The paper is organized as follows: first we will discuss the basic approach for extracting the line parameters. Then we will address the parametric models designed, followed by a discussion of the results obtained. Conclusions are drawn at the end.

2. NEW APPROACH ON PARAMETRIC MODELS FOR TRANSMISSION LINES

The downscaling of the dimensions to the nanometre domain has negative effects on the lithographic process and on the relative geometric deviations. The dimensions variability and the imperfections of the geometric forms are due to the fact that the realizations techniques are on the edge of maintaining control on the result. For realizing a robust product, the RFIC circuit designers need parametric models that enable the analysis of the variability and of the technological imperfections impact on the final product of large series. These models should be produced in a previous step. For example, during the electromagnetic simulation we should take into consideration the uncertainty caused by the variability of the processes when the structure of the model is generated. Also, the systems obtained after the extraction step should take in consideration the variability, presenting a dependence of certain forms with respect to the variable parameters. The new approach presented in this paper has three main steps:

Step 1: Solve field problems (2D electroquasistatic (EQS) and Full Wave Transversal Magnetic (FW,TM)) and compute per unit length (p.u.l.) parameters; compute admittance for the real length of the line and validate the model by comparing with measurements;

Step 2: Based on the semi-state representation of the parametric system, compute the p.u.l. parameters sensitivities with respect to the geometric parameters that vary;

Step 3: Develop the parametric models based on the sensitivities.

2.1. COMPUTATION OF LINE PARAMETERS

The parasitic effects of on-chip interconnects are important factors in the evolution of the integrated circuits of high frequencies. Because of interconnects complex geometries, analytical approximations do not estimate correctly the values of the line parameters and limit the validation of the results. In order to comply with designer's requirements, the model should include the field propagation along the line (distributed parameter – transmission line effects) as well as the non-uniform field distribution outside line, in its cross section and the skin effect inside line conductor. If one of these phenomena is missed or wrong modeled, the simulation results are far from the reality.

The extraction of line parameters is the main step in transmission lines modelling since the behaviour of a line of a given length can be computed from them. For instance, for a multi conductor transmission line, from the line parameters matrices $\boldsymbol{R}$, $\boldsymbol{L}$, $\boldsymbol{C}$ and $\boldsymbol{G}$ the transfer matrix can be computed as $\boldsymbol{T} = \exp(\boldsymbol{D} + \mathrm{j}\omega\boldsymbol{E})$, where $\boldsymbol{D} = \begin{bmatrix} \mathbf{0} & -\boldsymbol{R} \\ -\boldsymbol{G} & \mathbf{0} \end{bmatrix}$ and $\boldsymbol{E} = \begin{bmatrix} \mathbf{0} & -\boldsymbol{L} \\ -\boldsymbol{C} & \mathbf{0} \end{bmatrix}$. From them, other parameters (impedance, admittance or scattering) can be computed as shown for instance in [2]. When considering *geometric data*, the simplest model may consider uniform fields in steady-state electric conduction (EC), electrostatics (ES) and magneto statics (MS) to asses the line resistance, capacitance and inductance, respectively. Empirical formulas may also be found in the literature, such as the ones given in [3] for the line capacitance. None of them take the frequency dependence into account. A first attempt to take into consideration the frequency effect is to compute the skin depth in the conductor and to use a better approximation for the resistance.

The first step of the numerical approach is the computation of the line parameters. The method is based on solving two complementary problems [4]: **2D-EQS** field problem which describes the transversal behaviour of the line from which the p.u.l. admittance (so, consequently, the p.u.l. conductance and p.u.l. capacitance) will be extracted and **TM** simulation related to the longitudinal electric field and transversal magnetic field.

