LIQUID LEVEL CONTROL FOR INDUSTRIAL THREE TANKS SYSTEM BASED ON SLIDING MODE CONTROL

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Key words: Sliding mode control (SMC), Proportional integral differential (PID), Sliding surface, Coupled tank system, Level control.

Liquid level control in tanks is essential in petrochemical, paper, pharmaceutics and other industries; therefore, several papers on twin-tank systems consisting of two tanks coupled by a pipe have been published in the last years. In this paper, a three tanks system is investigated. At first, the authors develop a new non-linear multiple input, single output (MISO) state model of such a plant and determine its transfer function; a sliding mode controller (SMC) for liquid level control is designed and an interesting simulation in MATLAB\SIMULINK for testing the non-linear three coupled tank system controlled in two different ways is created. The performance of the system with a sliding mode controller is compared with the performance of the system using a PID controller.

1. INTRODUCTION

In petrochemical, paper, pharmaceutics and other industries processing liquids, the fluid is stored in tanks and transferred to other tanks; it is essential to maintain the liquid level range at desired values throughout the entire process. Therefore the liquid level must be controlled in a proficient way.

The twin tank system consisting of two tanks coupled by a pipe has been studied by several authors. In [1–4] the liquid is supplied to the first tank and is evacuated from the second; in [5] both tanks are supplied by pumps, but the liquid is evacuated from one of them; in [6–8] the two tanks are supplied and the liquid is evacuated from both of them. A system consisting in three cascade reservoirs is studied in [9].

In this paper, a three coupled tank system is investigated: two of them are supplied by pumps and are coupled to a third one from which the liquid is evacuated.

The tank systems control methods are different. A PI controller is used in [5], a PID in [4], in [9] the proposed solution is based on the state decoupling, but the most suitable solution is considered the sliding mode control (SMC) [1–3, 6, 7].

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The SMC has been proposed and developed by V.I. Utkin and his collaborators in [10] and other works; the idea seduced many specialists and was used in different fields [11–13].

In chapter 2, the coupled tanks plant is described, its mathematical model is developed in chapter 3, in chapter 4 the controller is designed, the simulation results are presented in chapter 5 and chapter 6 contains the paper conclusions.

2. **SYSTEM DESCRIPTION**

The system consists of three coupled tanks as shown in Fig. 1. The tanks 1 and 2 are supplied by two pumps controlled by voltages $v_1(t)$ and $v_2(t)$ and the third is supplied from tanks 1 and 2 by two conduits (pipes) of cross sectional areas a_1 and a_2. Through a third pipe of cross sectional area a_3, the liquid flows out. Manual valves are available between tanks 1 and 2 and the tank 3 and at the output of tank 3. Therefore, the effective areas of the conduits are $\beta_1 a_1$, $\beta_2 a_2$, $\beta_3 a_3$.

The liquid level in every tank is influenced by the input and output flows but, also, by the liquid level in the other tanks.

The main objective is to reach the desired level in tank 3 by controlling the input rate in tanks 1 and 2, where:

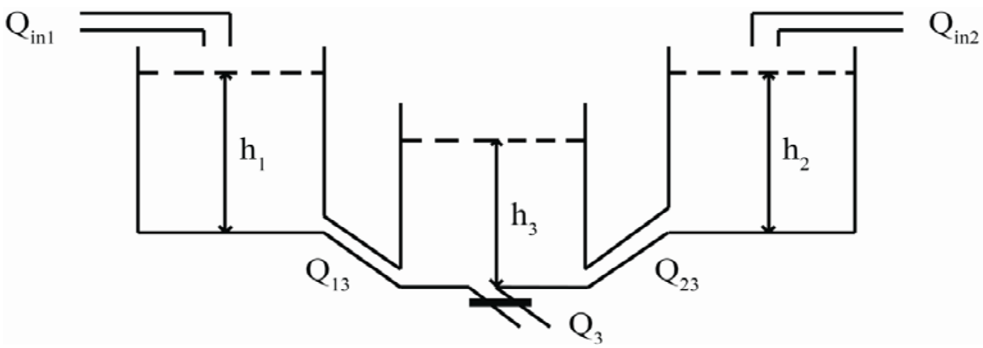


Fig. 1 – Schematic of the industrial three coupled tanks system.

