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A SIMPLE APPROACH FOR MODELING MULTILAYERED GROUND

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In this paper, an optimization procedure for modeling multilayered soil structure, consisting of finite number of horizontal layers, as a homogeneous soil of constant specific conductivity is presented. The equivalent specific conductivity value is determined using anew expression included into procedure. The expression is formed using the Green's functions for a point source in multilayered media obtained by solving the Poisson's *i.e.* the Laplace's equation for electric scalar potential and the method of moments. The approach is applied to the problem of a vertical ground electrode buried in soil structure consisting of three horizontal layers. The electrical parameters of the soil structure and thickness of each layer have been taken from experimental measurements.

1. INTRODUCTION

In order to ensure proper functioning of the grounding system, a necessary requirement is that the resistance value of the grounding system is as low as possible. Properly designed grounding system must lead fault current and lighting-induced currents into the surrounding soil, with maximum safety and without consequences for the working environment and electrical equipment.

One of the key factors in the process of design and production of a grounding system is determining distribution of the electrical conductivity in the surrounding soil. It depends on the soil temperature, soil type, moisture content, and depth and thickness of soil layers [1]. There are many previously published results of researches dealing with soil modeled as a homogeneous domain with uniform specific conductivity value [2–4]. In those researches, simple equations for fast calculation of the resistance of a ground electrode buried in the uniform surrounding soil have been proposed. These empirical equations can often lead to a large error of calculated resistance values compared to the ones obtained by experimental measurements [5, 6].

Some of previously published studies are related to the approach of inhomogeneous conductivity soil that is based on assuming specific conductivity of the ground as continuous variation function. In [7], the non-homogeneous soil having an exponentially decreasing conductivity is considered. In [8], the authors concluded that very good approximation of the variation law of the conductivity may be an exponentially decreasing.

Also, a large number of published researches deals with procedures for analyzing grounding systems in non-homogeneous ground approximated with homogeneous domains of various geometry. Among them, there are many researches which included approximation of non-homogeneous ground with a finite number of homogeneous horizontal layers. The approaches used in these researches are approximate summation or integration method combined with quasistationary image theory [9–11], numerical calculation using boundary elements method [12] and deriving approximate analytical expressions [13–15].

In this paper, a procedure for approximating multilayered soil structure consisting of a finite number of horizontal layers with a homogeneous domain of equivalent constant specific conductivity is presented and applied. The procedure includes using of the Green's functions for a point source in multilayered media obtained by solving the Poisson's *i.e.* the Laplace's equation for electric scalar potential [16] and the method of moments (MoM) [17]. So the problem of point current source in multilayered media is extended to vertical cylindrical rod placed in multilayered ground. Based on the proposed procedure, corresponding program packages were formed.

The results obtained using the above described procedure, are used for deriving approximate expression for the equivalent specific conductivity value. This value can be used for determining resistance of a vertical rod electrode placed in three-layered ground and treated as homogeneous media of equivalent specific conductivity. By that, the surface layer is assumed as thin and of small specific conductivity (which corresponds to usual ground structure). Consequently, its influence can be neglected. It makes it easier to design characteristics of a grounding electrode in terms of resistance, step voltage and touch voltage.

In order to validate the optimization procedure, numerical calculations were performed for N = 3 layers of soil having different specific conductivity values and thicknesses. The values for specific conductivity and thickness of layers have been taken from experimental measurements realized during research.

2. MULTILAYERED SOIL STRUCTURE

The procedure for characterizing influence of multilayered soil structure consisting of a finite number of horizontal layers on the grounding system is illustrated here with a problem of a vertical rod electrode placed in the ground (Fig. 1.). It includes using of the Green's functions for electric scalar potential and the MoM.,

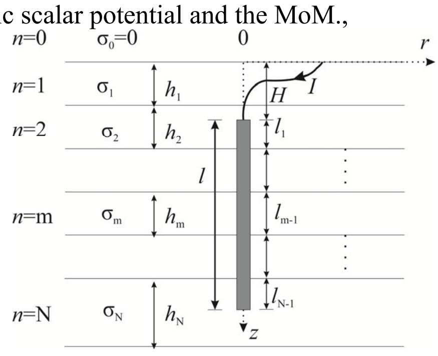

Fig. 1 – Ground electrode in multilayered ground.

