THE OUTAGE PROBABILITY OF MULTIBRANCH SELECTION COMBINING OVER CORRELATED WEIBULL FADING CHANNEL

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In this paper, we analyze the outage probability of a multibranch selection combining (SC) based on the signal to interference ratio (SIR) over Weibull fading channels in case of non-identical power distribution per branches. Both, desired and interference signal envelopes are exponentially correlated Weibull random variables. Also, an analysis of effects of fading parameter β, correlation coefficient ρ between branches, number of branches *L*, as well as difference between the average input SIR per branches Δ*S*, on performance of multibranch SC receiver was contributed.

1. INTRODUCTION

Wireless communications are subject to a complex propagation environment, like multipath fading and shadowing. Considerable efforts have been devoted to the channel modelling and characterization of these effects, resulting in a range of statistical channel models. Various techniques for reducing fading effect and influence of co-channel interference (CCI) have been proposed. Diversity reception is a very simple and effective approach based on the idea of providing the receiver with multiple faded replicas of the same information-bearing signal [1]. The goal of diversity techniques is to increase channel capacity and to upgrade transmission reliability without increasing transmission power and bandwidth [2]. Space diversity is an efficient method for amelioration systems quality-of-service (QoS) when multiple receiver antennas are used [1–3]. There are several types of combining techniques which entail various tradeoffs between performance and

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complexity. One of the least complex techniques is selection combining (SC). Switch and stay combining (SSC) is also low-complexity and very efficient technique that reduces fading and CCI influence. Combining techniques like equal gain (EGC) and maximal ratio (MRC) require all or some of the amount of the channel state information (CSI) available at the receiver [4–5].

When the noise power can be assumed to be equally distributed over branches, SC simply selects the branch with the highest signal to noise ratio (SNR), In fading environments where the level of interference is sufficiently high compared to the thermal noise, combiner must select the branch with highest signal to interference ratio (SIR). The case when SC processes the branch with the highest SIR [6] is called SIR-based selection diversity.

In order to simplify the analysis, it is convenient to assume that the diversity branches are uncorrelated and independent identically distributed (i.i.d). The branch correlation and non-identical distribution (n.i.d) can reduce the achievable diversity gain [4], which means that the uncorrelated and i.i.d branch assumption gives an optimistic result. In real life scenarios, the effect of correlation between the combined signals has to be taken into account for the accurate performance analysis of diversity systems.

Depending on the nature of the propagation environment, there are different models describing the statistical behaviour of the multipath fading envelope [4]. Despite the fact that Weibull distribution fits very well to experimental measurements, for both indoor [7] and outdoor environments [8], it has only very recently received a great interest. Very useful analytical expressions for the probability density function (PDF), cumulative density function (CDF) and moment-generating function (MGF) for the multivariate Weibull distribution are derived and applied to analyze the performance of diversity receivers [9-10]. The multibranch EGC and MRC operating over Weibull channel have been considered in [11]. The performance analysis of switched and generalized SC (GSC) over independent and n.i.d Weibull channels is presented in [12–13]. The performance analysis of diversity receivers over correlated Weibull and Hoyt channels in the presence of multiple CCI has been presented in [14–16], but the effect of non-identical power distribution per branches was not analysed. The performance of triple SC operating over correlated Weibull channel has been studied in [17] and some effects of non-identical power distribution per branches was contributed. In [18–19] some closed-form expressions for the system performance measures of multibranch SC over independent and correlated Weibull channels have been presented. The performance analysis of switched diversity operating over correlated Weibull channel with CCI has been studied in [20–21].

The remainder of this paper is as follows. After this Introduction, in Section II the SIR based SC receiver operating over correlated n.i.d Weibull fading channel in the presence of Weibull distributed interference is considered. Numerical performance evaluation results are presented in Section III, showing the effect of the average input SIR, fading severity, correlation coefficient and effect of non-identical distribution per branches on the system performance.

2. THE SIR-BASED SELECTION COMBINING

A multibranch SC operating over Weibull fading channel with interference influence will be analyzed with the assumption that both desired and interference signal envelopes are exponentially correlated Weibull distributed variables [10]. The exponential correlation model corresponds to the scenario of multichannel reception from equal-spaced diversity antennas among the pairs of combined signals, which fade if the distance between antennas increases [2].