For the first problem, the p.u.l. admittance $\boldsymbol{Y}_l$:

$$\boldsymbol{Y}_l = \boldsymbol{G}_l + \mathrm{j}\omega\boldsymbol{C}_l \tag{1}$$

can be computed from the EQS admittance as

$$\mathbf{Y}_l = \frac{\mathbf{Y}_{EQS}}{\Delta l}, \tag{2}$$

and then the p.u.l. conductance $\boldsymbol{G}_l$ and the p.u.l. capacitance $\boldsymbol{C}_l$ are:

$$\boldsymbol{G}_l = \mathrm{Re}(\boldsymbol{Y}_l), \quad \boldsymbol{C}_l = \frac{1}{\omega}\mathrm{Im}(\boldsymbol{Y}_l). \tag{3}$$

For the second model, the electric field is 3D, but the magnetic field is 2D. However, the p.u.l. impedance (and consequently, the p.u.l. resistance and p.u.l. inductance) can be extracted from a magneto-quasi-static (MQS) simulation, and not from the TM one. Since the TM includes the effects of transversal conductance and capacitance as well, we should subtract the EQS component out of the TM simulation to obtain the MQS component. Thus, the TM simulation gives the impedance $\boldsymbol{Z}_{TM}$ that corresponds to a line segment of length Δl. In order to compute the “longitudinal” MQS impedance, we have chosen a Π topology for the line equivalent circuit, so subtract half of the EQS component: $\boldsymbol{Z}_{MQS} = \left(\boldsymbol{Z}_{TM}^{-1} - \frac{1}{2} \boldsymbol{Y}_{EQS} \right)^{-1}$. The p.u.l. impedance:

$$\boldsymbol{Z}_l = \boldsymbol{R}_l + \mathrm{j}\omega \boldsymbol{L}_l \tag{4}$$

can be computed from the MQS impedance as

$$\boldsymbol{Z}_l = \frac{\boldsymbol{Z}_{MQS}}{\Delta l} \tag{5}$$

and then the p.u.l. resistance $\boldsymbol{R}_l$ and the p.u.l. inductance $\boldsymbol{L}_l$ are:

$$\boldsymbol{R}_l = \mathrm{Re}(\boldsymbol{Z}_l), \quad \boldsymbol{L}_l = \frac{1}{\omega} \mathrm{Im}(\boldsymbol{Z}_l). \tag{6}$$

The obtained values of the line parameters are frequency dependent. For a better accuracy of the solution, the line parameters matrices can be extracted using the dual Finite Integration Technique (dFIT) and the dual Equivalent Layer of Open Boundary Condition (dELOB) [5].

2.2. COMPUTATION OF FIRST ORDER SENSITIVITIES

For taking into consideration the effects of the variability of the processes, the systems are parameterized and their output depends on the parameterized values. For including the parameterized systems in an efficient simulation flux, the parametric dependence must be explicit, avoiding as much as possible recalculations of the parameterized system, meaning executing a new extraction.

Process uncertainties directly affect the geometric and electrical properties of the layout, and these variations can be represented as modifications of the values of the system matrices inside a state space descriptor [6]:

$$\boldsymbol{C}(\boldsymbol{p}) \frac{\mathrm{d}\, \boldsymbol{x}(\boldsymbol{p})}{\mathrm{d}\, t} + \boldsymbol{G}(\boldsymbol{p}) \boldsymbol{x}(\boldsymbol{p}) = \boldsymbol{B}\boldsymbol{u}, \tag{7}$$

$$\boldsymbol{y}(\boldsymbol{p}) = \boldsymbol{L}\boldsymbol{x}(\boldsymbol{p}), \tag{8}$$

where $\boldsymbol{x}$ is the state space vector, $\boldsymbol{B}$ is topological, sparse matrix and $\boldsymbol{u}$ is vector of input quantities.

The next level of the approximation in the modeling process is the computation of the first order sensitivities of the output quantity from the sensitivities of the state space matrices:

$$\frac{\partial \boldsymbol{y}(\boldsymbol{p})}{\partial \boldsymbol{p}} = \boldsymbol{L}(\boldsymbol{p})\frac{\partial \boldsymbol{x}(\boldsymbol{p})}{\partial \boldsymbol{p}}, \tag{9}$$

where

$$\frac{\partial \boldsymbol{x}(\boldsymbol{p})}{\partial \boldsymbol{p}} = -\left(\mathrm{j}\omega \boldsymbol{C}(\boldsymbol{p}) + \boldsymbol{G}(\boldsymbol{p})\right)^{-1}\left[\left(\mathrm{j}\omega\frac{\partial \boldsymbol{C}}{\partial \boldsymbol{p}} + \frac{\partial \boldsymbol{G}}{\partial \boldsymbol{p}}\right)\boldsymbol{x}(\boldsymbol{p})\right], \tag{10}$$

$$\boldsymbol{x}(\boldsymbol{p}) = \left(\mathrm{j}\omega \boldsymbol{C}(\boldsymbol{p}) + \boldsymbol{G}(\boldsymbol{p})\right)^{-1}\boldsymbol{Bu}. \tag{11}$$

