

RESISTANCE MEASUREMENT BY A WHEATSTONE BRIDGE USING A DIGITAL VOLTMETER AS DETECTOR

DANIEL BELEGA

Key words: Resistance measurement, Wheatstone bridge, Digital voltmeter.

In this paper the measurement of a resistance by means of a Wheatstone bridge in which the detector is a digital voltmeter (DVM) is investigated. The relationships for the determination of the Wheatstone bridge resistances in order to accurate measurement of a resistance are derived. To validate the derived relationships some experimental results are carried out.

1. INTRODUCTION

The resistance is one of the most important elements of an electrical circuit. From this reason it is very important to know with high accuracy the value of each resistance used in an electrical circuit. For accurate measurement of a resistance many methods exist. One of these is the well-known method based upon the Wheatstone bridge [1–3]. This method is used to measure the resistances with moderate values. A schematic view of the Wheatstone bridge is shown in Fig. 1. R_x is the resistance which follows to be measured and R_2 , R_3 and R_4 are the known resistances. The supply voltage V_s of the bridge is applied between the nodes a and b , which forms the supply diagonal. Between the nodes c and d , which forms the measurement diagonal, the detector D is connected. The detector D is characterized by the resistance R_D . Because of their high sensitivity the analog galvanometers are generally used as detector [1–3].

The measurement of the resistance R_x is carried out when the Wheatstone bridge is at balance.

At balance, as a function of the detector type, the current or voltage measured by the detector is zero. Thus, the nodes c and d are at the same potential. Hence, $i_1 R_x = i_2 R_2$ and $i_1 R_3 = i_2 R_4$. From these relationships it follows the balance condition:

$$R_x R_4 = R_2 R_3. \quad (1)$$

From the balance condition the resistance R_x is determined

“Politehnica” University of Timișoara, E-mail: daniel.belega@etc.upt.ro

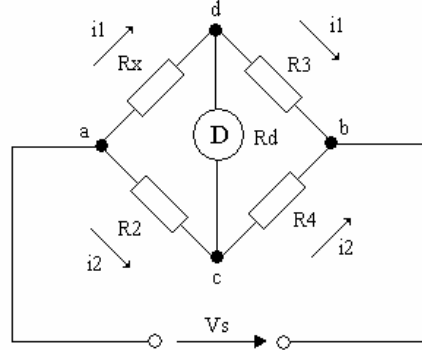


Fig. 1 – Wheatstone bridge.

$$R_x = \frac{R_2 R_3}{R_4}. \quad (2)$$

From (2) it follows that, at balance, the maximum of the modulus of relative error in the measurement of R_x , $\varepsilon_{R_{x \max_b}}$, is given by

$$\varepsilon_{R_{x \max_b}} = |\varepsilon_{R_2}| + |\varepsilon_{R_3}| + |\varepsilon_{R_4}|, \quad (3)$$

where ε_{R_2} , ε_{R_3} and ε_{R_4} are the relative errors of the resistances R_2 , R_3 and R_4 .

The balance condition is satisfied if one of the resistances R_2 , R_3 and R_4 is variable.

In the case of using an analog galvanometer as detector the relationships for the determination of the resistances R_2 , R_3 and R_4 in order to measure with high accuracy the resistance R_x are given in reference [1].

This paper is focused on the measurement of a resistance by means of a Wheatstone bridge which uses a digital voltmeter (DVM) as detector. In this case, the relationships for the determination of the resistances R_2 , R_3 and R_4 to accurately measure the resistance R_x are derived. Some experimental results are given in order to validate the derived relationships.