The joint PDF in correlated Weibull fading channel [10] can be described as:

$$p_z\left(r_1,\ldots,r_L\right)=\frac{1}{\left(1-\rho\right)^{L-1}}\prod_{l=1}^{L}\frac{\beta_l}{\Omega_l}r_l^{\beta_l-1}\cdot\exp\left[-\frac{1}{1-\rho}\left(\frac{r_1^{\beta_1}}{\Omega_1}+\left(1+\rho\right)\sum_{l=2}^{L-1}\frac{r_l^{\beta_l}}{\Omega_l}+\frac{r_L^{\beta_L}}{\Omega_L}\right)\right]\cdot$$
$$\cdot\prod_{l=1}^{L-1}I_0\left(\frac{2\sqrt{\rho}\;r_l^{\beta_l/2}r_{l+1}^{\beta_{l+1}/2}}{\left(1-\rho\right)\sqrt{\Omega_l}\sqrt{\Omega_{l+1}}}\right),\tag{1}$$

where $I_0(.)$ is modified Bessel function of the first kind and zero order, L is the number of diversity branches, $\Omega_l=\overline{r_l^2}$ is the average signal power at lth branch $l = 1, 2,..., L$, β is fading parameter and ρ is correlation coefficient. The fading intensity decreases with the increase of parameter β. The value $\beta = 2$ corresponds to Rayleigh fading, $\beta = 1.75$ corresponds to Nakagami-m ($m = 0.76$) and Hoyt ($q = 0.53$) fading; while $\beta = 3$ corresponds to Nakagami-m ($m = 2.17$) and Rice ($K = 2.76$) fading [4]. The value of ρ is $\rho = 0.10 \pm 0.06$ for urban and $\rho = 0.35 \pm 0.18$ for suburban areas.

Since both desired and interference signal envelopes follow the correlated Weibull distribution, after using expression for expanding $I_0(.)$ their joint PDFs respectively are:

$$p_{R_1\ldots R_L}\left(R_1,\ldots R_L\right)=$$
$$=\frac{1}{\left(1-\rho\right)^{L-1}}\cdot\prod_{l=1}^{L}\frac{\beta_l}{\Omega_{dl}}R_l^{\beta_l-1}\cdot\exp\left[-\frac{1}{1-\rho}\left(\frac{R_1^{\beta_1}}{\Omega_{dl}}+\left(1+\rho\right)\sum_{l=2}^{L-1}\frac{r_l^{\beta_l}}{\Omega_{dl}}+\frac{r_L^{\beta_L}}{\Omega_{dL}}\right)\right]\cdot\tag{2}$$
$$\cdot\sum_{i_1,\ldots i_{L-1}}^{\infty}\prod_{i=1}^{L-1}\frac{1}{\left(i_l!\right)^2}\left(\frac{\rho}{\left(1-\rho\right)^2}\right)^{i_l}\cdot\frac{R_l^{i_l\beta_l}}{\Omega_{dl}^{\;i_l}}\cdot\frac{R_{l+1}^{\;i_l\beta_{l+1}}}{\Omega_{d(l+1)}^{\;i_l}},$$

$$p_{r_1 \ldots r_L}\left(r_1,\ldots r_L\right)=$$
$$=\frac{1}{\left(1-\rho\right)^{L-1}}\cdot\prod_{l=1}^{L}\frac{\beta_l}{\Omega_{cl}}r_l^{\beta_l-1}\cdot\exp\left[-\frac{1}{1-\rho}\left(\frac{r_1^{\beta_1}}{\Omega_{c1}}+\left(1+\rho\right)\sum_{l=2}^{L-1}\frac{r_l^{\beta_l}}{\Omega_{cl}}+\frac{r_L^{\beta_L}}{\Omega_{cL}}\right)\right]\cdot \tag{3}$$
$$\cdot\sum_{i_1,\ldots i_{L-1}}^{\infty}\prod_{i=1}^{L-1}\frac{1}{\left(i_l!\right)^2}\left(\frac{\rho}{\left(1-\rho\right)^2}\right)^{i_l}\cdot\frac{{r_l}^{i_l\beta_l}}{{\Omega_{cl}}^{i_l}}\cdot\frac{{r_{l+1}}^{i_l\beta_{l+1}}}{{\Omega_{c(l+1)}}^{i_l}},$$

