Rev. Roum. Sci. Techn.– Électrotechn. et Énerg.
Vol. **62**, *2*, pp. 154–158, Bucarest, 2017

CYLINDRICAL INDUCTION HEATER – THE QUALITY FACTOR

ROMAN KIEROŃSKI

Key words: Electroheat. Induction heating. Parameters of induction furnaces and heaters. Crucible furnace.

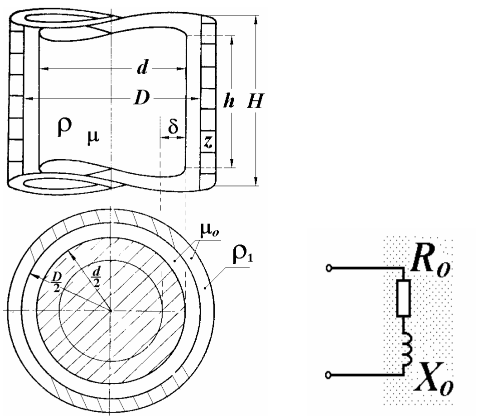
Derived formulas on quality factor cylindrical furnaces and induction heaters. In order to simplify the formulas introduced without dimensional relative energy absorption coefficient of the electromagnetic field relative to copper at 20°C. The values of this coefficient are summarized in the table as a function of temperature for the commonly used metals. Received simple formulas are additionally useful for rapid implementation of initial engineering projects and for teaching. Waveforms plotted efficiency. quality factor and compared with the values appearing in the literature.

1. INTRODUCTION

Determination of parameters of induction heaters and furnaces based on physical data is often necessary when dealing with the subject of induction heating. The article derived simple formulas for often used cylindrical induction heater. Designs can also be applied to cylindrical crucible induction furnaces.

2. DETERMINATION OF THE PARAMETERS ON THE BASIS OF THE PHYSICAL

On the basis of the physical data of the induction heater that is its geometric dimensions and material properties. can calculate the various parameters. *e.g.* resistance. reactance and therefore the power factor. quality factor. The article derived simple designs and are more accurate than those given in the literature [1] and [2].



a) b)

Fig. 1 – Construction (a) and an electric equivalent circuit (b) of the cylindrical induction heater inductor-charge system in diameters D, d and heights H, h, with the: – z coils inductor and the penetration depth δ to charge; μ, μ_0 – magnetic permeability, ρ, ρ_1 – resistivity, equivalent resistance R_o and equivalent reactance X_o.

Considered cylindrical induction heater has a geometrical dimensions and material properties of inductor_and charge as indicated in Fig. 1.

For clarity where it was possible to introduced simple without index codes (like Liwiński in [3]) in such a way that the large letters refer to the inductor and small to charge.

Equivalent resistance R_o and reactance X_o (fig. 1) also depend on the frequency *f*. The heater is supplied by generator values, *i*, u_o frequency *f*, as shown in schematic block-diagram of Fig. 2.

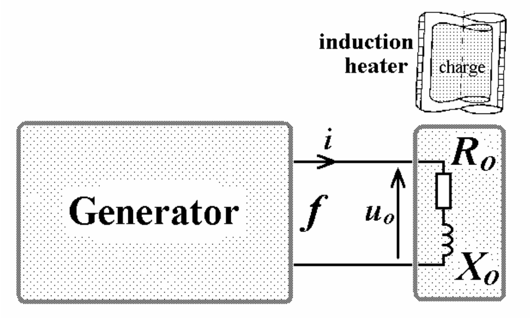


Fig. 2 – The block-diagram scheme of the supply induction heater with generator of frequency *f*.

The most useful is to calculate the electrical parameters of the heater resistance R_o and reactance X_o to determine the equivalent circuit (Fig. 1b) and the quality factor Q

$$Q = \frac{X_o}{R_o}. \tag{1}$$

The theoretical calculation is very much dependent formulas derived from only one parameter quality factor Q because they focus on themselves at the same time have two parameters R_o and X_o. Electrothermal literature [3–10] are often served formulas on R_o and X_o. Therefore, this article is derived formulas for the Q factor.

