EDDY-CURRENT INTEGRAL FORMULATION FOR ELECTROMAGNETIC FIELD AND FORCES COMPUTATION IN DOMAINS WITH PERMANENT MAGNETS, NONLINEAR MEDIA AND MOVING BODIES

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In applications that involve magnetizable media and moving bodies, the integral methods used for electromagnetic field computation have some major advantages in comparison with the differential methods. Polarization fixed point method allows the use of the eddy current integral formulation for computation of 3D electromagnetic field in nonlinear media. The integral equation numerical solution is obtained employing Galerkin procedures and using first order edge elements. By imposing the topological gauge condition the active unknowns are associated only with the cotree edges. A polyhedral mesh is used to discretize the polarization field. For the field of the current density a different discretization mesh can be employed. In domains with moving bodies only magnetizable and conductive media are discretized, their meshes remains unchanged and only the free term is reevaluated at each time step.

1. INTRODUCTION

The eddy current computation procedures are often based on finite element methods [1]. In this case the computation of problems with moving bodies encounters some difficulties. Most frequently, the rotation movement is taken into account (*e.g.* electrical machines) by employing a cylindrical separation surface for subdomains in relative motion located in the airgap area [2]. The mesh step on this surface is regular and it matches the adopted time step. For an arbitrary motion, the mesh should be deformed or rebuild and all the matrices have to be reevaluated at each time step.

In 1995, the famous professor C.W. Trowbridge presented the paper *Computing Electromagnetic Field for Research and Industry: major achievements*

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and future friends at the COMPUMAG'95 conference in Berlin and appreciated that the future research direction in computation of electromagnetic fields will use an integral approach in the treatment of the domains with rigid moving bodies. In 2D problems the eddy current integral formulation is a simple and efficient procedure for solving the electromagnetic field problem because we only have a scalar unknown [3]. 3D problems are more complicated because the current density is a vector unknown with zero divergence and the integral equation contains the gradient of an unknown scalar function. Albanese and Rubinacci [4] solved these two problems considering the edge elements shape functions $\boldsymbol{N}_i$ for electric potential vector $\boldsymbol{T}$. The current density can be written as

$$\boldsymbol{J} = \sum_{i=1}^{n} \alpha_i(t) \nabla \times \boldsymbol{N}_i \,, \tag{1}$$

which verifies zero convergence condition. The unknowns are associated only with the internal edges branches. The boundary condition $\boldsymbol{J} \cdot \boldsymbol{n} = 0$ is fulfilled by imposing zero values for the boundary edge elements. Projecting the integral equation on the functions $\nabla \times \boldsymbol{N}_k$, the component that contains the scalar function gradient vanishes. For domains with rigid bodies in motion the Faraday’s law in local coordinate systems is used, avoiding the term $\nabla \times (\boldsymbol{B} \times \boldsymbol{v})$ [5]. Unfortunately, the integral formulations for eddy current problems can be employed only for linear and homogeneous unbounded media.

The iterative polarization method [6, 7] replaces the nonlinear media with a fictive linear one and an additive term, having a magnetic polarization nature, which is corrected by the magnetic flux density at each iteration. The magnetic permeability remains unchanged during the iteration and can be chosen to be constant in the entire computing domain [8]. By using this method the integral formulations for eddy current problems can be extended to nonlinear media, the magnetic polarization contributing only to the right side term [3, 9–10].

This paper proposes an integral formulation for eddy current problems in 3D domains with permanent magnets, ferromagnetic media and moving bodies. The magnetic forces, that depend on the ferromagnetic media nonlinearities, permanent magnets and eddy current, are efficiently computed. A tetrahedral mesh and first order edge elements are used for the current density discretization. The active unknowns are only those associated with the internal cotree edges. The tree-cotree spanning starts from the boundary edges. The polarization field discretization is preformed using a polyhedral mesh (which may be different from the tetrahedral one of the current density) and volume elements. In each polyhedron an average value of magnetic flux is considered.