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2.1. GREEN'S FUNCTION

The point source of current I placed in layer m of non-homogenous domain approximated with total of N+1 homogenous horizontal layers of specific conductivity σ_n, n=0,1,…N is observed (Fig. 2). The potential of the system from Fig. 2 satisfies a piecewise function containing both Laplace's equation and Poisson's equation, respectively [18],

$$\Delta\varphi = \begin{cases} 0, \; n \neq m, \; n = 0,1,...N \\ -\dfrac{I}{2\pi\sigma_n r}\delta(r)\delta(z-H), n = m, \; n = 0,1,...N. \end{cases} \quad (1)$$

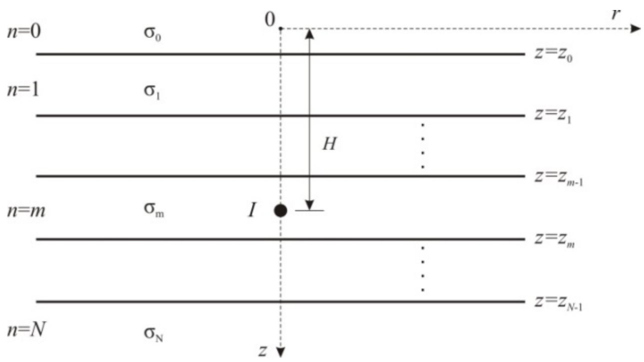

Fig. 2 – Point current source in multilayered media.

In (1), r and z and denote cylindrical coordinates, while δ is a one-dimensional Dirac delta function defined as

$$\delta(x) = \begin{cases} 0, \; x \neq 0 \\ \infty, \; x = 0. \end{cases} \quad (2)$$

General solution of the differential equation (1) can be assumed in the form [19],

$$\varphi_{gnm}(r,z) = \int_0^\infty f_{nm}(z,k) J_0(kr) k \, \mathrm{d}k, n = 0,1,...N \quad (3a)$$

$$f_{nm}(z,k) = \begin{cases} A_n \mathrm{e}^{kz} + B_n \mathrm{e}^{-kz}, n = 0, N, \; n \neq m \\ A_m \mathrm{e}^{kz} + B_m \mathrm{e}^{-kz}, \; n = m, \; z_{m-1} \leq z \leq H \\ \left(A_m - \dfrac{I}{4\pi\sigma_m k}\mathrm{e}^{-kH}\right)\mathrm{e}^{kz} + \left(B_m + \dfrac{I}{4\pi\sigma_m k}\mathrm{e}^{kH}\right)\mathrm{e}^{-kz}, \\ n = m, \; H \leq z \leq z_m. \end{cases} \quad (3b)$$

In (3), $\varphi_{g\,nm}$, n, m = 0,1,…, N is the potential at points in the n-th layer, when source is in layer m, A_n, B_n, n = 0,1,…, N are unknown coefficients, while J_0 denotes the Bessel function of the first kind of zero order.

Unknown coefficients A_n, B_n, n = 0,1,…, N from (3) can be determined from the boundary conditions for the potential, normal component of the conducting current and the finite value of the potential, *i.e.*:

$$\varphi(z = z_n^-) = \varphi(z = z_n^+) \quad (4a)$$

$$\sigma_n \frac{\partial\varphi}{\partial z}(z = z_n^-) = \sigma_{n+1}\frac{\partial\varphi}{\partial z}(z = z_n^+), (\text{for } n = 0,...,N\text{-}1) \quad (4b)$$

$$\lim_{z\to\pm\infty} \varphi = 0. \quad (4c)$$

2.2. GREEN'S FUNCTION

To determine parameters of a vertical ground electrode from Fig. 1, it is necessary to obtain the Green's function of a point source placed in non-homogeneous ground approximated with three horizontal homogeneous layers n = 1, 2, 3 (N=3), while layer labeled by n = 0 corresponds to air, (σ_0 = 0), as it is shown in Figs. 3 and 4. The Green's function can be obtained based on the general solution (3) and boundary conditions (4).

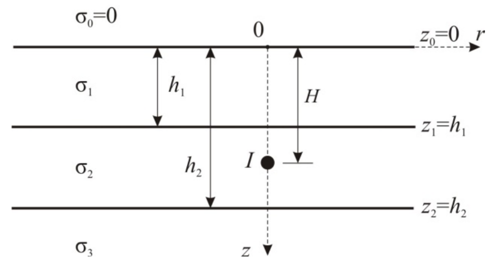

Fig. 3 – Point current source in three-layered ground (layer 2).