Sensitivities of the p.u.l. parameters can be expressed as follows:

$$\frac{\partial \boldsymbol{G}_l}{\partial \boldsymbol{p}} = \frac{1}{l}\mathrm{Re}\frac{\partial \boldsymbol{Y}_{EQS}}{\partial \boldsymbol{p}}, \quad \frac{\partial \boldsymbol{C}_l}{\partial \boldsymbol{p}} = \frac{1}{\omega l}\mathrm{Im}\frac{\partial \boldsymbol{Y}_{EQS}}{\partial \boldsymbol{p}}, \tag{12}$$

$$\frac{\partial \boldsymbol{R}_l}{\partial \boldsymbol{p}} = \frac{1}{l}\mathrm{Re}\frac{\partial \boldsymbol{Z}_{MQS}}{\partial \boldsymbol{p}}, \quad \frac{\partial \boldsymbol{L}_l}{\partial \boldsymbol{p}} = \frac{1}{\omega l}\mathrm{Im}\frac{\partial \boldsymbol{Z}_{MQS}}{\partial \boldsymbol{p}}. \tag{13}$$

2.3. PARAMETRIC MODELS BASED ON FIRST ORDER SENSITIVITIES

Taylor series expansion is the most appropriate representation basis when considering the objectives related to parameter variability. The computation of the derivatives of the device characteristics with respect to the design parameters is needed [7, 8]. We assume that $y(p_1, p_2,..., p_n) = y(\boldsymbol{p})$ is the device characteristic which depends on the design parameters $\boldsymbol{p} = [p_1, p_2,..., p_n]$. The quantity y may be, for instance the real or the imaginary part of the device admittance at a given frequency. In our case this quantity is any of the p.u.l. parameters. This approach has also been considered in [9, 10]. The novelty of this paper consists in consideration of the parametric models designed for the longitudinal p.u.l. parameters (p.u.l. resistance and p.u.l. inductance) with respect to the variability of multiple geometric dimensions. A new aspect is observed, meaning the dependence of these parameters w.r.t. the frequency.

The parameter variability is completely described by the real function $y : S \rightarrow \mathbb{R}$, defined over the design space S, a subset of $\mathbb{R}^n$. The nominal design corresponds to the particular choice $y_0 = y(\boldsymbol{p_0})$ where $\boldsymbol{p}_0 = (p_{01}, p_{02},..., p_{0n})$ represents the vector of the nominal values of the design parameters. If y is smooth enough then its truncated Taylor series expansion is the best polynomial approximation in the vicinity of the expansion point p_0.

2.3.1. ADDITIVE MODEL (A)

Based on the additive model described in [9, 10] available for one variable parameter, in the multiparametric case, one gets:

$$y(\boldsymbol{p})=y(\boldsymbol{p}_0)+\nabla y(\boldsymbol{p}_0)\cdot(\boldsymbol{p}-\boldsymbol{p}_0)=y_0+\sum_{k=1}^{n}\frac{\partial y}{\partial p_k}(\boldsymbol{p}_0)(p_k-p_{0k}). \tag{14}$$

The relative sensitivities with respect to each parameter are denoted by $\frac{\partial y}{\partial p_k}(\boldsymbol{p}_0)\frac{p_{0k}}{y_0}=S_{p_k}^{y}$ and the relative variations of the parameters by $\frac{p_k-p_{0k}}{p_{0k}}\overset{not}{=}\delta p_k$.

The additive model (A) for *n* parameters is given by:

$$\hat{\boldsymbol{y}}(\boldsymbol{p})=y_0\left(1+\sum_{k=1}^{n}S_{p_k}^{y}\delta p_k\right). \tag{15}$$

2.3.2. RATIONAL MODEL (R)

The rational model is the additive model for the reverse quantity $1/y$. It is obtained from the first order truncation of the Taylor Series expansion for the function $1/y$. For the multiparametric case, the rational model is:

$$\hat{\boldsymbol{y}}(\boldsymbol{p})=\frac{y_0}{1+\sum_{k=1}^{n}S_{p_k}^{1/y}\delta p_k}. \tag{16}$$

2.3.3. MULTI-PARAMETRIC MODEL (M)

We assume that in the multiparametric case the output quantity can be expressed as a product of functions with separated variables:

$$\boldsymbol{y}(\boldsymbol{p})=y_1(p_1)y_2(p_2)\ldots y_n(p_n). \tag{17}$$

Each component function, y_k depends only on a single parameter, p_k and for each one we can use either an additive or a rational model. Although this product (17) seems to be a particular case, it fits perfectly the variation of RLC parameters w.r.t. geometric parameters extracted from uniform electric or magnetic field.