2. THE EDDY CURRENT INTEGRAL FORMULATION

The eddy current integral equation is

$$\rho \boldsymbol{J} + \frac{\mu_0}{4\pi}\frac{\mathrm{d}}{\mathrm{d}t}\int_{\Omega_C}\frac{\boldsymbol{J}}{r}\mathrm{d}v + \nabla V = -\frac{\mathrm{d}\boldsymbol{A}_0}{\mathrm{d}t}\ , \tag{2}$$

with $$\boldsymbol{A}_0 = \frac{\mu_0}{4\pi}\int_{\Omega_0}\frac{\boldsymbol{J}_0}{r}\mathrm{d}v + \frac{\mu_0}{4\pi}\int_{\Omega_F}\frac{\nabla\times\boldsymbol{M}_F}{r}\mathrm{d}v + \frac{\mu_0}{4\pi}\int_{\Omega_{M_0}}\frac{\nabla\times\boldsymbol{M}_0}{r}\mathrm{d}v\ , \tag{3}$$

where ρ is the resistivity, $\boldsymbol{J}_0$ is the imposed current density in the nonconductive domain Ω_0, $\boldsymbol{M}_0$ is the magnetization in the domains Ω_{M_0} with permanent magnets, $\boldsymbol{M}_F$ is the magnetization in the ferromagnetic domains Ω_F, which is iteratively corrected using the fixed point polarization method. The conductive domain boundary condition is given by $\boldsymbol{J}\cdot\boldsymbol{n} = 0$.

3. THE SPATIAL DISCRETIZATION AND THE SOLUTION OF THE INTEGRAL EQUATION

The electric vector potential $\boldsymbol{T}$ is written as a linear combination of $\boldsymbol{N}_i$ vector shape functions and the current density results as presented in (1). By projecting the equation (2) on the functions $\nabla\times\boldsymbol{N}_k$, the following system of equations is obtained

$$\{R\}\ [I] + \frac{\mathrm{d}\{L\}[I]}{\mathrm{d}t} = -\frac{\mathrm{d}}{\mathrm{d}t}[\Phi]\ , \tag{4}$$

where the entries of the matrices $\{R\}$, $\{L\}$ and $[\Phi]$ are

$$R_{ik} = \int_{\Omega}\rho\left(\nabla\times\boldsymbol{N}_i\right)\cdot\left(\nabla\times\boldsymbol{N}_k\right)\mathrm{d}v\ ,$$
$$L_{ik} = \frac{\mu_0}{4\pi}\int_{\Omega}\int_{\Omega}\frac{1}{r}\left(\nabla\times\boldsymbol{N}_i\right)\cdot\left(\nabla\times\boldsymbol{N}_k\right)\mathrm{d}v\mathrm{d}v\ , \tag{5}$$

$$\Phi_k = \int_{\Omega}\left(\nabla\times\boldsymbol{N}_k\right)\cdot\boldsymbol{A}_0\,\mathrm{d}v\ , \tag{6}$$

and the unknown term is $[I] = (\alpha_1, \alpha_2, \ldots, \alpha_n)^{\mathrm{T}}$. By projecting the equation (2) on the functions $\nabla \times \boldsymbol{N}_k$, the term containing ∇V vanishes.

4. THE COTREE EDGE ELEMENTS

The most convenient shape functions choice is that of edge elements [4]. We consider a tree-cotree spanning of the entire tetrahedral mesh [11]. It's obvious that the flux of ***J*** across any elementary facets is given by the line integral of the vector potential along the edges surrounding the facet. We can add any particular values to the tree edges if we add values on the cotree edges which verify Kirchhoff's second law. Therefore we nullify all tree edge values, and so, the uniqueness of the potential ***T*** along the edges is insured. As a result, the active unknowns are only the α_i terms that correspond to the inner cotree edge elements. The boundary condition is fulfilled by imposing zero value of the boundary edge elements. For a better conditioning of the equation system matrices it is recommended to begin the tree-cotree spanning with the boundary edges.