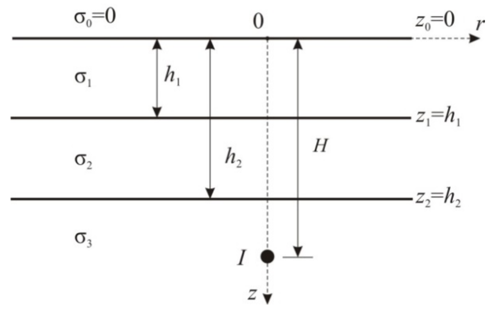

Fig. 4 – Point current source in three-layered ground (layer 3).

The potential in the n-th layer from the point source placed in the layer with specific conductivity (m = 2), Fig. 3, is

$$\varphi_{gn2}(r,z) = \int_0^\infty f_{n2}(z,k) J_0(kr) k \, \mathrm{d}k, n = 0,1,2,3, \quad (5a)$$

$$\begin{aligned} & f_{02}(z,k) = A_0 \mathrm{e}^{kz} + B_0 \mathrm{e}^{-kz}, z \leq 0;, \\ & f_{12}(z,k) = A_1 \mathrm{e}^{kz} + B_1 \mathrm{e}^{-kz}, 0 \leq z \leq h_1; \\ & f_{22}(z,k) = \begin{cases} \left(A_2 - \dfrac{I}{4\pi\sigma_2 k}\mathrm{e}^{-kH}\right)\mathrm{e}^{kz} + \left(B_2 + \dfrac{I}{4\pi\sigma_2 k}\mathrm{e}^{kH}\right)\mathrm{e}^{-kz}, H \leq z < h_2, \\ A_2 \mathrm{e}^{kz} + B_2 \mathrm{e}^{-kz}, h_1 \leq z < H, \end{cases} \\ & f_{32}(z,k) = \left(A_3 \mathrm{e}^{kz} + B_3 \mathrm{e}^{-kz}\right), h_2 \leq z \leq \infty. \end{aligned} \quad (5b)$$

$$A_0=\frac{I}{2\pi\sigma_2 k}\mathrm{e}^{2kh_1}\frac{(\mathrm{e}^{k(2h_2-H)}+t_2\mathrm{e}^{kH})(1-t_1)}{\mathrm{e}^{2kh_2}(\mathrm{e}^{2kh_1}-t_1)+\mathrm{e}^{2kh_1}(\mathrm{e}^{2kh_1}t_1-1)t_2},B_0=0,$$
$$A_1=B_1=\frac{I}{4\pi\sigma_2 k}\mathrm{e}^{2kh_1}\frac{(\mathrm{e}^{k(2h_2-H)}+t_2\mathrm{e}^{kH})(1-t_1)}{\mathrm{e}^{2kh_2}(\mathrm{e}^{2kh_1}-t_1)+\mathrm{e}^{2kh_1}(\mathrm{e}^{2kh_1}t_1-1)t_2},$$
$$A_2=\frac{I}{4\pi\sigma_2 k}\frac{(\mathrm{e}^{2kh_1}-t_1)(\mathrm{e}^{k(2h_2-H)}+t_2\mathrm{e}^{kH})}{\mathrm{e}^{2kh_2}(\mathrm{e}^{2kh_1}-t_1)+\mathrm{e}^{2kh_1}(\mathrm{e}^{2kh_1}t_1-1)t_2}, \tag{5c}$$
$$B_2=\frac{I}{4\pi\sigma_2 k}\frac{\mathrm{e}^{2kh_3}(\mathrm{e}^{k(2h_2-H)}+\mathrm{e}^{kH}t_2)(1-\mathrm{e}^{2kh_1}t_1)}{\mathrm{e}^{2kh_2}(\mathrm{e}^{2kh_1}-t_1)+\mathrm{e}^{2kh_1}(\mathrm{e}^{2kh_1}t_1-1)t_2},$$
$$A_3=0,\ B_3=\frac{I}{4\pi\sigma_2 k}\frac{2\mathrm{e}^{k(2h_2-H)}\mathrm{e}^{2kh_1}(1-\mathrm{e}^{2kh_1}t_1)+\mathrm{e}^{kH}\mathrm{e}^{2kh_2}(\mathrm{e}^{2kh_1}-t_1)}{\mathrm{e}^{2kh_2}(\mathrm{e}^{2kh_1}-t_1)+\mathrm{e}^{2kh_1}(\mathrm{e}^{2kh_1}t_1-1)t_2}.$$