2. DETERMINATION OF THE RESISTANCES R_2 , R_3 AND R_4 OF A WHEATSTONE BRIDGE WHICH USE A DIGITAL VOLTMETER AS DETECTOR

Assume the detector is a DVM which measures voltages of microvolts order. The voltage V_{cd} between the nodes c and d is given by

$$V_{cd} = \frac{(R_x R_4 - R_2 R_3) R_D V_s}{R_D (R_x + R_3) (R_2 + R_4) + R_x R_3 (R_2 + R_4) + R_2 R_4 (R_x + R_3)}. \quad (4)$$

The voltmeter indicates 0 V when the module $|V_{cd}|$ is smaller than the half of the voltmeter resolution (or half of one Least Significant Digit value 0.5 LSD).

In this case the Wheatstone bridge is an apparent balance.

Suppose the bridge is an apparent balance and $V_{cd} = V_{cdm}$, with $|V_{cdm}| < 0.5 \text{ LSD}$.

After some algebra, from (4) it can be established that

$$\begin{aligned} R_x &= \frac{R_2 R_3}{R_4} \frac{1 + \frac{V_{cdm}}{V_s R_D R_2} [R_D (R_2 + R_4) + R_2 R_4]}{1 - \frac{V_{cdm}}{V_s R_D R_4} [(R_D + R_3) (R_2 + R_4) + R_2 R_4]} = \\ &= \frac{R_2 R_3}{R_4} \frac{1 + \frac{V_{cdm}}{V_s} \left(1 + \frac{R_4}{R_2}\right) \left(1 + \frac{R_2 R_4}{R_D (R_2 + R_4)}\right)}{1 - \frac{V_{cdm}}{V_s} \left(1 + \frac{R_2}{R_4}\right) \left(1 + \frac{R_3}{R_D} + \frac{R_2 R_4}{R_D (R_2 + R_4)}\right)}. \end{aligned} \quad (5)$$

In practice the voltmeter resistance R_D is often much higher than the resistances R_2 , R_3 and R_4 . In this case from (5) we obtain

$$R_x \cong \frac{R_2 R_3}{R_4} \frac{1 + \frac{V_{cdm}}{V_s} \left(1 + \frac{R_4}{R_2}\right)}{1 - \frac{V_{cdm}}{V_s} \left(1 + \frac{R_2}{R_4}\right)}. \quad (6)$$

From the above expression it follows that, at the apparent balance, the maximum of the modulus of relative error in the measurement of R_x , $\varepsilon_{R_{\max_ab}}$, can be approximated by

$$\varepsilon_{R_{\max_ab}} \cong |\varepsilon_{R_3}| + \frac{|\varepsilon_{R_2}| + |\varepsilon_{R_4}| + \frac{|V_{cdm}|}{V_s} \left(2 + \frac{R_4}{R_2} + \frac{R_2}{R_4}\right) |\varepsilon_{V_s}|}{\left[1 + \frac{|V_{cdm}|}{V_s} \left(1 + \frac{R_4}{R_2}\right)\right] \left[1 - \frac{|V_{cdm}|}{V_s} \left(1 + \frac{R_2}{R_4}\right)\right]}, \quad (7)$$

where ε_{V_s} is the relative error of the voltage V_s .

From the expression (7) the following conclusions are obtained:

- $|\varepsilon_{R_3}|$ is not affected by any multiplicative factor;
- $|\varepsilon_{R_2}|$, $|\varepsilon_{R_4}|$ and $|\varepsilon_{V_s}|$ are affected by some factors which depends upon the rapports $|V_{cdm}|/V_s$ and R_4/R_2 ;
- for the values used in practice for the rapports $|V_{cdm}|/V_s$ and R_4/R_2 , the influence of the error $|\varepsilon_{V_s}|$ on the $\varepsilon_{R_{x \max_{ab}}}$ is practical neglected;
- the difference between the errors $\varepsilon_{R_{x \max_{ab}}}$ and $\varepsilon_{R_{x \max_{b}}}$ decreases when the rapport $|V_{cdm}|/V_s$ decreases.