The most common formula is given in the literature on resistance R_o (Fig. 1):

$$R_o = R_1 + R_w = \frac{R_w}{\eta} = \pi z^2 \frac{\rho d}{h\delta} \frac{F_r K_r}{\eta}. \tag{2}$$

where: R_1 – resistance of the inductor; R_w – resistance of the charge; F_r – charge cylindrical shape coefficient (Fig. 3); K_r – correction coefficient (Fig. 4); η – electrical efficiency of the induction heating system (4).

Reactance X_o (Fig. 1 and Fig. 2):

$$X_o = \pi z^2 \left[\frac{\omega_o \mu_o}{4} \left(\frac{D^2}{H} K_N - \frac{d^2}{h} K_x \right) + \frac{\rho d}{h\delta} F_x K_x \right]. \tag{3}$$

where: F_x – charge cylindrical shape coefficient (Fig. 3); K_x – correction coefficient (Fig. 4); K_N – Nagaoka coefficient (Fig. 5); $\omega_o = 2\pi f$ – the pulsation.

The coefficients for the charge cylindrical given in formulas (2) and (3), respectively shape F_r, F_x, correction K_r, K_x and Nagaoka K_N are drawings in Figs. 3 to 5.

The shape coefficients F_r, F_x, of the cylindrical charge is

AGH Akademia Górniczo-Hutnicza w Krakowie, Poland, E-mail: kieronsk@kaniup.agh.edu.pl

plotted as a function $\frac{d}{\delta}$ (based on the literature [3, 5, 9]) in Fig. 3.

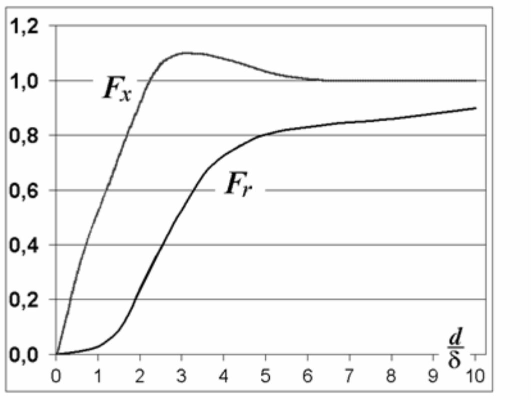


Fig. 3 – The shape coefficients F_r and F_x of the cylindrical charge as a function $\frac{d}{\delta}$.

Correction coefficients K_r, K_x of charge as a function $\frac{d}{h}$ of various parameters $\frac{D}{d}$ determined by [3] are shown in Fig. 4.

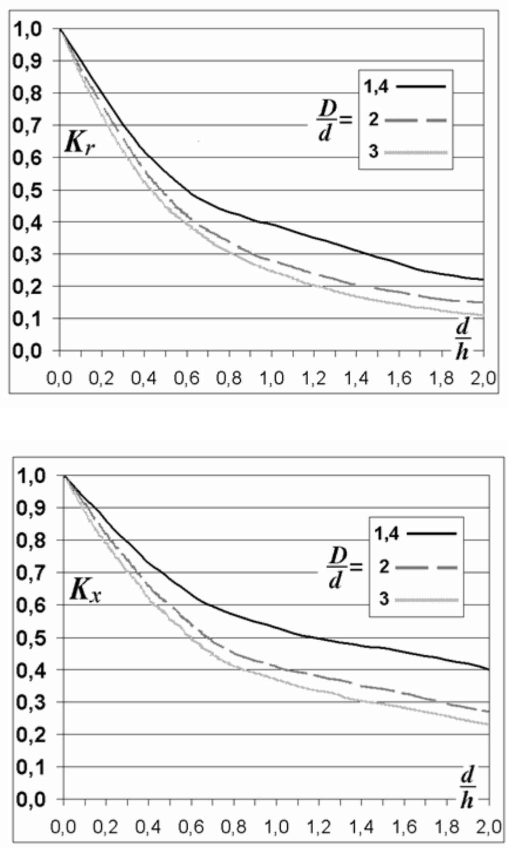


Fig. 4 – Correction coefficients K_r, K_x of charge as a function $\frac{d}{h}$ of various parameters $\frac{D}{d}$.