where $\Omega_{dl}=\overline{R_l^2}$ and $\Omega_{cl}=\overline{r_l^2}$ are the average powers of desired and interference signals at lth branch. The SC chooses the branch with largest instantaneous SIR, defined as $\varsigma_l = R_l^2/r_l^2$ for lth branch. The average SIR at lth branch is $S_l=\Omega_{dl}/\Omega_{cl}$. Furthermore, the effect only of the strongest interference signal is considered, like [6]. The joint PDF of instantaneous SIR at the SC output is:

$$\begin{aligned}p_{\varsigma_1\ldots\varsigma_L}\left(t_1,\ldots,t_L\right)=\int_0^\infty\int_0^\infty\left|J\right|p_{R_1\ldots R_L}\left(x_1\sqrt{t_1},\ldots,x_L\sqrt{t_L}\right)\cdot\\ \cdot p_{r_1\ldots r_L}\left(x_1,\ldots,x_L\right)\cdot x_1\ldots\cdot x_L\cdot\mathrm{d}x_1\ldots\mathrm{d}x_L.\end{aligned}\tag{4}$$

We will use short hand notations $f_l\left[I,K,t_l\right]$ and $F_l\left[I,K,t_l\right]$ as follows:

$$f_l\left(I,K,t_l\right)\Leftrightarrow\left(\frac{t_l^{\beta_l/2}}{S_l}\right)^{1+I}\cdot\left(1+\frac{t_l^{\beta_l/2}}{S_l}\right)^{-2-I-K},$$
$$F_l\left(I,K,t_l\right)\Leftrightarrow{}_2F_1\left(1+I,2+I+K,2+I,-\frac{t_l^{\beta_l/2}}{S_l}\right)\cdot\left(\frac{t_l^{\beta_l/2}}{S_l}\right)^{1+I},$$

where ${}_2F_1$ (a,b;c;d) is the Gaussian hypergeometric function and Γ (.) is the Gamma function [22]. Substituing (2) and (3) in (4) yields:

$$p_\varsigma(t)=\frac{(1-\rho)^2}{(1+\rho)^{2(L-1)}}\cdot\frac{\prod_{l=1}^{L}\beta_l t_l^{-1}}{2^L}\cdot\sum_{\substack{i_1,\ldots,i_{L-1}=0\\k_1,\ldots,k_{L-1}=0}}^{\infty}\left\{\frac{\rho^{\sum_{l=1}^{L}i_l+k_l}}{(1+\rho)^{2\sum_{l=1}^{L}i_l+k_l}}\cdot\frac{\Gamma(2+i_1+k_1)}{i_l!\,k_l!}\cdot f_1[i_1,k_1,t_1]\cdot\right.$$
$$\left.\cdot\prod_{l=2}^{L-1}\left[\frac{\Gamma(2+i_{l-1}+i_l+k_{l-1}+k_l)}{i_{l-1}!\,i_l!k_{l-1}!k_l!}\cdot f_l[i_{l-1}+i_l,k_{l-1}+k_l,t_l]\right]\cdot\frac{\Gamma(2+i_{L-1}+k_{L-1})}{i_{L-1}!\,k_{L-1}!}\cdot f_L[i_{L-1},k_{L-1},t_L]\right\}.\tag{5}$$

After L-fold integrations, generalized CDF can be expressed as:

$$P_\varsigma(\gamma)=\underbrace{\int_0^{r_1}\ldots\int_0^{r_L}}_{L-fold} p_{\varsigma_1\ldots\varsigma_L}(t_1,\ldots,t_L)\mathrm{d}t_1\,\mathrm{d}t_L =$$

$$=\frac{(1-\rho)^2}{(1+\rho)^{2(L-1)}}\cdot\sum_{\substack{i_1,\ldots,i_{L-1}=0\\k_1,\ldots,k_{L-1}=0}}^{\infty}\left\{\frac{\rho^{\sum_{l=1}^{L} i_l+k_l}}{(1+\rho)^{i_1+k_1+i_{L-1}+k_{L-1}+2\sum_{l=2}^{L-2} i_l+k_l}}\cdot\frac{\Gamma(2+i_1+k_1)}{i_l!\,k_l!}\cdot F_1[i_1,k_1,\gamma_1]\cdot\right. \quad . \tag{6}$$

$$\left.\cdot\prod_{\substack{l=2\\L\geq 3}}^{L-1}\frac{\Gamma(2+i_{l-1}+i_l+k_{l-1}+k_l)}{i_{l-1}!\,i_l!k_{l-1}!k_l!}\cdot F_l[i_{l-1}+i_l,k_{l-1}+k_l,\gamma_l]\cdot\frac{\Gamma(2+i_{L-1}+k_{L-1})}{i_{L-1}!\,k_{L-1}!}\cdot F_L[i_{L-1},k_{L-1},\gamma_L]\right\}$$