– h_1, h_2, and h_3 are the liquid heights in tanks 1, 2 and 3;
– A_1, A_2, A_3, are the cross sectional areas of the tanks;
– Q_3 is the outflow from tank 3;
– Q_{13} and Q_{23} are the flows from tanks 1 and 2 to tank 3;
– Q_{in1} and Q_{in2} are the inflows of tanks 1 and 2; $Q_{in1}=k \times v_1(t)$, $Q_{in2}=k \times v_2(t)$, k is the gain of the two pumps.

3. MATHEMATICAL MODEL OF THE THREE COUPLED TANKS SYSTEM

Assuming that the liquid used is steady and non-viscous, the volume balance equation commonly used in hydrodynamics gives for the three tanks:

$$\begin{aligned}
\frac{\mathrm{d}h_1}{\mathrm{d}t} &= \frac{Q_{in} - Q_{13}}{A_1}, \\
\frac{\mathrm{d}h_2}{\mathrm{d}t} &= \frac{Q_{in} - Q_{23}}{A_2}, \\
\frac{\mathrm{d}h_3}{\mathrm{d}t} &= \frac{Q_{13} + Q_{23} - Q_3}{A_3}.
\end{aligned} \tag{1}$$

Due to the liquid characteristics mentioned above, the Bernoulli's equation can be used to get a set of non-linear equations:

$$\begin{aligned}
Q_{12} &= \beta_1 a_1 \sqrt{2g\left(h_1(t) - h_3(t)\right)}, \\
Q_{23} &= \beta_2 a_2 \sqrt{2g\left(h_2(t) - h_3(t)\right)}, \\
Q_3 &= \beta_3 a_3 \sqrt{2g h_3(t)}.
\end{aligned} \tag{2}$$

Introducing the expressions (2) in (1), the following equations are obtained:

$$\begin{aligned}
\frac{\mathrm{d}h_1}{\mathrm{d}t} &= \frac{k \times u(t) - \beta_1 a_1 \sqrt{2g\left(h_1(t) - h_3(t)\right)}}{A_1}, \\
\frac{\mathrm{d}h_2}{\mathrm{d}t} &= \frac{k \times u(t) - \beta_2 a_2 \sqrt{2g\left(h_2(t) - h_3(t)\right)}}{A_2}, \\
\frac{\mathrm{d}h_3}{\mathrm{d}t} &= \frac{\beta_1 a_1 \sqrt{2g\left(h_1(t) - h_3(t)\right)} - \beta_2 a_2 \sqrt{2g\left(h_2(t) - h_3(t)\right)} - \beta_3 a_3 \sqrt{2g h_3(t)}}{A_3}.
\end{aligned} \tag{3}$$

Using the Taylor series, relations (3) become:

$$\begin{aligned}
\frac{\mathrm{d}h_1}{\mathrm{d}t} &= \frac{k}{A_1} v_1(t) - \frac{1}{T_1}\left(h_1(t) - h_3(t)\right), \\
\frac{\mathrm{d}h_2}{\mathrm{d}t} &= \frac{k}{A_1} v_2(t) - \frac{1}{T_2}\left(h_2(t) - h_3(t)\right),
\end{aligned} \tag{4}$$

$$\frac{\mathrm{d}h_3}{\mathrm{d}t} = \frac{1}{T_1}\left(h_1(t) - h_3(t)\right) + \frac{1}{T_2}\left(h_2(t) - h_3(t)\right) - \frac{1}{T_3} h_3(t).$$

where:

$$T_1 = \frac{A_1}{\beta_1 a_1}\sqrt{\frac{2\left(h_1(t)-h_3(t)\right)}{g}};\ T_2 = \frac{A_2}{\beta_2 a_2}\sqrt{\frac{2\left(h_2(t)-h_3(t)\right)}{g}};\ T_3 = \frac{A_3}{\beta_3 a_3}\sqrt{\frac{2h_3(t)}{g}}\ . \tag{5}$$

As the liquid heights have small variations, further, T_1, T_2, T_3 will be considered constant.