Similarly, the potential in the n-th layer from the point source placed in the layer with specific conductivity (m = 3), Fig. 4 is

$$\varphi_{gn3}(r,z)=\int_0^\infty f_{n3}(z,k)J_0(kr)k\,\mathrm{d}k, n=0,1,2,3 \tag{6a}$$

$$\begin{aligned}
&f_{03}(z,k)=A_0\mathrm{e}^{kz}+B_0\mathrm{e}^{-kz}, z\le 0\\
&f_{13}(z,k)=A_1\mathrm{e}^{kz}+B_1\mathrm{e}^{-kz}, 0\le z\le h_1\\
&f_{23}(z,k)=A_2\mathrm{e}^{kz}+B_2\mathrm{e}^{-kz},\ h_1\le z<h_2\\
&f_{33}(z,k)=\begin{cases}A_3\mathrm{e}^{kz}+B_3\mathrm{e}^{-kz},\ h_2\le z<H\\ \left(A_3-\frac{I}{4\pi\sigma_3 k}\mathrm{e}^{-kH}\right)\mathrm{e}^{kz}\left(B_3+\frac{I}{4\pi\sigma_3 k}\mathrm{e}^{kH}\right)\mathrm{e}^{-kz}\ H\le z\le\infty,\end{cases}
\end{aligned} \tag{6b}$$

$$A_0=\frac{I}{2\pi\sigma_3 k}\frac{\mathrm{e}^{2kh_1}(1-t_1)(1-t_2)\mathrm{e}^{k(2h_2-H)}}{(\mathrm{e}^{2kh_1}-t_1)\mathrm{e}^{2kh_2}+\mathrm{e}^{2kh_1}(\mathrm{e}^{2kh_1}t_1-1)t_2},B_0=0$$
$$A_1=B_1=\frac{I}{4\pi\sigma_3 k}\frac{\mathrm{e}^{2kh_1}(1-t_1)(1-t_2)\mathrm{e}^{k(2h_2-H)}}{(\mathrm{e}^{2kh_1}-t_1)\mathrm{e}^{2kh_2}+\mathrm{e}^{2kh_1}(\mathrm{e}^{2kh_1}t_1-1)t_2}$$
$$A_2=\frac{I}{4\pi\sigma_3 k}\frac{(\mathrm{e}^{2kh_1}-t_1)(1-t_2)\mathrm{e}^{k(2h_2-H)}}{(\mathrm{e}^{2kh_1}-t_1)\mathrm{e}^{2kh_2}+\mathrm{e}^{2kh_1}(\mathrm{e}^{k2h_1}t_1-1)t_2}, \tag{6c}$$
$$B_2=\frac{I}{4\pi\sigma_3 k}\frac{(1-\mathrm{e}^{2kh_1}t_1)(1-t_2)\mathrm{e}^{k(2h_2+2h_1-H)}}{(\mathrm{e}^{2kh_1}-t_1)\mathrm{e}^{2kh_2}+\mathrm{e}^{2kh_1}(\mathrm{e}^{k2h_1}t_1-1)t_2}$$
$$A_3=\frac{I}{4\pi\sigma_3 k}\mathrm{e}^{-kH},\ B_3=\frac{I\,\mathrm{e}^{k(2h_2-H)}}{4\pi\sigma_3 k}\frac{\mathrm{e}^{2kh_1}(1-\mathrm{e}^{2kh_1}t_1)+\mathrm{e}^{2kh_2}t_2(t_1-\mathrm{e}^{2kh_1})}{(\mathrm{e}^{2kh_1}-t_1)\mathrm{e}^{2kh_2}+\mathrm{e}^{2kh_1}(\mathrm{e}^{2kh_1}t_1-1)t_2}.$$

In (5c) and (6c) $t_1=\frac{\sigma_1-\sigma_2}{\sigma_1+\sigma_2}$ and $t_2=\frac{\sigma_2-\sigma_3}{\sigma_2+\sigma_3}$, while other parameters are previously defined and/or can be observed from Figs. 3 and 4. The Green's function can now be defined as

$$G_{nm}(r,z)=\varphi_{gnm}(r,z)/I, n=0,1,2,3 \text{ and } m=2,3. \tag{7}$$

The obtained Green's functions obtained by (7) can be also used for low frequency problems, replacing specific conductivity σ_m with complex specific conductivity $\underline{\sigma}_m=\sigma_m+\mathrm{j}\omega\varepsilon_m$, m = 1, 2, 3 ($\varepsilon_m=\varepsilon_0\,\varepsilon_{rm}$ – permittivity, ω – angular frequency)