3. **THE PERFORMANCE ANALYSIS AND NUMERICAL RESULTS**

The outage probability (P_{out}) is very important system performance measure, which is, at the interference-limited environment, defined as the probability of failing to achieve a specified output SIR value γ sufficient for satisfactory reception, which simplifies the CDF of instantaneous SIR at the SC output:

$$P_{\text{out}} = P_R(\varsigma < \gamma) = \int_0^{\gamma} p_\varsigma(t)\,\mathrm{d}t = P_\varsigma(\gamma). \tag{7}$$

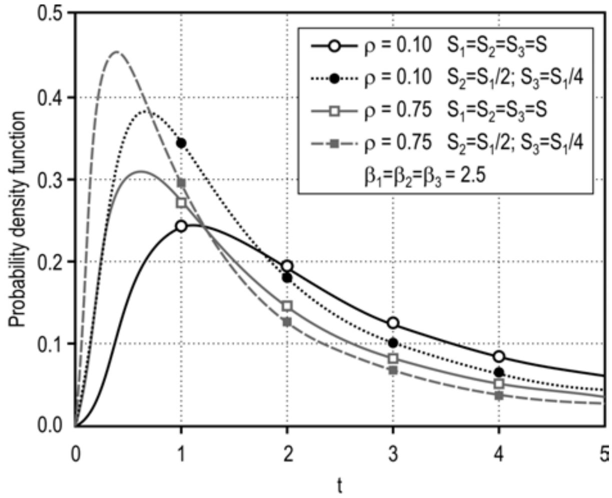

Fig. 1 – PDF of instantaneous SIR at the 3-SC output.

The PDF of instantaneous SIR at the 3-SC output for different values of ρ and β, and for different average SIR values per branch is shown in Fig. 1, while the CDF evaluated at threshold γ, presenting P_{out}, is presented in Figs. 2–8.

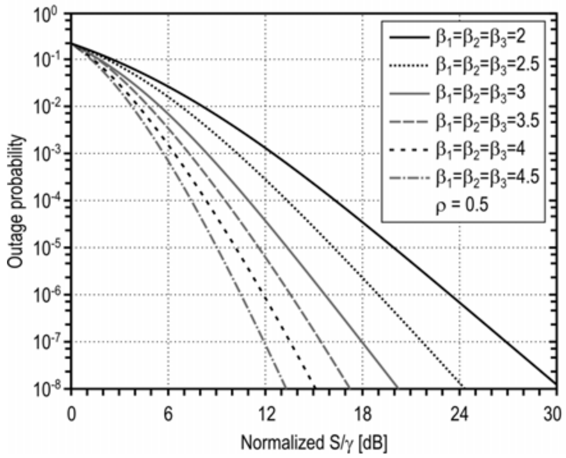

Fig. 2 – P_{out} *versus* S/γ for identically faded diversity branches and constant ρ.

Fig. 2 and Fig. 3 present the P_{out} in case of identically and non-identically faded branches. The P_{out} decreases rapidly as β increases, for both cases, where the average value of β for non-identically faded 3-SC is $\beta_{av} = (\beta_1+\beta_2+\beta_3)/3$. We can also conclude that the P_{out} for non-identically faded branches is approximately as in case of identically faded branches characterized with β_{av}.