When the first order edge elements are used in the tetrahedral mesh, the entries of matrices $\{R\}$, $\{L\}$ from relation (5) become

$$R_{ik} = \sum_{p \in \{i\} \cap \{k\}} \left(\nabla \times \boldsymbol{N}_{i_p}\right) \cdot \left(\nabla \times \boldsymbol{N}_{k_p}\right) v_p \ , \tag{7}$$

where $p \in \{i\} \cap \{k\}$ is the index of the ω_p subdomain which contains the edges *i* and *k*, respectively, and $\nabla \times \boldsymbol{N}_{i_p}$ is the value of $\nabla \times \boldsymbol{N}_i$ in ω_p,

$$L_{ik} = \frac{\mu_0}{4\pi} \sum_{p \in \{i\}} \sum_{q \in \{k\}} \left(\nabla \times \boldsymbol{N}_{i_p}\right) \cdot \left(\nabla \times \boldsymbol{N}_{k_q}\right) \int_{\omega_p} \int_{\omega_q} \frac{1}{r} \mathrm{d}v_p \mathrm{d}v_q \ , \tag{8}$$

where $p \in \{i\}$ are the indexes of the subdomains containing the edge *i*. Using Gauss formulas the double volume integral in (8) becomes $-\frac{1}{2} \sum_{m \in \{p\}} \sum_{n \in \{q\}} \boldsymbol{n}_{pm} \boldsymbol{n}_{qn} \int_{\partial\omega_p} \int_{\partial\omega_q} r \, \mathrm{d}s_p \mathrm{d}s_q$ and it is numerical evaluated over subdomains facets.

The elements of the free vector $[\Phi]$ from (6) can be written as $\Phi_k = \Phi_{M_k} + \Phi_{J_k}$, where $\Phi_{M_k} = \sum_{l=1}^{n_M} \boldsymbol{a}_{kl} \boldsymbol{M}_l$, $\boldsymbol{M}_l$ being the magnetization in ferromagnetic and permanent magnet domains, considered to be constant over each subdomain ω_l, n_M the number of subdomains in magnetizable media,

$$\boldsymbol{a}_{kl} = \frac{\mu_0}{4\pi} \sum_{p\in\{k\}} \boldsymbol{a}'_{pl} \times \left(\nabla \times \boldsymbol{N}_{k_p}\right), \quad \boldsymbol{a}'_{pl} = \int_{\partial\omega_p} \boldsymbol{n}_p \int_{\partial\omega_l} \frac{\boldsymbol{n}_l \boldsymbol{r}}{r} \mathrm{d}s_p \mathrm{d}s_l , \tag{9}$$

and $\Phi_{J_k} = \sum_{z=1}^{n_0} \boldsymbol{b}_{kz} \boldsymbol{J}_{0_z}$, with $\boldsymbol{J}_0$ considered to be constant over each tetrahedral subdomain and $\boldsymbol{b}_{kz} = \frac{\mu_0}{8\pi} \sum_{k\in\{p\}} \left(\nabla \times \boldsymbol{N}_{k_p}\right) \int_{\partial\omega_p} \int_{\partial\omega_z} r\, \boldsymbol{n}_p \boldsymbol{n}_z \mathrm{d}s_p \mathrm{d}s_z$.