When the resistance R_D is very high, based upon the relationships (2) and (6), the relative error of R_x , δ_{R_x} , can be approximated by

$$\delta_{R_x} \cong \frac{\frac{V_{cdm}}{V_s} \left(2 + \frac{R_4}{R_2} + \frac{R_2}{R_4} \right)}{1 - \frac{V_{cdm}}{V_s} \left(1 + \frac{R_2}{R_4} \right)}. \quad (8)$$

From the expression (8) it follows that the relative error δ_{R_x} practically not depends upon the resistance R_3 , but depends upon the rapports V_{cdm}/V_s and R_4/R_2 . From this reason to satisfy the bridge balance condition it is suited to chose R_3 as the variable resistance. For a fine variation of the resistance R_3 , often this is a decade resistance.

If $R_4/R_2 = n$, the expression (6) becomes

$$R_x \cong \frac{R_3}{n} \frac{1 + \frac{V_{cdm}}{V_s} (n+1)}{1 - \frac{V_{cdm}}{V_s} \frac{(n+1)}{n}} \quad (9)$$

and the expression (8) becomes

$$\delta_{R_x} \cong \frac{\frac{V_{cdm}}{V_s} (n+1)^2}{n \left(1 - \frac{V_{cdm}}{V_s} \right) - \frac{V_{cdm}}{V_s}}. \quad (10)$$

In practice we have $|V_{cdm}|/V_s \ll 1$ and $n \gg |V_{cdm}|/V_s$. Thus, δ_{R_x} can be approximated by $\hat{\delta}_{R_x}$, which has the expression

$$\hat{\delta}_{R_x} = \frac{(n+1)^2}{n} \frac{V_{cdm}}{V_s}. \quad (11)$$

From the above expression the following conclusions are obtained:

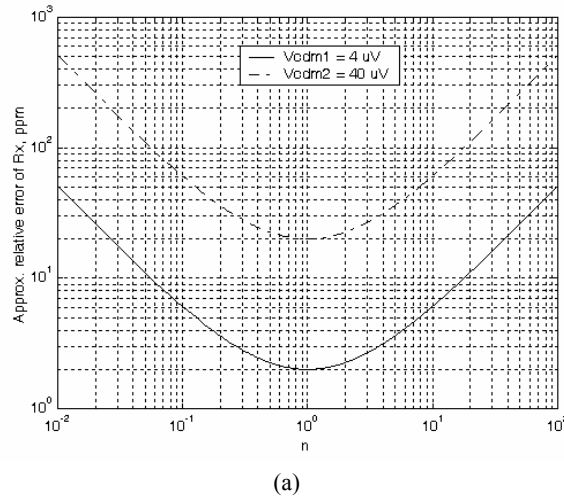
- $|\hat{\delta}_{R_x}|$ decreases when the rapport $|V_{cdm}|/V_s$ decreases;
- minimum of $|\hat{\delta}_{R_x}|$ is obtained for $n = 1$ and is given by

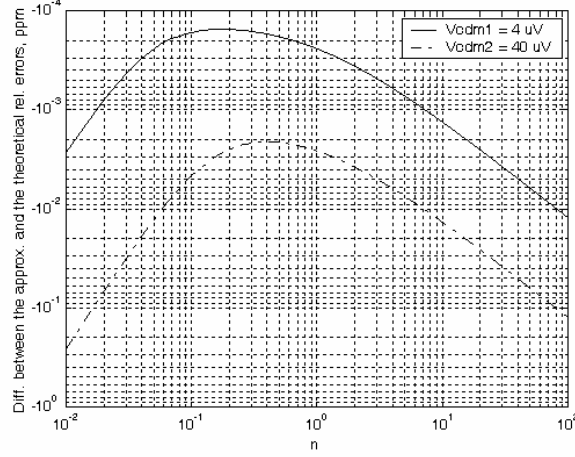
$$|\hat{\delta}_{R_x}|_{\min} = 4 \frac{|V_{cdm}|}{V_s}. \quad (12)$$