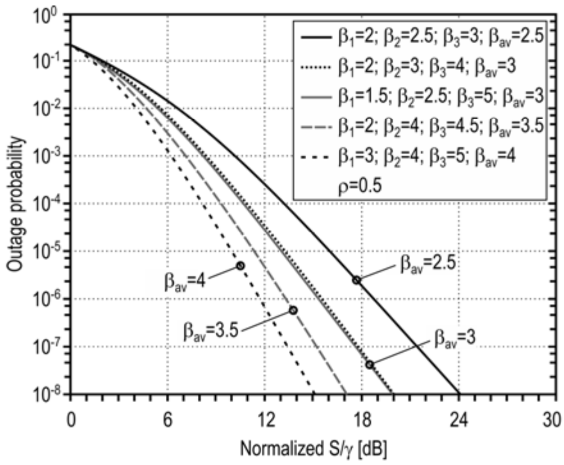

Fig. 3 – P_{out} *versus* S/γ for non-identically faded diversity branches and constant ρ.

Fig. 4 and Fig. 5 present the P_{out} *versus* β and ρ, respectively, for predetermined values of normalized first branch S/γ = 6dB and 12dB. If β increases the P_{out} decreases slowly for S/γ = 6dB and rapidly for S/γ = 12dB, as in Fig. 4.

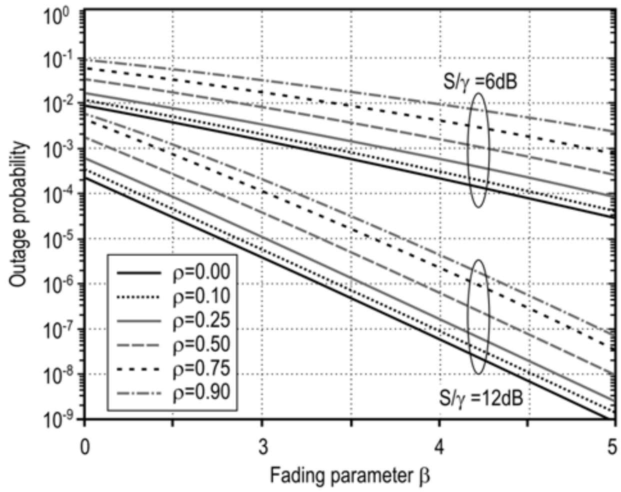

Fig. 4 – P_{out} *versus* β, for different values of ρ.

If ρ increases the P_{out} increases slowly for both values of S/γ, as in Fig. 5.

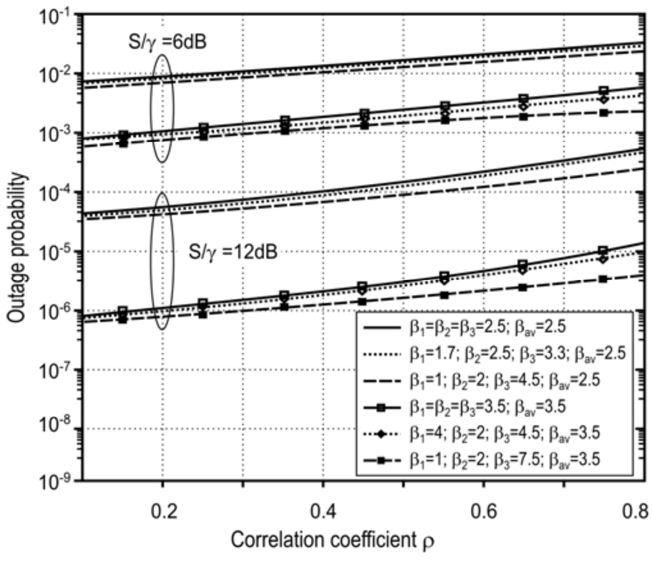

Fig. 5 – P_{out} *versus* ρ, for different values of β.

The P_{out} characteristics for non-identically faded branches are approximately as in case of identically faded branches characterized with β_{av}, for the values $\rho < 0.5$. For the values $\rho > 0.5$, the difference between P_{out} curves in the cases of identically and non-identically faded branches exists, but it is not significant, as in Fig. 5.

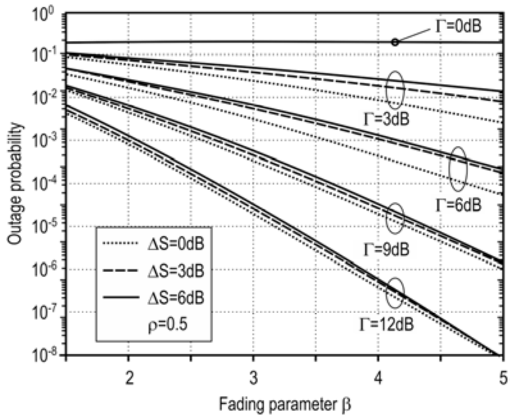

Fig. 6 – P_{out} *versus* β for different values of ΔS and $\rho = 0.5$.