5. POLARIZATION FIXED POINT METHOD

The nonlinear relationship ***H*=F(*B*)** is replaced by $\boldsymbol{B} = \mu(\boldsymbol{H} + \boldsymbol{M}_F)$, where μ can be chosen to be the permeability of free space μ_0 and the magnetization $\boldsymbol{M}_F$ is iteratively corrected as a function of ***B***

$$\boldsymbol{M}_F = (\boldsymbol{B} - \mathbf{F}(\boldsymbol{B}))/\mu = \mathbf{G}(\boldsymbol{B}) . \tag{10}$$

For the spatial discretization of the magnetization a polyhedral mesh is defined in ferromagnetic bodies and volume elements are employed. If the ferromagnetic domain is also a conductive domain, we have two distinct discretization meshes: a tetrahedral one for the current density and a hexahedral one for the magnetization. Initially an arbitrarily value $\boldsymbol{M}^{(0)}$ is chosen. Assuming $\boldsymbol{M}^{(k)}$ has known value at iteration *k*, we can calculate the value for the current density from (4), and then calculate the average value for the flux density ***B*** using $\widetilde{\boldsymbol{B}} = \widetilde{\boldsymbol{B}}_J + \widetilde{\boldsymbol{B}}_M$, where $\widetilde{\boldsymbol{B}}_J$ is the average flux density due to eddy current density ***J*** and $\widetilde{\boldsymbol{B}}_M$ is the average flux density due to magnetization $\boldsymbol{M}_F$ in ferromagnetic domain and magnetization $\boldsymbol{M}_0$ in permanent magnets

$$\widetilde{\boldsymbol{B}}_{J_{ik}} = \frac{1}{v_i}\left(\beta_{ik} \times \boldsymbol{J}_k\right), \text{ with } \beta_{ik} = \frac{\mu_0}{4\pi} \int_{\partial\omega_i} \int_{\partial\omega_k} \frac{\left(\boldsymbol{n}_k \boldsymbol{r}_{ik}\right)\boldsymbol{n}_i}{r} \mathrm{d}s_k \mathrm{d}s_i , \tag{11}$$

$$\widetilde{\boldsymbol{B}}_{M_{il}} = \frac{1}{v_i} \overline{\overline{\gamma}}_{il} \boldsymbol{M}_l , \text{ with } \overline{\overline{\gamma}}_{il} = \frac{\mu_0}{4\pi} \int_{\partial\omega_i} \int_{\partial\omega_l} \frac{\left(\boldsymbol{n}_l \boldsymbol{n}_i\right)\overline{\overline{1}} - \left(\boldsymbol{n}_l ; \boldsymbol{n}_i\right)}{r} \mathrm{d}s_l \mathrm{d}s_i . \tag{12}$$

Magnetization $\boldsymbol{M}$ has a nonlinear dependence and it's iteratively corrected with (10). When $\left\| \boldsymbol{M}_F{}^{(n)} - \boldsymbol{M}_F{}^{(n-1)} \right\|$ is enough small, the convergence is achieved and we stop the iterative process.

6. ANALYSIS OF STRUCTURES WITH MOVING BODIES

The time discretization is performed using a Crank Nicholson method. Starting with the eddy current vector $[I_0]$ initialized to zero at time step t_0, we compute the entries of matrices $\{R\}$, $\{L\}$ and the free vector term $[\Phi]$ using the next iterative procedure:

Time step t_1: *the moving bodies come into* t_1 *time step position, velocity being known.*

a) The vector $[\Phi_1^{(0)}]$ is computed at t_1 time step, where $\boldsymbol{M}$ has the t_0 time step value $\boldsymbol{M}_1^{(0)}$. In ferromagnetic media we assume for simplicity zero initial value for magnetization $\boldsymbol{M}_{F_1}^{(0)} = 0$. The field sources are imposed current density and magnetization $\boldsymbol{M}_0$ from permanent magnets.

b) The vector $[I_1^{(1)}]$ is computed by solving the following system of equations

$$\left(\frac{\Delta t}{2} \{R\} + \{L\} \right) [I_1^{(1)}] = -[\Phi_1^{(0)}] + [\Phi_0] \ .$$

c) The average value of the flux density $\boldsymbol{B}_1^{(1)}$, magnetization values $\boldsymbol{M}_{F_1}^{(i)}$, $\boldsymbol{M}_1^{(1)}$, and the vector free term's new value $[\Phi_1^{(1)}]$ are computed.