Nagaoka coefficient K_N as a function $\frac{D}{H}$ by [5] is shown in Fig. 5.

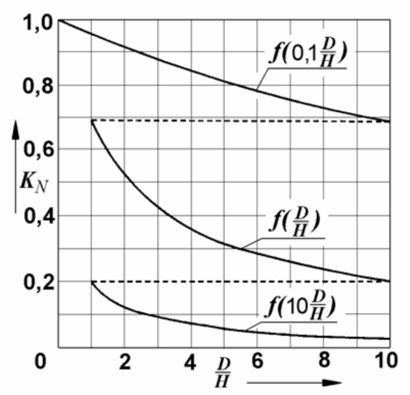


Fig. 5 – Nagaoka coefficient K_N for inductor cylindrical on the basis of [5].

3. EFFICIENCY

Electrical efficiency η of the induction heating system according to [5–9] show a formula:

$$\eta = \frac{1}{1 + \frac{1}{F_r K_r k_w} \frac{HD}{hd} \sqrt{\frac{\rho_1}{\mu_r \rho}}} = \frac{1}{1 + \frac{1}{F_r K_r k_w} \frac{HD}{hd} \frac{\sqrt{\rho_1}}{k_p}}, \quad (4)$$

where: $k_p = \sqrt{\rho \mu_r}$ is the coefficient of the energy absorption of the electromagnetic field, k_w – the fill coefficient of the winding.

The literature is rare designation $k_p = \sqrt{\rho \mu_r}$ called energy absorption coefficient of the electromagnetic field according to [9], because it has a dimension and is uncomfortable range of values. Therefore introduced without dimensional designation $k_{pr} = \sqrt{\rho_r \mu_r}$ – relative energy absorption coefficient of the electromagnetic field by introducing the relative resistivity $\rho_r = \frac{\rho}{\rho_{Cu20}}$ (proposed in relation to copper at 20° C). The values of this coefficient k_{pr} for the most commonly used metals are calculated in Table 1 by [11].

Table 1

Relative coefficient k_{pr} of electromagnetic field energy absorption as a function of temperature for various metals with respect to the copper at 20°C

Temperature	Coefficient k_{pr} for various metals						
°C	Cu	Al	Ag	Au	Sn	Steel * $\mu_r = 20$	Steel * $\mu_r = 100$
0	0.93	1.22	0.91	1.13	2.51	10.38	23.20
20	1.00	1.28	0.95	1.17	2.51	10.58	23.66
100	1.12	1.45	1.08	1.39	3.03	13.13	29.35
200	1.27	1.65	1.23	1.56	3.25	16.33	36.51
300	1.42	1.83	1.36	1.74	**5.29**	19.65	43.94
400	1.55	2.01	1.49	1.90	**5.37**	23.09	51.64
500	1.67	2.19	1.61	2.08	**5.49**	26.46	59.16
600	1.84	2.35	1.71	2.21	**5.61**	29.81	66.67
700	1.88	**3.94**	1.81	2.37	**5.71**	33.98	75.98
800	1.99	**4.06**	1.90	2.52	**5.83**	**8.16**	**8.16**
900	2.08	**4.18**	1.99	2.65	**5.92**	**8.26**	**8.26**
1000	2.16	**4.25**	**3.28**	2.79	**6.03**	**8.26**	**8.26**
1100	**3.38**	**4.34**	**3.39**	**4.61**	**6.13**	**8.42**	**8.42**

(table continuated)

1200	**3.44**	**4.44**	**3.49**	**4.71**	**6.24**	**8.47**	**8.47**
Melting point °C	**1083**	**660**	**960**	**1063**	**232**	**~ 1500**	**~ 1500**

* Steel with low carbon content

Table 1 shows that the steel of course. has the largest value of the coefficient due $\mu >> 1$. However, above the Curie point above approx. 750°C according to [10] after the loss of the magnetic properties of other metals (particularly in a liquid state) absorb energy nearly as steel. Data indicated in bold: for metals in liquid and for steel when the loss of magnetic properties.