2.3. DETERMINATION OF THE SYSTEM'S POTENTIAL AND RESISTANCE

The vertical electrode from Fig. 5 is observed. It is assumed that the leakage current density per unit length is uniform on the electrode's part in layer 2 (I_{leak2}) and on the part placed in layer 3 (I_{leak3}). This approach is justified for quasi-stationary regime. Consequently,

$$I_{\text{leak }2}=I_2/(h_2-H_1), I_{\text{leak }3}=I_3/(H_2-h_2), \tag{8}$$

where I_2 and I_3 are total currents leaking from the part of the conductive rod placed in layer n = 2 (I_2), *i.e.* n = 3 (current I_3). Now, the potential φ_n in layer n = 0, 1, 2, 3 can be expressed as

$$\varphi_n=\int_{H_1}^{h_2}I_{\text{leak }2}G_{n2}(r,z)\,\mathrm{d}z+\int_{h_2}^{H_2}I_{\text{leak }3}G_{n3}(r,z)\,\mathrm{d}z. \tag{9}$$

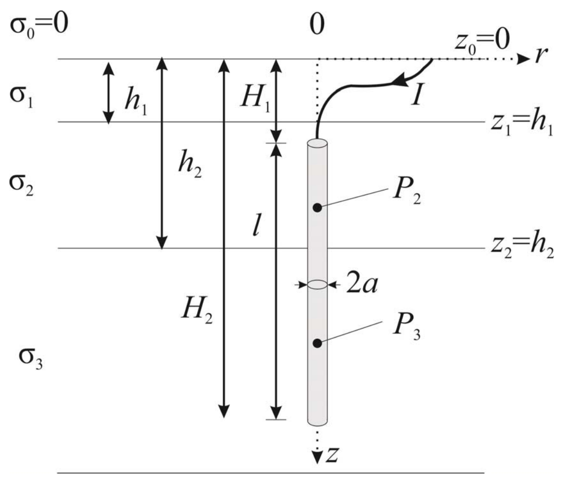

Fig. 5 – Ground electrode in multilayered ground.

Applying the MoM and matching the potential value φ=U at the middle point of the surface of the conductive rod segment placed in layer 2 defined by the field vector $\vec{R}_2$ (point P_2), and also in the middle point of the surface part placed in layer 3 defined by the field vector $\vec{R}_3$ (point P_3),

$$\begin{aligned}\vec{R}_2&=a\hat{r}+0.5(H_1+h_2)\hat{z}=r_2\hat{r}+z_2\hat{z},\\ \vec{R}_3&=a\hat{r}+0.5(H_2+h_2)\hat{z}=r_3\hat{r}+z_3\hat{z}\end{aligned} \tag{10}$$

it is possible to form linear equation system

$$\begin{aligned}U&=\int_{H_1}^{h_2}I_{\text{leak}2}G_{22}(r_2,z_2)\,\mathrm{d}z+\int_{h_2}^{H_2}I_{\text{leak}3}G_{23}(r_2,z_2)\,\mathrm{d}z,\\ U&=\int_{H_1}^{h_2}I_{\text{leak}2}G_{32}(r_3,z_3)\,\mathrm{d}z+\int_{h_2}^{H_2}I_{\text{leak}3}G_{33}(r_3,z_3)\,\mathrm{d}z.\end{aligned} \tag{11}$$

Solutions of equation systems (11) are currents I_2 and I_3. The resistance of the grounding system is now

$$R_{\mathrm{g}}=U/(I_2+I_3) \tag{12}$$

and the potential can be obtained from (9).

3. **MEASUREMENTS OF SOIL CONDUCTIVITY**

As it has already been emphasized, the information about the soil's specific conductivity is very important in the process of design and realization of the grounding system, which follows predefined requirements (values of touch voltage, step voltage, resistance *etc*.). More generally, it also includes information about thicknesses and position of the layers as well as about their inner structure and specific conductivity values.

In order to determine the soil structure, *i.e.* order of soil layers and corresponding specific conductivity values, the geotechnical research of surroundings soil has been performed for a certain area of interest in Niš, Serbia. This research includes the following:

- Wenner method [20] application;
- method of specific electric resistance (SER) with short (Sn) probe application;
- method of electrical potential application;
- measurement of the borehole diameter;
- measurement of temperature; and
- measurement of natural radioactivity.

Different soil structures having various specific conductivity and thickness values are given in Table 1.