- if $n \geq 1$, $|\hat{\delta}_{R_x}|$ increases when n increases and if $0 < n < 1$, $|\hat{\delta}_{R_x}|$ decreases when n increases;
- because $|V_{cdm}| < 0.5 \text{ LSD}$ we have

$$\hat{\delta}_{R_x} < \frac{(n+1)^2}{2n} \frac{1LSD}{V_s}. \quad (13)$$

Fig. 2 presents, in logarithmic scales, $\hat{\delta}_{R_x}$ as a function of n (Fig. 2a) and the difference between $\hat{\delta}_{R_x}$ and δ_{R_x} as a function of n (Fig. 2b) in the case of using two DVMs as detector. The first DVM has $V_{cdm1} = 4 \mu\text{V}$ (fine resolution DVM) and the other has $V_{cdm2} = 40 \mu\text{V}$ (coarse resolution DVM). Both DVMs have $R_D = 10 \text{ M}\Omega$. The bridge supply voltage is $V_s = 8 \text{ V}$. The resistances of the bridge are: $R_3 = R_4 = nR_2$, with $R_2 = 1200 \Omega$. n varies in the range $[0.01, 100]$ with an increment of 0.01.





(b)

Fig. 2 – a) $\hat{\delta}_{Rx}$ as a function of n ; b) difference between $\hat{\delta}_{Rx}$ and δ_{Rx} as a function of n .

Fig. 2a shows that for a given n , $\hat{\delta}_{Rx}$ obtained when the fine resolution DVM is employed is smaller than the $\hat{\delta}_{Rx}$ obtained when the coarse resolution DVM is employed. However, the values $\hat{\delta}_{Rx}$ obtained when the coarse resolution DVM is used are small. Fig. 2b shows that for a given n , δ_{Rx} is better approximate by $\hat{\delta}_{Rx}$ when the fine resolution DVM is employed than when the coarse resolution DVM is employed.

Also, from Fig. 2a it can be observed that minimum $\hat{\delta}_{Rx}$ is obtained for $n = 1$.

Another conclusion which can be obtained from Fig. 2a is the fact that for a high n the R_x measurement accuracy obtained when the fine resolution DVM is employed can be smaller than the one obtained for a small n when the coarse resolution DVM is employed as detector.

The relationships used for the determination of the resistances R_2 , R_3 and R_4 are

$$R_2 = R'_x, \quad R_3 = nR'_x, \quad R_4 = nR'_x, \quad (14)$$

where R'_x is an approximate value of R_x , known *a priori* and n is an integer. R'_x must be rounded to the nearest integer for a proper measurement of the resistance R_x .

We consider that R_3 is variable and this is a decade resistance characterized by the smallest step increment $\Delta R_{3\min}$.

As it is precised above the minimum value of $\hat{\delta}_{R_x}$ is obtained for $n = 1$. However, from $R_3 = nR'_x$, to obtain a proper representation of the R_x value, *i.e.* to obtain a high number of significant digits, n must be higher than 1.

Based upon (9) it can be established

$$V_{cd} \cong \frac{nR'_x - R_3}{(n+1)(R'_x + R_3)} V_s. \quad (15)$$

When the resistance R_3 varies with $\pm \Delta R_{3\min}$ from the above relationship we obtain

$$V_{cd\pm} \cong \frac{nR'_x - R_3 \mp \Delta R_{3\min}}{(n+1)(R'_x + R_3 \pm \Delta R_{3\min})} V_s \quad (16)$$

or

$$V_{cd\pm} \cong V_{cd} \mp \frac{\Delta R_{3\min}}{(n+1)(R'_x + R_3)} V_s. \quad (17)$$

Based upon (14) the voltages $V_{cd\pm}$ can be approximated by

$$V_{cd\pm} \cong V_{cd} \mp \frac{\Delta R_{3\min}}{(n+1)^2 R'_x} V_s. \quad (18)$$

From the above approximation it can be deduced that for high n , $V_{cd\pm}$ can be smaller than 0.5 LSD. Thus, the balance condition can be meet for two or more values of the resistance R_3 yielding a R_x measurement affected by uncertainties.