Take into account that the inductor is a little warmer even though it is cooled and has up to 60°C. Then expression in the formula (4) is a $\sqrt{\mu_r \frac{\rho}{\rho_1}} = \frac{7}{8} k_{pr}$ and by giving the winding fill coefficient $k_w = \frac{8}{10}$ and assuming $H = h$ a formula efficiency:

$$\eta = \frac{1}{1 + \frac{1}{F_r K_r} \cdot \frac{1}{\frac{8}{10}} \cdot \frac{D}{d} \cdot \frac{1}{\frac{7}{8} \cdot k_{pr}}} = \frac{1}{1 + \frac{1}{F_r K_r} \cdot \frac{D}{d} \cdot \frac{1}{0.7 \cdot k_{pr}}} \qquad (5)$$

which waveforms are drawing in Fig. 6 as a function $\frac{d}{h}$ of various values parameter k_{pr}, assuming values $F_r = 0.9$ (Fig. 3) and $\frac{D}{d} = 1.4$.

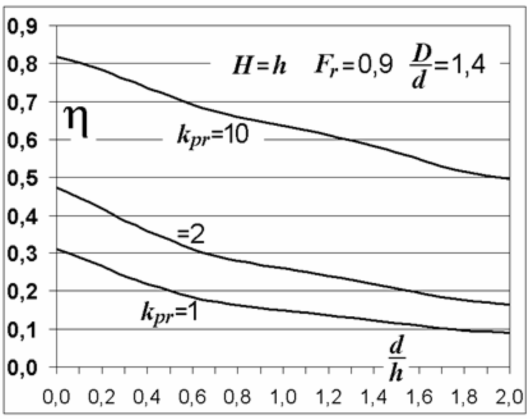


Fig. 6 – Efficiency of heater as a function $\frac{d}{h}$ of various values k_{pr}, assuming values $F_r = 0.9$ and $\frac{D}{d} = 1.4$.

4. QUALITY FACTOR *Q*

After insertion of (2) and (3) to (1) and continues to $H = h$ is derived formula on Q factor induction heater:

$$Q = \frac{\frac{\omega_o \mu_o}{4}\left(\frac{D^2}{H} K_N - \frac{d^2}{h} K_x\right) + \frac{\rho d}{h\delta} F_x K_x}{\frac{\rho d}{h\delta} \frac{F_r K_r}{\eta}} = \frac{\frac{\omega_o \mu_o}{4}\left(D^2 K_N - d^2 K_x\right) + \frac{\rho d}{\delta} F_x K_x}{\frac{\rho d}{\delta} \frac{F_r K_r}{\eta}}. \qquad (6)$$

On the basis of the conversion formula (5) have the relationship

$$\frac{F_r K_r}{\eta} = F_r K_r + \frac{D}{d} \cdot \frac{1}{0.7 \cdot k_{pr}}. \qquad (7)$$

After inserting the expression (7) to (6) is obtained by the formula

$$Q = \frac{\frac{\omega_o \mu_o}{4}\left(D^2 K_N - d^2 K_x\right) + \frac{\rho d}{\delta} F_x K_x}{\frac{\rho d}{\delta}\left(F_r K_r + \frac{D}{d} \cdot \frac{1}{0.7 \cdot k_{pr}}\right)} = \frac{\frac{\delta \omega_o \mu_o \mu_r}{4 \rho \mu_r}\left(D^2 K_N - d^2 K_x\right) + d F_x K_x}{d\left(F_r K_r + \frac{D}{d} \cdot \frac{1}{0.7 \cdot k_{pr}}\right)}. \qquad (8)$$