Table 1

Structure of the surrounding soil.

Type of soil	Thickness [m]	σ [S/m]
The surface decomposed (Peat, loam and mud)	0.20	0.00625
Clay and sand mixture	1.80	0.0142
Clay	∞	0.052
The surface decomposed (Peat, loam and mud)	0.18	0.00648
Clayed sand	2.00	0.0235
Clay	∞	0.0426
Asphalt	0.23	0.000753
Sandstone (fine grained, clayed)	1.60	0.00834
Sandstone (coarse-grained, sandy)	∞	0.00356
Asphalt	0.25	0.000753
Sandy and clayed gravel	2.35	0.0112
Clay	∞	0.0844
The surface decomposed (Peat, loam and mud)	0.21	0.00568
Clay and sand mixture	2.8	0.0182
Clayed sand	∞	0.0263

4. OPTIMIZATION PROCEDURE AND EQUIVALENT SPECIFIC CONDUCTIVITY

Based on the results obtained using the procedure from Section 2, and taking into consideration the fact that specific conductivity of the layer at the ground surface, σ_1, is usually very small compared to specific conductivity of other layers, the following expressions for equivalent specific conductivity of the homogeneous ground which approximate multi-layered soil σ_e (Fig. 6), are proposed,

$$\sigma_e = \frac{A_1 l_1 \cdot \sigma_2 + A_2 l_2 \cdot \sigma_3}{l_1 + l_2}, \text{for } \sigma_2 > \sigma_3 \tag{13a}$$

$$\sigma_e = \frac{B_1 \cdot l_1 \sigma_2 + B_2 \cdot l_2 \cdot \sigma_3}{l_1 + l_2}, \text{for } \sigma_2 < \sigma_3 \tag{13b}$$

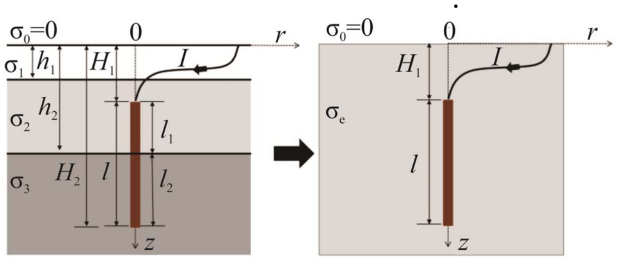

Fig. 6 – Equivalent homogeneous surrounding soil.

The expressions (13) are empirical, based on the assumption that the influence of the specific electrical conductivity of the ground surface on conductive rod resistance is negligible. Also, it is justified to expect that the influence of specific conductivities σ_2 and σ_3 are proportionate to length l_1 and l_2 respectively.

In order to determine constants values A_1, A_2, B_1, and B_2, the set of the data varying σ_2 and σ_3 values as well as the value $h_2 - h_1$ (thickness of the layer having specific conductivity σ_2) and depth H_1 is formed. The influence of σ_1 and corresponding layer thickness on resistance is practically neglected, and its influence on potential distribution at the ground surface is very small. Because of that, the thickness of the first layer h_1 was assumed as constant value. The data values were changing in the intervals which are chosen based on the values from Table 1 and [3,4] and resistance of the grounding electrode from Fig. 5, R_g is determined using the Green's function and the MoM as it has been explained in Section 2.

The resistance R_{gh} of the vertical rod having dimension as the one from Fig. 5 and placed in homogeneous ground normalized with specific resistance of homogeneous ground $\rho_{eh}=1/\sigma_{eh}$ (where $R_{ghnorm}= R_{gh}/\rho_{eh}= R_{gh}\sigma_{eh}$, i.e. $\sigma_{eh}= R_{ghnorm}/R_{gh}$) is determined using the segment method [21] with 100 segments. Then, the difference function between σ_e determined from (14) and σ_{eh} is formed. Thereafter, the optimization method based on the minimization of sum of squared error for data sets values,

$$F = \sum_{\substack{\text{data set}\\ \text{element}}} (\sigma_e - \sigma_{eh})^2 = \sum_{\substack{\text{data set}\\ \text{element}}} \left(\sigma_e - \frac{R_{\text{gh norm}}}{R_{gh}} \right)^2 \tag{14}$$

is applied. The result of this optimization are the constant values A_1, A_2, B_1, and B_2.

5. NUMERICAL RESULTS

The optimization procedure has been applied for characterization of the rod electrode placed in three-layered soil. The rod electrode length is l = 3 m, while diameter of the electrode is 0.035 m.