Also, from (18) it can be deduced that for small n , $V_{cd\pm}$ can be much higher than 1 LSD. In these situations it is possible do not meet the balance condition.

Based upon the aforementioned observations it follows that when the second term of $V_{cd\pm}$ from the expression (18) (*i.e.* $\frac{\Delta R_{3\min}}{(n+1)^2 R'_x} V_s$) is close, but smaller than 1.5 LSD, a fine adjustment is obtained. This fine adjustment permits to meet the balance condition. To satisfy the above constraint the most suitable value of n is

$$\tilde{n} = \left\lceil \sqrt{\frac{\Delta R_{3\min}}{1.5 R'_x} \frac{V_s}{LSD}} - 1 \right\rceil, \quad (19)$$

where $[y^*]$ returns the largest integer smaller or equal to y .

The resistances R_2 , R_3 and R_4 are given by (14) in which $n = \tilde{n}$

$$R_2 = R'_x, \quad R_3 = \tilde{n}R'_x, \quad R_4 = \tilde{n}R'_x. \quad (20)$$

A proper R_x measurement supposes $\tilde{n} > 1$. From (19) it can be established that

$$R_x < \frac{\Delta R_{3\min}}{6} \frac{V_s}{1LSD}. \quad (21)$$

Thus, the maximum value of the resistance R_x up to which this can be measured, $R_{x\max}$, is given by

$$R_{x\max} = \frac{\Delta R_{3\min}}{6} \frac{V_s}{1LSD}. \quad (22)$$

For a given $\Delta R_{3\min}$, from (22) it follows that $R_{x\max}$ increases when the rapport $V_s/1 \text{ LSD}$ increases, *i.e.* when the supply voltage of the bridge increases or the DVM resolution decreases.

3. EXPERIMENTAL RESULTS

Three resistances of approximate 250 Ω , 570 Ω and 1200 Ω are measured. The resistances R_2 , R_3 and R_4 of the Wheatstone bridge are decade resistances. These resistances have six decades ($\times 0.1\Omega$, $\times 1\Omega$, $\times 10\Omega$, $\times 100\Omega$, $\times 1000\Omega$ and $\times 10000\Omega$), which permit to obtain a value between $0.1 \div 111111 \Omega$. Two DVMs were used as detector – the digital multimeters (DMMs) Beha 93447 and Voltcraft M-4650CR. Beha 93447 is a 3½-digits DMM and M-4650CR is 4½-digits DMM. These DMMs were used on the smallest measurement interval of c.c. voltage, 200 mV. In this measurement interval we have 1 LSD = 0.1 mV for Beha 93447 and 1 LSD = 0.01 mV for M-4650 CR.

The supply voltage of the Wheatstone bridge is $V_s = 8 \text{ V}$. For both detectors, all resistances which follow to be measured satisfy the condition (21). The resistors R_2 , R_3 and R_4 are calculated by the relationships (20). The values of the resistances R_3 , obtained when the balance condition is meet, R_{3b} , and the voltages V_{cd} obtained at the resistances $R_{3b} \pm \Delta R_{3\min}$, $V_{cdb\pm}$ are presented in Table 1 when the Beha 93447 is used as detector and in Table 2 when the M-4650CR is used as detector. The results are given for the $\tilde{n}$ values and, also for some n values situated around the $\tilde{n}$ values. When the balance condition is obtained for two or more resistances R_{3b} , V_{cdb-} corresponds to the minimum value of R_{3b} minus $\Delta R_{3\min}$ and V_{cdb+} corresponds to the maximum value of R_{3b} plus $\Delta R_{3\min}$. With the gray background of tables cells the results obtained for the $\tilde{n}$ values are indicated.