After inserting $\delta^2 = \frac{2\rho}{\omega_o \mu_o \mu_r}$ into the formula (8) was obtained:

$$Q = \frac{\frac{\frac{d}{\delta}}{2\mu_r}\left[\left(\frac{D}{d}\right)^2 K_N - K_x\right] + F_x K_x}{F_r K_r + \frac{D}{d} \cdot \frac{1}{0.7 \cdot k_{pr}}}. \qquad (9)$$

or

$$Q = \frac{\frac{1}{\mu_r} \frac{d}{\delta}\left[\left(\frac{D}{d}\right)^2 - \frac{K_x}{K_N}\right] + 2 \cdot F_x \frac{K_x}{K_N}}{2 \cdot \left(F_r + \frac{D}{d} \cdot \frac{1}{0.7 K_r \cdot k_{pr}}\right) \frac{K_r}{K_N}}. \qquad (10)$$

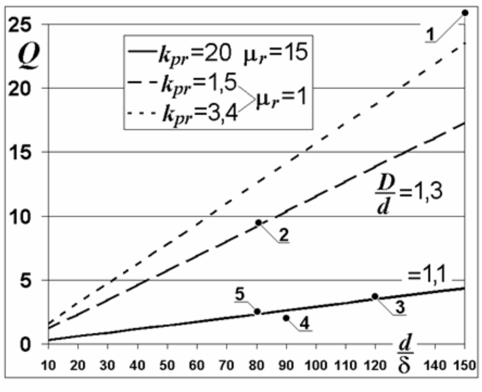


Fig. 7 – Waveforms the Q factor of heater as a function $\frac{d}{\delta}$ of various parameters k_{pr}, μ_r, $\frac{D}{d}$.

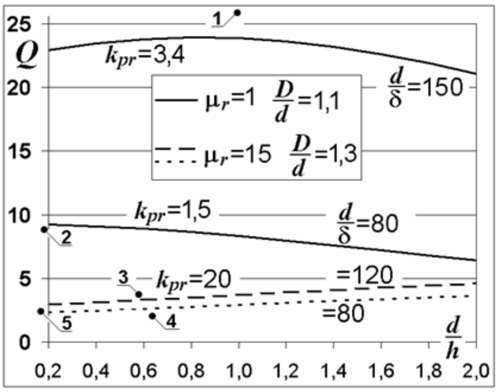


Fig. 8 – Waveforms the Q factor of heater as a function $\frac{d}{h}$ of various parameters k_{pr}, μ_r, $\frac{D}{d}$ and $\frac{d}{\delta}$.

For a rough calculations by [2] can be coefficients with formula (10) approximated:

$$K_r \approx \exp\left(-\frac{d}{h}\right)$$
$$\frac{K_r}{K_N} \approx \exp\left(-\frac{d}{2h}\right) \tag{11}$$
$$\frac{K_x}{K_N} \approx 0.9.$$

Waveforms example of the formulas (9) and (10) are drawing in Fig. 7 as a function $\frac{d}{\delta}$ of various values parameters k_{pr}, μ_r, $\frac{D}{d}$.

It is preferred to operate with low values $\frac{d}{\delta}$ of those shown in Fig. 7. Waveforms the Q factor as a function $\frac{d}{h}$ of various values k_{pr}, μ_r, $\frac{D}{d}$ and $\frac{d}{\delta}$ plotted in Fig. 8.

From Fig. 8 can be seen that the values little change for the analyzed range $\frac{d}{h}$ and that the smallest quality factor Q are present at high values $\mu_r = 15$.

Often there is a chance that this formulas can be simplified when $F_r = 1$. $F_x = 1$ for $\frac{d}{\delta} > 10$ (Fig. 3) and after inserting formulas (11) to equation (10) was obtained:

$$Q = \frac{\frac{1}{2\mu_r}\frac{d}{\delta}\cdot\left[\left(\frac{D}{d}\right)^2 - 0.9\right] + 0.9}{\frac{\frac{D}{d}}{0.7\cdot k_{pr}}\cdot\exp\left(+\frac{d}{2h}\right) + \exp\left(-\frac{d}{2h}\right)}. \tag{12}$$

From formula (12) further shows that heater quality factor Q decreases with the increase coefficient μ_r and increases with increasing $\frac{d}{\delta}$ and $\frac{D}{d}$ and depending on the charge size $\frac{d}{h}$ for the analyzed range $\frac{d}{h} \in (0, 2)$.