A set of data is formed for $\sigma_1 = 7\times10^{-3}\,\text{S/m}$, while values of σ_2 and σ_3 were changed in the interval $2\times10^{-2}\,\text{S/m} \div 10^{-1}\,\text{S/m}$. Simultaneously, the thickness of the layer having specific conductivity σ_2 is changing in the interval 1.5 m < $h_2 - h_1$ < 2.5 m while parameter H_1 is taking values 0.5 m and 1 m. (see Table 2). Thickness of the surface soil is assumed as constant h_1 < 0.2 m.

Using the above mentioned procedure for minimization of the value of the function given with (14), the following constants values in (13) are obtained,

$$\begin{aligned} A_1 &= 0.9085, A_2 = 1.0953, \\ B_1 &= 1.0759, B_2 = 1.0195. \end{aligned} \tag{15}$$

The resistances of the rod electrode R_g from Fig. 6. obtained using procedure from Section 2, and described optimization procedure based on data set from Table 2 for total 90 data sets are given in Fig. 7. The maximum error is between – 5.3 % and 12.16 %, while the mean absolute error is 2.07 % of the resistance obtained using procedure from Section 2. Those results justify application of the optimization procedure proposed in this paper.

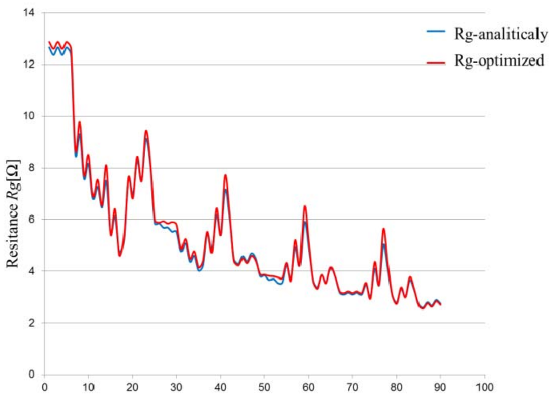

Fig. 7 – The resistances of the rod electrode (Fig. 6), obtained using procedure from Section 2 and optimization procedure from Section 4 for total 90 data sets.