Rearranging the equations (4), yields:

$$\begin{aligned}
\frac{\mathrm{d}h_1}{\mathrm{d}t} &= -\frac{1}{T_1}h_1(t)+\frac{1}{T_1}h_3(t)+\frac{k}{A_1}v_1(t),\\
\frac{\mathrm{d}h_2}{\mathrm{d}t} &= -\frac{1}{T_2}h_2(t)+\frac{1}{T_2}h_3(t)+\frac{k}{A_2}v_2(t),\\
\frac{\mathrm{d}h_3}{\mathrm{d}t} &= \frac{1}{T_1}h_1(t)+\frac{1}{T_2}h_2(t)-\left(\frac{1}{T_1}+\frac{1}{T_2}+\frac{1}{T_3}\right)h_3\ .
\end{aligned} \tag{6}$$

Further, the three tank system space model is developed:

$$\begin{aligned}
\dot{\boldsymbol{x}} &= \boldsymbol{A}\overline{\boldsymbol{x}} + \boldsymbol{B}\overline{\boldsymbol{u}}\ ,\\
y &= \boldsymbol{C}\overline{\boldsymbol{x}} + \boldsymbol{D}\overline{\boldsymbol{u}}\ ,
\end{aligned} \tag{7}$$

where the notations are:

$$\overline{\boldsymbol{x}} = \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix};\quad \overline{\boldsymbol{u}} = \begin{pmatrix} v_1(t) \\ v_2(t) \end{pmatrix};\quad y = h_3\ . \tag{8}$$

$$\boldsymbol{A} = \begin{pmatrix} -\frac{1}{T_1} & 0 & \frac{1}{T_1} \\ 0 & -\frac{1}{T_2} & \frac{1}{T_2} \\ \frac{1}{T_1} & \frac{1}{T_2} & -\frac{1}{T_1}-\frac{1}{T_2}-\frac{1}{T_3} \end{pmatrix};\ \boldsymbol{B} = \begin{pmatrix} \frac{k}{A_1} & 0 \\ 0 & \frac{k}{A_2} \\ 0 & 0 \end{pmatrix};\ \boldsymbol{C} = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix};\ \boldsymbol{D} = 0\ . \tag{9}$$

The transfer function matrix of the plant may be derived in terms of the state space model using the formula:

$$\overline{\boldsymbol{H}}(s) = \begin{pmatrix} H_1(s) \\ H_2(s) \end{pmatrix} = \begin{pmatrix} \frac{H_3(s)}{V_1(s)} \\ \frac{H_3(s)}{V_2(s)} \end{pmatrix} = \boldsymbol{C}\left(s\boldsymbol{I}_3 - \boldsymbol{A}\right)^{-1} + \boldsymbol{D}\ , \tag{10}$$

where $H_3(s)$, $V_1(s)$ and $V_2(s)$ are the Laplace transforms of the height $h_3(t)$ and the voltages $v_1(t)$ and $v_2(t)$.