This procedure is repeated, resulting $[I_1^{(2)}]$, $\boldsymbol{B}_1^{(2)}$, $\boldsymbol{M}_1^{(2)}$, $[\Phi_1^{(2)}]$. The iterative correction will run until convergence is achieved ($\left\| \boldsymbol{M}_F^{(n)} - \boldsymbol{M}_F^{(n-1)} \right\|$ is small enough), then we will move to the next time step.

Time step t_2: *the moving bodies come into* t_2 *time step position*.

a) The vector $[\Phi_2^{(0)}]$ is computed at t_2 time step, where $\boldsymbol{M}$ has the t_1 time step value $\boldsymbol{M}_2^{(0)} = \boldsymbol{M}_1^{(n)}$.

b) The vector $[I_2^{(1)}]$ is computed by solving the following system of equations

$$\left(\frac{\Delta t}{2}\{R\}+\{L\}\right)[I_2^{(1)}]=-\left(\frac{\Delta t}{2}\{R\}-\{L\}\right)[I_1^{(n)}]-[\Phi_2^{(0)}]+[\Phi_0^{(n)}] .$$

c) The average value for flux density $\boldsymbol{B}_2^{(1)}$, magnetization value $\boldsymbol{M}_2^{(1)}$, and the vector free term's new value $[\Phi_2^{(1)}]$ are computed.

The procedure is repeated, resulting $[I_2^{(2)}]$, $\boldsymbol{B}_2^{(2)}$, $\boldsymbol{M}_2^{(2)}$, $[\Phi_2^{(2)}]$. The iterative correction will run until convergence is achieved ($\left\|\boldsymbol{M}_F^{(n)}-\boldsymbol{M}_F^{(n-1)}\right\|$ is small enough), then we move to the next time step.

The presented iterations are repeated for all steps covering the studied time interval.

7. THE ELECTROMAGNETIC FORCES COMPUTATION

The force computation using the Maxwell stress tensor implies its integration over a closed surface Σ which surrounds the conductive domain Ω_C. The integrating surface Σ must be oriented to the outside domain on which the force is exerted and for the numerical approach this surface should be subdivided in a discretization mesh who can take any surface form.

The total force is given by $\boldsymbol{F}=\frac{1}{\mu_0}\oint_{\Sigma}\left[(\boldsymbol{n}\cdot\boldsymbol{B})\boldsymbol{B}-\boldsymbol{n}\frac{B^2}{2}\right]\mathrm{d}s$. The integral is numerically evaluated by taking into account the flux density $\boldsymbol{B}_i$ computed in the weight center of the surface Σ facets, $\boldsymbol{B}_i=\boldsymbol{B}_{J_i}+\boldsymbol{B}_{M_i}$, where $\boldsymbol{B}_{J_i}$ and $\boldsymbol{B}_{M_i}$ are the flux density given by the current density $\boldsymbol{J}$ and by the magnetization $\boldsymbol{M}$, respectively

$$\boldsymbol{B}_{J_i}=\sum_{k=1}^{n_J}\boldsymbol{\beta}_{ik}^{J}\times\boldsymbol{J}_k\ ,\quad \text{where}\quad \boldsymbol{\beta}_{ik}^{J}=-\frac{\mu_0}{4\pi}\int_{\partial\omega_k}\frac{\boldsymbol{n}_k}{r}\mathrm{d}s_k\ , \tag{13}$$

$$\boldsymbol{B}_{M_i}=\sum_{l=1}^{n_M}\overline{\overline{\gamma}}_{il}^{M}\boldsymbol{M}_l\ ,\quad \text{where}\quad \overline{\overline{\gamma}}_{il}^{M}=\frac{\mu_0}{4\pi}\int_{\partial\omega_l}\frac{(\boldsymbol{n}_l\cdot\boldsymbol{r})\overline{\overline{1}}-(\boldsymbol{n}_l;\boldsymbol{r})}{r}\mathrm{d}s_l\ . \tag{14}$$