Table 2

Data set

No.	σ_2 [S/m]	σ_3 [S/m]	$h_2 - h_1$ [m]	H_1 [m]	R_g [Ω]
1	0.02	0.02	1.5	0.5	12.661
2	0.02	0.02	1.5	1	12.362
3	0.02	0.02	2	0.5	12.661
4	0.02	0.02	2	1	12.362
5	0.02	0.02	2.5	0.5	12.661
6	0.02	0.02	2.5	1	12.362
7	0.02	0.05	1.5	0.5	8.5152
8	0.02	0.05	1.5	1	9.3034
9	0.02	0.05	2	0.5	7.5716
10	0.02	0.05	2	1	8.156
11	0.02	0.05	2.5	0.5	6.8152
12	0.02	0.05	2.5	1	7.2762
13	0.02	0.08	1.5	0.5	6.4646
14	0.02	0.08	1.5	1	7.4813
15	0.02	0.08	2	0.5	5.4633
16	0.02	0.08	2	1	6.1218
17	0.02	0.08	2.5	0.5	4.734
18	0.02	0.08	2.5	1	5.2041
19	0.04	0.02	1.5	0.5	7.6
20	0.04	0.02	1.5	1	6.96
21	0.04	0.02	2	0.5	8.2857
22	0.04	0.02	2	1	7.5099
23	0.04	0.02	2.5	0.5	9.1147
24	0.04	0.02	2.5	1	8.1643
25	0.04	0.05	1.5	0.5	5.8539
26	0.04	0.05	1.5	1	5.8606
27	0.04	0.05	2	0.5	5.6854
28	0.04	0.05	2	1	5.6897
29	0.04	0.05	2.5	0.5	5.5238
30	0.04	0.05	2.5	1	5.529
31	0.04	0.08	1.5	0.5	4.7761
32	0.04	0.08	1.5	1	5.0694
33	0.04	0.08	2	0.5	4.3636
34	0.04	0.08	2	1	4.5963
35	0.04	0.08	2.5	0.5	4.015
36	0.04	0.08	2.5	1	4.2091
37	0.06	0.02	1.5	0.5	5.4508
38	0.06	0.02	1.5	1	4.8459
39	0.06	0.02	2	0.5	6.1737
40	0.06	0.02	2	1	5.3973
41	0.06	0.02	2.5	0.5	7.1609
42	0.06	0.02	2.5	1	6.1076
43	0.06	0.05	1.5	0.5	4.4756
44	0.06	0.05	1.5	1	4.2825
45	0.06	0.05	2	0.5	4.583
46	0.06	0.05	2	1	4.3809
47	0.06	0.05	2.5	0.5	4.6985
48	0.06	0.05	2.5	1	4.4845
49	0.06	0.08	1.5	0.5	3.8076
50	0.06	0.08	1.5	1	3.8409
51	0.06	0.08	2	0.5	3.6666
52	0.06	0.08	2	1	3.6955
53	0.06	0.08	2.5	0.5	3.5338
54	0.06	0.08	2.5	1	3.5612
55	0.08	0.02	1.5	0.5	4.2456
56	0.08	0.02	1.5	1	3.7171
57	0.08	0.02	2	0.5	4.923
58	0.08	0.02	2	1	4.2132
59	0.08	0.02	2.5	0.5	5.9063
60	0.08	0.02	2.5	1	4.8814
61	0.08	0.05	1.5	0.5	3.626
62	0.08	0.05	1.5	1	3.3752
63	0.08	0.05	2	0.5	3.8468
64	0.08	0.05	2	1	3.5644
65	0.08	0.05	2.5	0.5	4.1039
66	0.08	0.05	2.5	1	3.7787
67	0.08	0.08	1.5	0.5	3.1709
68	0.08	0.08	1.5	1	3.0933
69	0.08	0.08	2	0.5	3.1709
70	0.08	0.08	2	1	3.0933
71	0.08	0.08	2.5	0.5	3.1709
72	0.08	0.08	2.5	1	3.0933
73	0.1	0.02	1.5	0.5	3.477
74	0.1	0.02	1.5	1	3.015
75	0.1	0.02	2	0.5	4.0947
76	0.1	0.02	2	1	3.4554
77	0.1	0.02	2.5	0.5	5.029
78	0.1	0.02	2.5	1	4.0661
79	0.1	0.05	1.5	0.5	3.0486
80	0.1	0.05	1.5	1	2.7854
81	0.1	0.05	2	0.5	3.3173
82	0.1	0.05	2	1	3.0053
83	0.1	0.05	2.5	0.5	3.6494
84	0.1	0.05	2.5	1	3.2672
85	0.1	0.08	1.5	0.5	2.7184
86	0.1	0.08	1.5	1	2.59
87	0.1	0.08	2	0.5	2.7981
88	0.1	0.08	2	1	2.6622
89	0.1	0.08	2.5	0.5	2.8847
90	0.1	0.08	2.5	1	2.7391

It is interesting to emphasize that application of the same approach as in Section 2, for $\sigma_1 = \sigma_2 = \sigma_3 = \sigma_e$ (homogeneous ground of specific resistance σ_e) for vertical rod having dimension as the one from Fig. 5 results with the expression

$$R_g = \frac{1}{4\pi l \sigma_e} \ln \frac{(l+\sqrt{l^2+4a^2})^2}{4a^2} \cdot \frac{3l+4H_1+\sqrt{(3l+4H_1)^2+4a^2}}{l+4H_1+\sqrt{(l+4H_1)^2+4a^2}} \quad (16)$$

Comparing results obtained by using expressions (13) (for constant values given with (14)) and (16) gives maximal error value between – 4.98 % and 12.57 %, while the mean absolute error is 1.36 %.

6. **CONCLUSIONS**

One optimization procedure, which makes simpler analysis of a grounding system placed in multilayered soil having horizontal layers, is presented in the paper. Based on characterization of the vertical rod electrode placed in the multilayered ground, optimization procedure provides approximation of multilayered soil as homogeneous ground having uniform specific conductivity. By that, the expression for the equivalent specific conductivity value is proposed. The results are compared with the ones obtained using the Green's function and the MoM. The comparison of the results and very good agreement between them justifies application of the optimization procedure, which allows avoiding analysis of a grounding system in multi-layered soil and reduces it to analysis of the grounding electrode in homogeneous ground. It is important to emphasize that the obtained Green's functions can be also used for low frequency problems, replacing specific conductivity σ with complex specific conductivity $\underline{\sigma} = \sigma + j\omega\varepsilon$, where $\varepsilon = \varepsilon_0\varepsilon_r$ is permittivity, while ω is corresponding angular frequency.

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