Replacing matrices $\boldsymbol{A}-\boldsymbol{D}$ in (10) yields:

$$\boldsymbol{H}(s)=\frac{T_3}{T_1T_2T_3s^3+(2T_2T_3+2T_1T_3+T_1T_2)s^2+(3T_3+T_1+T_2)s+1}\begin{pmatrix}\dfrac{T_2s+1}{A_1}\\ -\dfrac{T_1s+1}{A_2}\end{pmatrix}. \quad (11)$$

In practice, in most cases $A_1 = A_2 = A$, $a_1 = a_2$, $v_1(t) = v_2(t) = v(t)$ and, therefore, $h_1 = h_2$, $T_1 = T_2$. The first two state equations in (6) are identical and (6) becomes:

$$\begin{aligned}\frac{\mathrm{d}h_1}{\mathrm{d}t}&=-\frac{1}{T_1}h_1(t)+\frac{1}{T_1}h_3(t)+\frac{k}{A}v(t),\\ \frac{\mathrm{d}h_3}{\mathrm{d}t}&=-\frac{2}{T_1}h_1(t)-\left(\frac{2}{T_1}+\frac{1}{T_3}\right)h_3(t).\end{aligned} \quad (12)$$

The matrices $\boldsymbol{A}-\boldsymbol{D}$ of this state model are:

$$\boldsymbol{A}=\begin{pmatrix}-\dfrac{1}{T_1} & \dfrac{1}{T_1}\\ \dfrac{2}{T_1} & -\dfrac{2}{T_1}-\dfrac{1}{T_3}\end{pmatrix};\ \boldsymbol{B}=\begin{pmatrix}\dfrac{k}{A}\\ 0\end{pmatrix};\boldsymbol{C}=(0\quad 1);\ \boldsymbol{D}=0. \quad (13)$$

There is only one transfer function given always by (10). Its expression is:

$$H(s)=\frac{H_3(s)}{V(s)}=\frac{k}{A}\frac{2T_3}{T_1T_3s^2+(3T_3+T_1)s+1}. \quad (14)$$

4. CONTROLLER DESIGN

The closed loop block diagram of the SMC three tank system is represented in Fig. 2.

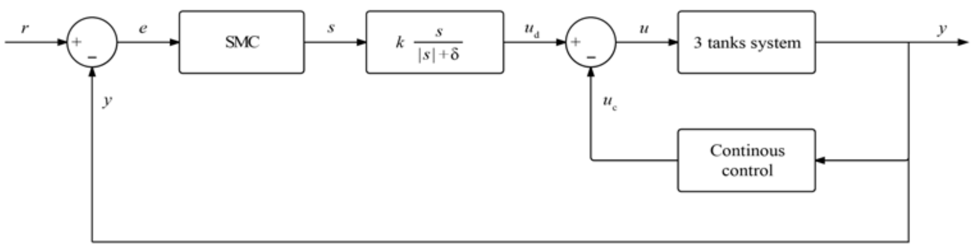


Fig. 2 – Closed loop block diagram for SMC three tanks system.

According to [7], the control variable $u(t)$ is composed of a continuous part $u_c(t)$ and a discontinuous part $u_d(t)$: $u(t) = u_c(t) + u_d(t)$.

The discontinuous part is [7]:

$$u_d = K\frac{s}{|s| + \delta}, \tag{15}$$

where: s is the sliding function; the second order sliding function is: $s = \dot{e} + \lambda e$, with $\lambda > 0$ its slope; e is the error: $e(t) = h_d(t) - h_3(t)$, $h_d(t)$ and $h_3(t)$ are the desired, respectively the actual liquid heights in tank 3.

The continuous part of the control variable is [7]:

$$u_c(t) = \frac{1}{b}\left[(c_1 - \lambda)\frac{\mathrm{d}h_3}{\mathrm{d}t} + c_0 h_3(t)\right]. \tag{16}$$

The equation is known in SMC as the equivalent control procedure. The constant b is [1]:

$$b = \frac{a_1\sqrt{2g}}{2A^2\sqrt{h_1(t) - h_3(t)}}. \tag{17}$$

The constants c_0 and c_1 may be deduced, according to [1]:

$$c_0 = \frac{2a_1\sqrt{2g}}{A^2\sqrt{h_1(t) - h_3(t)}}; \quad c_1 = a_1 \times c_0 . \tag{18}$$

The control variable $u(t)$ is:

$$u(t) = u_c(t) + u_d(t) = \frac{1}{b}\left[(c_1 - \lambda)\frac{\mathrm{d}h_3}{\mathrm{d}t} + c_0 h_3(t)\right] + K\frac{s}{|s| + \delta}. \tag{19}$$

5. **SIMULATION RESULTS**

The tank system and sliding mode controller parameters used in simulation are given in Tables 1 and 2.