5. VERIFICATION OF THE RESULTS

The calculated Q factor are compared in the graphs (Fig. 7–8) with the calculated examples in the literature [12–15] (Table 2) based on measurements of electrical quantities (no mechanical dimensions). Points 1–5 in Figs. 7–8 results show good agreement with the given literature.

Table 2

Results verification of the data and the dimensions of figures 1 and 2

Charge	f	d	D	$H=h$	$\frac{D}{d}$	$\frac{d}{h}$	$\frac{d}{\delta}$	Q	Point Fig. 7–8	Literature
	kHz	mm	mm	mm					nr	
Cu	1	1100	1220	1100	1.11	1.00	150	26.32	1	[12]
Cu	1000	7.5	8.5	40	1.13	0.19	80	9.43	2	[13]
Stal	100	35	45	60	1.29	0.58	120	3.57	3	[13]
Stal	100	25	30	40	1.20	0.63	90	1.95	4	[14]
Stal	500	10	13	60	1.30	0.17	80	2.45	5	[15]

6. SPECIAL CASES

When there is no charge in the inductor that is $\frac{D}{d} \to \infty$ obtained by

$$Q \approx \frac{1}{2}\frac{D}{\delta_1}. \tag{13}$$

where δ_1–penetration depth of the wave to the inductor.

This is the basic formula for Q factor heater without load. It also follows from the primary literature [3–10].

When the load is a ferromagnetic charge and at the best size for heating: $\frac{D}{d} \to 1$, $\frac{d}{h} \to 0$, Q factor depends mainly on charge and formula (10) simplifies to:

$$Q \approx \frac{F_x K_x}{F_r K_r} \approx \frac{F_x}{F_r}. \tag{14}$$

This formula is as in basic literature [3–10] to the Q factor of an electromagnetic wave incident on the conductive material.

Verification of the results shows that in practice the most common occurs $\frac{d}{\delta} > 20$. Then for charges of ferromagnetic this is often true $\frac{1}{\mu_r}\frac{d}{\delta} \approx 2$ that when inserted into equation (12) gives the result

$$Q \approx \frac{\left(\frac{D}{d}\right)^2}{\frac{\frac{D}{d}}{0.7\cdot k_{pr}}\cdot\exp\left(\frac{d}{2h}\right) + \exp\left(-\frac{d}{2h}\right)}. \tag{15}$$

Especially for thin ferromagnetic charges ($\frac{D}{d} > 5$) can occur: $\frac{\frac{D}{d}}{0.7 \cdot k_{pr}} \approx 1$ and then equation (15) is the easiest:

$$Q \approx \frac{\left(\frac{D}{d}\right)^2}{2 \cdot \cosh\left(\frac{d}{2h}\right)}. \quad (16)$$

Special cases are simple formulas (13–16) and (13–14). are the same as in the electroheat primary literature [3–10].

7. CONCLUSIONS

The derived formulas are simpler than those available in the literature (for example [11, 16–21] and algorithms in [22, 23]). Some of them are somewhat simplified but correct to make quick calculations and preliminary engineering projects. In addition have to the value of teaching for teachers and students because of the formula can be seen directly as efficiency and quality factor depends on the dimensions and coefficients induction heater with Fig. 1. and the frequency of its supply by Fig.2.

Collected values efficiencies and quality factor generally agree with those administered in the literature concerning induction heating (*e.g.* [5, 9]).

Verification of results and special cases confirm the correct derivation of formulas for Q factor.