In order to make the explanations easy to understand, we look at the case of two variable parameters p_1, p_2 and four versions of model M are possible:

– M_{AA} – additive models for both parameters:

$$\hat{y}(\boldsymbol{p}) = y_0\left(1 + S_{p_1}^{y}\right)\left(1 + S_{p_2}^{y}\right), \tag{18}$$

– M_{RR} – rational models for both parameters:

$$\hat{y}(\boldsymbol{p}) = y_0 \frac{1}{\left(1 + S_{p_1}^{1/y}\right)\left(1 + S_{p_2}^{1/y}\right)}, \tag{19}$$

– M_{AR} – additive model for the first parameter and rational model for the second one:

$$\hat{y}(\boldsymbol{p}) = y_0 \frac{\left(1 + S_{p_1}^{y}\right)}{\left(1 + S_{p_2}^{1/y}\right)}, \tag{20}$$

– M_{RA} – rational model for the first parameter and additive model for the second one

$$\hat{y}(\boldsymbol{p}) = y_0 \frac{\left(1 + S_{p_2}^{y}\right)}{\left(1 + S_{p_1}^{1/y}\right)}. \tag{21}$$

Together with the two "classical" A and R models, there are six possible parametric models for the two parameter case.

3. **NUMERICAL EXAMPLE**

In order to get a good level of confidence in our proposed first order modeling approach, several experiments have been performed on a test structure that consists of a microstrip (MS) transmission line having one Aluminum conductor embedded in a SiO_2 layer. The line has a rectangular cross – section, parameterized by the parameters p_2 and p_3 (Fig. 1). Its position is parameterized by p_1. The return path is the grounded surface placed at y = 0. The nominal values used are: $x_{\max} = 264\ \mu\text{m}$, $h_3 = 10\ \mu\text{m}$, $h_1 = 1\ \mu\text{m}$, $p_1 = 1\ \mu\text{m}$, $p_2 = 0.69\ \mu\text{m}$, $p_3 = 3\ \mu\text{m}$, $\sigma_{Si} = 10000\ \text{MS/m}$, $\sigma_{Al} = 3.3\ \text{MS/m}$, $\varepsilon_{\text{r-SiO2}} = 3.9$.

In order to comply with designer's requirements, the model should include the field propagation along the line, taking into consideration the distributed parameters and the high frequency effects. The line capacitance is extracted using the dualFIT technique [5] and optimal value obtained is:

$$C_l = 169.6 \text{ pF/m}. \tag{22}$$

The longitudinal parameters (p.u.l. resistance and p.u.l. inductance) are frequency dependent. They were extracted using two different numerical techniques: FIT – Finite Integration Technique (Chamy software developed in the Numerical Methods Laboratory [11]) and FEM – Finite Element Method (COMSOL software [12]). The results obtained with the two methods are similar for different frequencies (Table 1).

Table 1

Comparison of the results obtained for the longitudinal line parameters

Frequency (GHz)	Method	p.u.l. Resistance (Ohm/m)	p.u.l. Inductance (H/m)
1	FIT	1.50e+4	3.41e-7
	FEM	1.62e+4	4.27e-7
15	FIT	1.56e+4	3.38e-7
	FEM	1.86e+4	2.92e-7
30	FIT	1.67e+4	3.31e-7
	FEM	2.17e+4	2.82e-7

In order to validate the results obtained for the nominal values of p.u.l. parameters, we derived from them the scattering parameters (S) [2] and compare the results with the measurements provided within the European project (Fig. 2) FP5/Codestar (www.imec.be/codestar).