Table 1

Parameters of the tank system

A	a_1, a_2	β_1, β_2	h_1, h_3	h_2	$U(t)$	k
250 cm2	5 cm^2	0.1	22 cm	20 cm	3V	10

Table 2

Parameters of SMC controller, calculated on the basis of the approximated model

b	c_0	c_1	λ	k	δ
0.0008854	0.0035416	0.017708	0.1	1000	0.1

The transfer function of the system is:

$$H(s) = \frac{228.56s + 8.08}{103424s^3 + 4256s^2 + 367s + 1}.$$

In Figs. 3–8, the performance of SMC system and the conventional PID system are compared using the performance terms: IAE (integral of the absolute error), ITAE (integral time absolute error), ISE (integral square error), SMC showed better tracking results than PID controller.

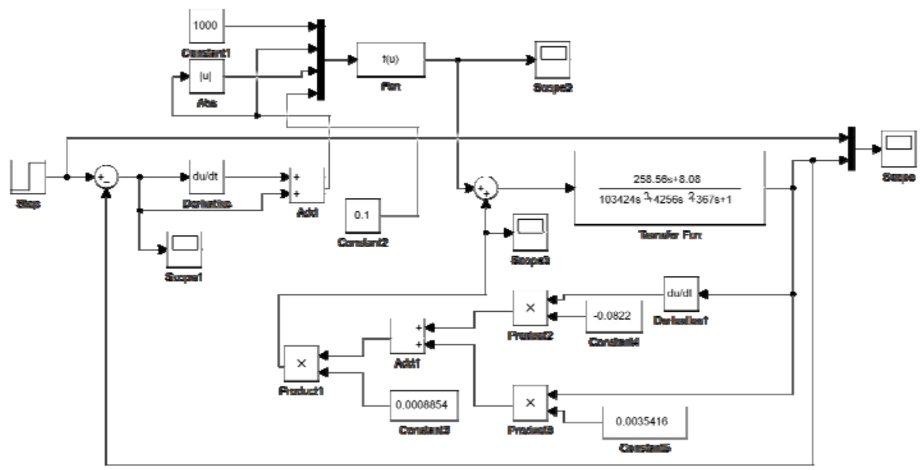
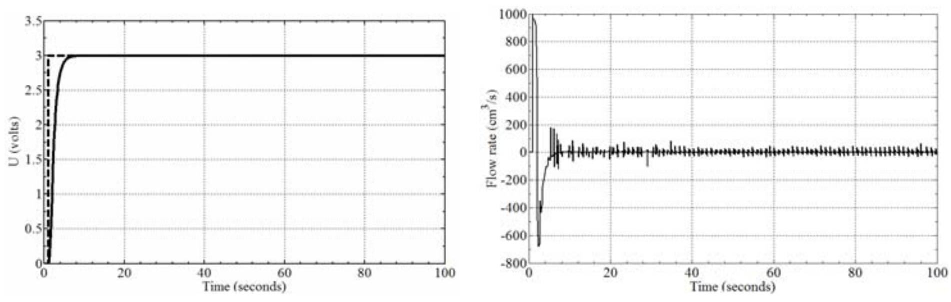


Fig. 3 – Block diagram of the SMC system.

Next the SMC control and flow rate for disturbances and different command signals (sinusoidal and trapezoidal) are simulated.

All the simulations are performed in a 100 seconds time interval. The voltage (volts) and the flow rate (cm^3) are represented on Y axis and the time (seconds) on X axis.