In the case of n.i.d branch scenario, we analyze two cases: the average SIR per branch decreases successively for ΔS = 3dB and 6dB. The Fig. 6 presents the P_{out} *versus* β ($\beta_1 = \beta_2 = \beta_3 = \beta$) and n.i.d branches with $S_{i+1} = S_i - \Delta S$, for several values of $\Gamma = S_1/\gamma$. When parameter Γ takes lower values (Γ < 9dB), it is evident that ΔS has a great influence on P_{out}, for higher values of β ($\beta > 4$), as shown in Fig. 6.

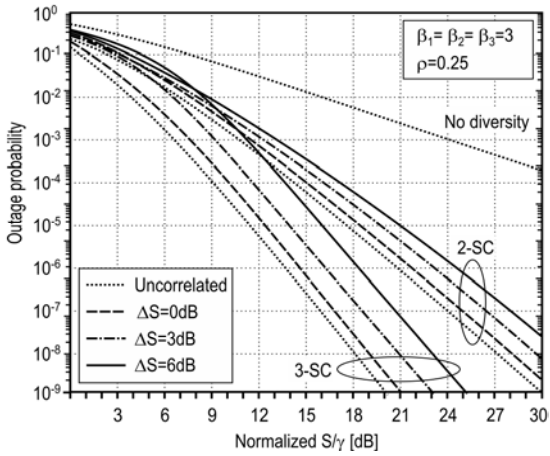

Fig. 7 – P_{out} *versus* S/γ for 2-SC and 3-SC, for different values of ΔS.

Fig. 7. presents comparative P_{out} graphics for 2-SC and 3-SC in cases of i.i.d and n.i.d branches, for several values of ΔS. It is evident that the increase of ΔS makes the system performance worse, but this influence is greater for 3-SC than for 2-SC. In Fig. 7. it is shown that for smaller values of S/γ (S/γ<10dB) the performance of 3-SC is worse than the performance of 2-SC for ΔS = 6dB.

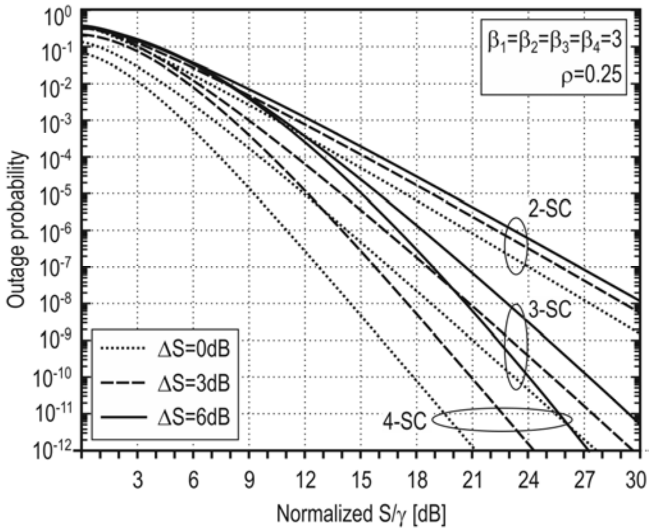

Fig. 8 – P_{out} *versus* S/γ for 2-SC, 3-SC and 4-SC, for different values of ΔS.

Fig. 8 presents comparative P_{out} graphics for 2-SC, 3-SC and 4-SC in cases of i.i.d and n.i.d branches, for several values of ΔS. It is evident that for constant values of S/γ, the increase of ΔS makes the system performance worse. It is indicated that this influence is the greatest for 4-SC and the smallest for 2-SC.

4. **CONCLUSIONS**

The performance of SIR based multibranch SC receiver operating over correlated Weibull fading channel, was studied in this paper. Signal reception in case of non-identical power distribution per SC branches was considered. Also, an anlysis of effects of fading parameter β, correlation coefficient ρ between branches, number of branches L, as well as difference between the average input SIR per branches ΔS, on performance of multibranch SC receiver was contributed.

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