For all $\tilde{n}$ values the balance condition is meeting. Also, for all $\tilde{n}$ values, excepting the measurement of the resistance $R'_x = 250 \Omega$ when M-4650CR is used as detector the finest adjustment is obtained ($V_{cdb-} = -1\text{LSD}$ and $V_{cdb+} = 1 \text{ LSD}$).

Table 1

The results obtained with Beha 93447 as detector

R'_x [Ω]	n	R_{3b} [Ω]	V_{cdb-} [mV]	V_{cdb+} [mV]
250	1	the balance is not meet	–	–
	2	the balance is not meet	–	–
	3	803.6	–0.1	0.2
	4	1071.1 and 1071.2	–0.1	0.1
	5	1339.0, 1339.1 and 1339.2	–0.1	0.1
570	1	567.9	–0.4	0.2
	2	1135.8	–0.1	0.1
	3	1703.6 and 1703.7	–0.1	0.1
	4	2271.4, 2271.5, 2271.6 and 2271.7	–0.1	0.1
1200	1	1266.4	–0.1	0.1
	2	2432.5, 2532.6 and 2532.7	–0.1	0.1

All the resistances are also measured by means of the HP-3455A DMM. This is a 5½-digits DMM. The results obtained are: 267.98 Ω , 568.31 Ω and 1265.66 Ω . We assume that these are the true values of the measured resistances. Thus, the relative errors obtained in the measurement of R_x are smaller than 0.1%.

For a given R'_x , the values of R_x calculated by the formula (20) when both DMMs are used as detectors are very close. From this reason and due to the fact that the relative errors in the measurement of R_x are determined, in principal, by the relative errors of the resistances R_2 , R_3 and R_4 (see the relationship (7)), it follows, that to measure with high accuracy the resistance R_x , it is sufficient to use the Beha93447 as detector.

Table 2

The results obtained with M-4650CR as detector

R'_x [Ω]	n	R_{3b} [Ω]	V_{cdb-} [mV]	V_{cdb+} [mV]
250	10	2677.8	–0.02	0.03
	11	2945.7	–0.01	0.02
	12	3213.3	–0.01	0.01
	13	3481.1	–0.01	0.02
	14	3748.8	–0.01	0.01
	15	4015.9 and 4016.0	–0.01	0.02
	16	4283.8 and 4283.9	–0.01	0.01
570	5	2839.7	–0.04	0.03
	6	3407.5	–0.01	0.03
	7	3975.6	–0.01	0.02
	8	4542.7	–0.01	0.01
	9	5111.6 and 5111.7	–0.01	0.02
1200	3	3798.8	–0.04	0.03
	4	5065.2	–0.01	0.03
	5	6331.3	–0.01	0.01
	6	7597.1 and 7597.2	–0.01	0.02

4. CONCLUSION

In this paper has been shown that a resistance can be measured with high accuracy by means of a well-known Wheatstone bridge in which the detector is a DVM. The relationships for the determination of the resistances of Wheatstone bridge in order to obtain the balance condition and a fine adjustment are derived. The experimental results presented validate the derived relationships.

The errors which affect the measurement of the resistance R_x introduced by the DMM which is used as detector are very small and so are negligible compared with the resistances R_2 , R_3 and R_4 inaccuracies.

By using a DVM as detector, the voltage from the Wheatstone bridge measuring diagonal can be easily read and is more accurately than the one obtained when an analogic galvanometer is used as detector.

Received on 11 July 2007

REFERENCES

1. G. Gherman *et al.*, *Laboratory Exercises in Electrical Measurements* (in Romanian), "Politehnica" University of Timisoara Publishing House, pp. 129–138 (1972).
2. * * *, *Yokogawa, Precision Measuring Instruments*, available: <http://www.yokogawa.com/gmi/pdf/Bulletin/Bull-2700-E.pdf>
3. * * *, *The Measurement, Instrumentation, and Sensors Handbook*, CRC Press and IEEE Press, 1998, pp. 37.4–37.6.