8. **ILLUSTRATIVE EXAMPLE**

We study the problem of a moving permanent magnet over a nonferromagnetic conducting plate. The plate dimensions are $10^{-1}\times10^{-1}$ m, is placed in xOy plane and its thickness is $5\cdot10^{-3}$ m. The plate is made of aluminum with a resistivity of $2.6\cdot10^{-8}$ Ωm. Under the aluminum conductive plate is a nonconductive ferromagnetic plate, which has the same size as the conductive plate. Its ***B***-***H*** characteristic is presented in Fig. 1. The distance between the two plates is $5\cdot10^{-3}$ m. The permanent magnet dimensions are $5\cdot10^{-2}\times5\cdot10^{-2}$ m in xOy plane and the thickness is 10^{-1} m and it is moving with an imposed velocity v = 1.25 m/s along z direction, being driven away from the conductive plate. The magnet has an imposed magnetization M_0 = $7.95\cdot10^{5}$ A/m along the z direction. The entire computation domain will have a fictitious permeability chosen to be the permeability of free space μ_0.

The discretization meshes are shown in Fig. 2, where the conductive plate was divided in 450 tetrahedral elements resulting a total of 505 active edges, the ferromagnetic plate was divided in 72 hexahedrons and the permanent magnet is divided only on its surface as 90 rectangular subdomains. Using a time step of $\Delta t = 1$ ms the eddy current density field increases in time achieving its maximum value at 3 ms and then decreases with the movement of the permanent magnet. The eddy current density distribution acquired after 10 time steps is plotted in Fig. 3. The arrows give the size and direction of the density current vector in the weight centers of each tetrahedral element.

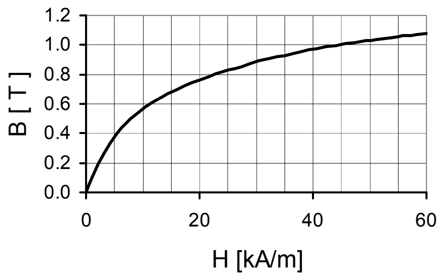

Fig. 1 – ***B***-***H*** characteristic of the ferromagnetic plate.

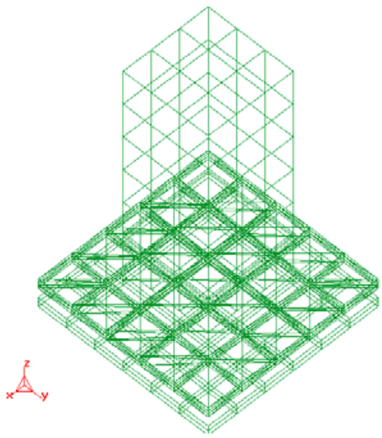

Fig. 2 – Tetrahedral, hexahedral and rectangular discretization mesh.

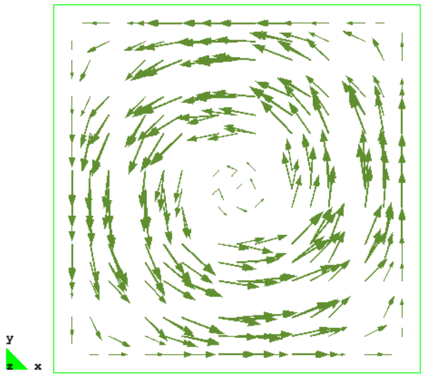

Fig. 3 – Eddy current distribution at t = 10 ms.

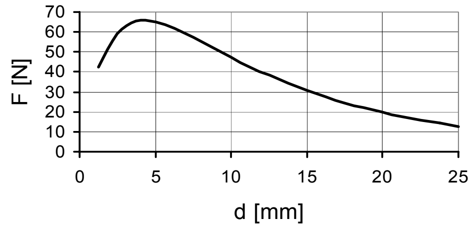

Fig. 4 – The force along z direction depending on the distance between conductive plate and permanent magnet.