(a) (b)

Fig. 4 – SMC control (a) and flow rate (b) for step command.

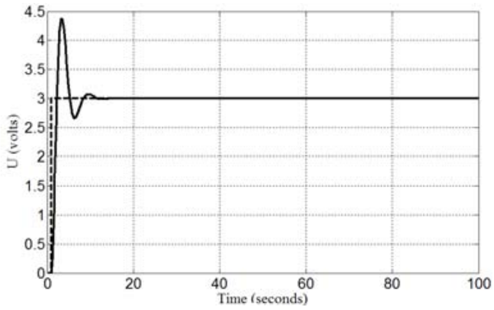
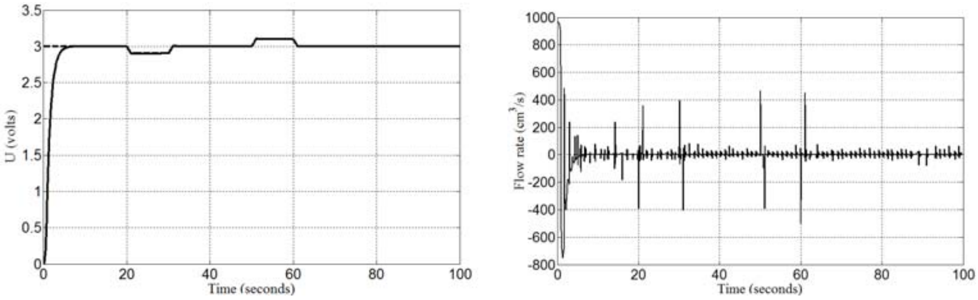


Fig. 5 – PID control.



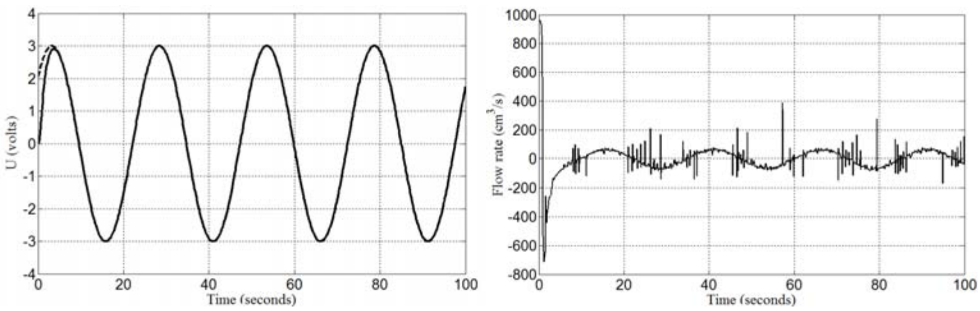
a) b)

Fig. 6 – SMC control (a) and flow rate (b) for step command with disturbance.

Table 3

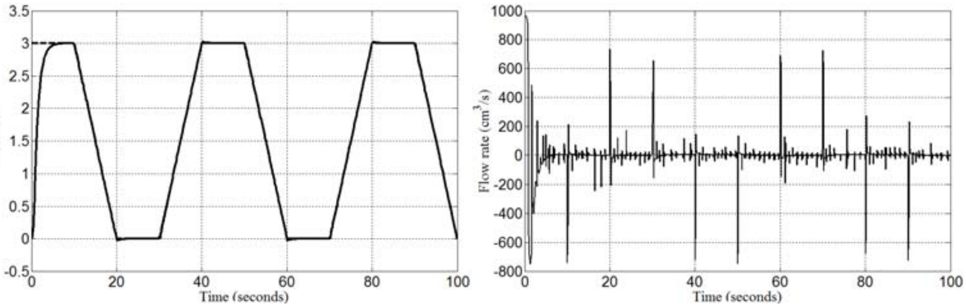
Comparison of performance measures

Parameters	**PID**	**SMC**
Settling time	19.3 s	9.45 s
Peak overshot	0.55	0
ITAE	–3.218	4.974
IAE	5.564	4.563
ISE	7.739	8.736



a) b)

Fig. 7 – SMC control (a) and flow rate (b) for sinusoidal command.



a) b)

Fig. 8 – SMC control (a) and flow rate (b) for trapezoidal command.