Received on May 5, 2016

REFERENCES

1. R. Kieroński, *Uproszczenie wzorów na parametry cylindrycznych pieców i nagrzewnic indukcyjnych zasilanych z falownika*. Elektrotechnika i Elektronika, **26**, *1–2*, pp. 37–39 (2007).
2. R. Kieroński, *Nagrzewnica indukcyjna cylindryczna, wzory na parametr tłumienia i dobroć*, Przegląd Elektrotechniczny, **91**, *7*, pp. 101–105 (2015).
3. W. Liwiński, *Nagrzewnice indukcyjne*, WNT, 1968.
4. J. Hauser, *Elektrotechnika*, *Podstawy elektrotermii i techniki świetlnej*, Wyd, Politechniki Poznańskiej, 2006.
5. M. Hering, *Podstawy elektrotermii cz*, II, WNT, 1998.
6. M. Hering, *Poradnik inżyniera elektryka*. *T.* 1. *Elektrotermia*, WNT, 2009.
7. P. Novak, *Elektricke Teplo*, Vydala Equilibria, 2010.
8. T. Rodacki, A. Kandyba, *Urządzenia elektrotermiczne*, Wydawnictwo Politechniki Śląskiej, 2002.
9. C. Sajdak, E. Samek, *Nagrzewanie indukcyjne*, Wyd, Śląsk, 1985.
10. J. Turowski, *Elektrodynamika techniczna*, WNT, 1968.
11. R. Kieroński, *Nagrzewnica indukcyjna prostopadłościenna. Sprawność. wzory na parametr tłumienia i dobroć*, Przegląd Elektrotechniczny, **91**, *8*, pp. 176–179 (2015).
12. C. Sajdak, J. Barglik, L. Pieszczek, R. Przyłucki, *Wspomaganie komputerowe projektowania pieców indukcyjnych tyglowych do topienia metali nieżelaznych i stopów żelaza*, Konferencja Wspomaganie komputerowe projektowania urządzeń elektrotermicznych, Politechnika Śląska, Szczyrk, 1994, pp. 176–182.
13. R. Kieroński, *Analiza pracy i sposób sterowania dwułącznikowego falownika napięcia do nagrzewania indukcyjnego*, Doktorat, Kraków, 2012.
14. B. Grzesik, Z. Kaczmarczyk, M. Kasprzak, *Falowniki tranzystorowe o częstotliwości powyżej 100 kHz do nagrzewania indukcyjnego*, Konferencja Naukowo-Techniczna Energoelektronika – Energooszczędność, Politechnika Śląska, Gliwice, 1995.
15. A. Skała, *Falownik w.cz. klasy E do zastosowania w procesach elektrotermicznych*, Doktorat, Kraków, 2014.
16. K. Kurek, M. Niklewicz, *Ocena przydatności nagrzewania indukcyjnego w procesie hartowania kół zębatych*, Przegląd Elektrotechniczny, **7**, pp. 86–88 (2008).
17. K. Kurek, A. Smalcerz, *Badanie ekranów wielowarstwowych tłumiących pole magnetyczne wokół nagrzewnic indukcyjnych*. Przegląd Elektrotechniczny, **84**, *7*, pp. 89–91 (2008).
18. R. Przyłucki, A. Kachel, *Model obliczeniowy indukcyjnego układu grzejnego z uwzględnieniem radiacyjnej wymiany ciepła*, Przegląd Elektrotechniczny, **84**, *7*, pp. 94–96 (2008).
19. M. Wciślik, T. Kwaśniewski, *Indukcyjność cewki jednowarstwowej. Wzory analityczne*, Przegląd Elektrotechniczny, **84**, *7*, pp. 103–106 (2008).
20. J. Barglik, T. Wieczorek, A. Smalcerz. *Modeling of the surface induction hardening process*, Przegląd Elektrotechniczny, **90**, *2*, pp. 1–4 (2014).
21. M. Wesołowski, R. Niedbała, *Dwuwzbudnikowa nagrzewnica indukcyjna do krystalizacji metodą PVT – koncepcja i symulacja*. Przegląd Elektrotechniczny, **90**, *2*, pp. 36–39 (2014).
22. T. Leuca, Ş. Nagy, N.D. Trip, H. Silaghi, C. Mich-Vancea, *Optimal design for induction heating using genetic algorithms*, Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., **60**, *2*, pp. 133–142 (2015).
23. T. Leuca, Ş. Nagy, N.D. Trip, H. Silaghi, A. Burca, *Design of experiments approach for induction heating optimization*, Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., **61**, *2*, pp. 169–172 (2016).