The total force acting on the conductive plate is computed for each time step. The force components along x and y directions are negligible. The force along z direction reflects the permanent magnet attraction and the ferromagnetic plate rejection. The attraction force versus the distance between the conductive plate and the permanent magnet is presented in Fig. 4. The force of attraction between the conductive plate and permanent magnet has the same value but opposite direction to the repelling force between the conductive plate and ferromagnetic plate.

9. CONCLUSIONS

The analysis of the quasi-stationary electromagnetic field involves the computation of an eddy current problem which is very efficiently solved using integral methods. Such methods have become ever more popular in electromagnetic field computation, due to the following advantages: the small size of the system of equations, does not require the introduction of an artificial boundary and allows parallel programming. The polarization fixed point iterative method treatment of nonlinear media allows the use of eddy currents integral formulation. The matrices of the equation system remains unchanged during the iterative process, only free vector terms are reevaluated. The advantages become more obvious when the integral methods are used in problems that involve moving bodies or ferromagnetic media: only the conductive and ferromagnetic domains are meshed (no mesh in the air), during the movement some entries of the matrices of the equations system remains unchanged.

By imposing the topological gauge condition and tree-cotree spanning we obtain a substantial decrease in number of unknowns that is reflected in a small computation effort and usage of low memory resources. In order to reduce the computation time even more we use Gaussian formulas which have changed the volume-volume integral operator (that shows up in matrices and vector system of equations) in a surface-surface integral operator.

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REFERENCES

1. T. Nakata, N. Takahashi, K. Fujiwara, K. Muramatsu, *Investigation of effectiveness of various methods with different unknown variables for 3D eddy current analysis*, IEEE Trans. on Magn., **26**, *2*, pp. 442–445, 1990.
2. S. Wiliamson, E.K.C. Chan, *Three-dimensional finite-element formulation for problems involving time-varying fields, relative motion, and magnetic saturation*, IEE Proc. A, **140**, *2*, pp. 121–130, 1993.
3. I.R. Ciric, F.I. Hantila, M. Maricaru, S. Marinescu, *Efficient analysis of the solidification of moving ferromagnetic bodies with eddy-current control*, IEEE Trans. on Magn., **45**, *3*, pp. 1238–1241, 2009.
4. R. Albanese, G. Rubinacci, *Integral formulation for 3D eddy-current computation using edge elements*, IEE Proc. A, **135**, *7*, pp. 457–462, 1988.
5. G. Preda, F. Hănţilă, *Integral equation for 3-D eddy current in moving bodies*, Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., **43**, *3*, pp. 301–306, 1998.
6. F. I. Hănţilă, *A method for solving stationary magnetic field in nonlinear media*, Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., **20**, *3*, pp. 397–407, 1975.
7. F. Hantila, G. Preda, M. Vasiliu, *Polarization method for static fields*, IEEE Trans. on Magn., **36**, *4*, pp. 672–675, 2000.
8. F. I. Hănţilă, *Mathematical models of the relation between B and H for nonlinear media*, Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., **19**, *3*, pp. 429–448, 1974.
9. R. Albanese, F. Hănţilă, G. Rubinacci, *Eddy current integral formulation for nonlinear media*, Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., **40**, *2*, pp. 151–158, 1995.
10. R. Albanese, F. Hănţilă, G. Preda, G. Rubinacci, *Integral formulation for 3-D eddy current computation in ferromagnetic moving bodies*, Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., **41**, *4*, pp. 421–429, 1996.
11. F. Hănţilă, G. Preda, M. Vasiliu, T. Leuca, E. Della Giacomo, *Calculul numeric al curenţilor turbionari*, Editura ICPE, 2001.