6. **CONCLUSIONS**

The sliding mode control technique was implemented for a nonlinear three tanks system. New state model and the transfer function of the plant were developed and a sliding mode controller for liquid level control has been designed.

The closed loop system was tested with various types of input signals and disturbances through simulation. SMC is able to control the system with its robust behaviour for different input signals or parameter values. Compared with PID controller for the step command, SMC has shown better performances for all the indicators.

Further research will be done in order to perfect the proposed SMC controller in various conditions and in comparison to other types of existing control methods.

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REFERENCES

1. H. Delavari, A. R. Noiey, *Robust Intelligent Control of Coupled Tanks*, WSEAS International Conference on Automatic Control, Modeling & Simulation, Istanbul, Turkey, May 27–29, 2007.
2. B.J. Parvat, V.K. Jadhav, N.N. Lokhande, *Design and Implementation of Sliding Mode Controller for Level Control*, IOSR Journal of Electronics and Communication Engineering (IOSR-JECE), pp. 51–54.
3. N. B. Almutairi, M. Zribi, *Sliding mode control of coupled tanks*, Mechatronics, **16**, *7*, pp. 427–441, 2006.
4. E. Rani, S. Isaac J, *Modelling and Design Aspects of PI Controller for Coupled Tank Process*, IJCA Proceedings on International Conference on Innovations In Intelligent Instrumentation, Optimization and Electrical Sciences ICIIIOES 10–14, December 2013.
5. M. Kh. Khan, S. K. Spurgeon, *Robust MIMO water level control in interconnected twintanks using second order sliding mode control*, Control Engineering Practice, **14**, *4*, pp. 375–386, 2006.
6. M. N. Mahyuddin, M. R. Arshad, Z. Mohamed, *Simulation of Direct Model Reference Adaptive Control on a Coupled-Tank System using Nonlinear Plant Model*, International Conference on Control, Instrumentation and Mechatronics Engineering (CIM'07), Malaysia, May 28–29, 2007.
7. H. Abbas, S. Asghar, S. Qamar, *Sliding Mode Control for Coupled-Tank Liquid Level Control System*, 10th International Conference on Frontiers of Information Technology (FIT), 17–19 Dec. 2012, pp. 325–330.
8. I. Holic, V. Vesely, *Robust PID Controller Design For Coupled-Tank Process*, 18th International Conference on Process Control, June 14–17, Tatranska Lomnica Slovakia, 2011.
9. C. Lupu, Doinita Chirita, *Performance Control for Cascade Reservoirs Systems*, 13th International Conference on Engineering of Modern Electric Systems (EMES), Oradea, Romania, 11–12 June 2015, pp. 63–68.
10. V. I. Utkin, *Variable structure systems with sliding modes* Transactions of IEEE on Automatic Control, pp. 212–222, 1997.
11. M. Adjoudj, M. Abid, A. Aissaoui, Y. Ramdani, H. Bounoua, *Sliding Mode Control Of A Doubly Fed Induction Generator For Wind Turbines,* Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., **56**, *1*, pp. 15–24, 2011.
12. B. Bouchiba, A. Hazzab, H. Glaoui, M. K. Fellah, I. K. Bousserhane, P. Sicard, *Multiple-Input Multiple-Output Fuzzy Sliding Mode Controller For Multi-Motors System,* Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., **57**, *1*, pp. 202–211, 2012.
13. A. I. Gal, L. Vlădăreanu, R. I. Munteanu, *Sliding Motion Control With Bond Graph Modeling Applied On A Robot Leg,* Rev. Roum. Sci. Techn. – Électrotechn. et Énerg., **60**, *2*, pp. 215–224, 2015.