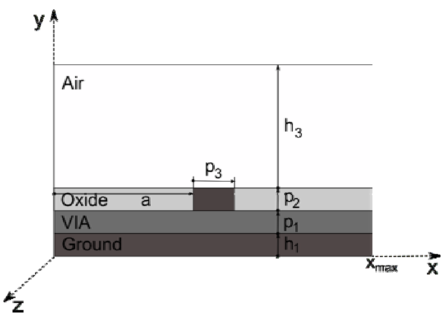

Fig. 1 – Stripline parameterized conductor.

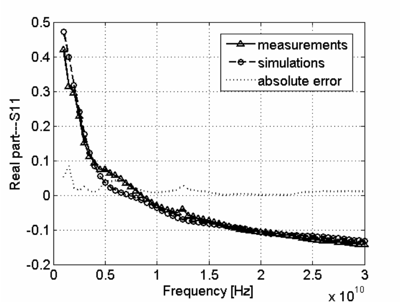

Fig. 2 – Frequency characteristic Re(S11): measurements vs numerical model.

The problem described above is subject to variation of the geometric parameters: p_1, p_2 and p_3. We consider the case when two parameters are variable.

For the first test we have studied the approximation of the line resistance using the additive, rational and multi-parametric models and parameters p_2 and p_3 variable. Considering the relative variations of the geometrical parameters less than 15 %, a set of samples in $[0.59,\ 0.79]\ \mu\text{m}\ \times[2.4,\ 3.6]\ \mu\text{m}$ have been chosen. In this case, model M is computed using rational models for both parameters for a single frequency, f = 1 GHz. Fig. 3 compares the relative variation of the errors w.r.t. a relative variation of parameter p_2 for a variation of p_3 of 10 %. Model M provides lower errors (maximum error is 3.24 %) than models R (5.12 %) and A (7.33 %).

For the second test we have chosen to approximate the line inductance when parameters p_1 and p_3 are variable. Model M is computed using an additive model for parameter p_1 and a rational one for p_3. A set of samples in $[0.8,\ 1.2]\ \mu\text{m}\ \times[2.4,\ 3.6]\ \mu\text{m}$ were considered.

Fig. 4 illustrates that for a variation of p_1 of 10 %, the rational model with respect to the relative variation of p_3 registers the lowest errors. Though, in a range from –10 % to 0 % model M is the best one. Thus, the appropriate multi-parametric model may eliminate the necessity of higher order approximations.

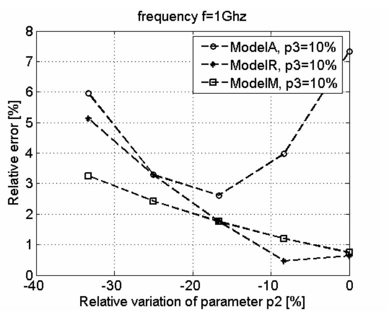

Fig. 3 – Relative error of the p.u.l. resistance w.r.t. the relative variation of parameter p_2, for a variation of p_3 of 10 %.

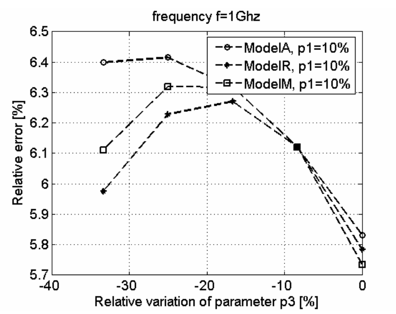

Fig. 4 – Relative error for p.u.l. inductance w.r.t. the relative variation of parameter p_3, for a variation of p_1 of 10 %.

4. **CONCLUSIONS**

New variability models based on first order sensitivities for transmission lines have been analyzed. For the multiple parameter case, a multiplicative model (M) based on Taylor Series expansion has been proposed as an alternative to the additive and rational models. The numerical experiments with the proposed algorithm for all the particular structures investigated (for example a variation of the geometric dimensions of ±15 % – typical for the 65 nm node) can be modelled with acceptable accuracy (relative errors under 5 %) using only first order parametric models for the line parameters. So, the analysis of the technological variability can be realized simple and efficient without the necessity of estimating higher orders derivatives. The longitudinal line parameters depend on the frequency, so the line length is a special geometric parameter which should be modeled with the exponential relations that include the propagation along the line. This idea is subject to a new approach for obtaining parametric models dependent on the frequency as well.

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