FAULT DIAGNOSIS IN ANALOG CIRCUITS USING APPROXIMATE SYMBOLIC POLE/ZERO EXPRESSIONS

CONSTANTIN VIOREL MARIN, FLORIN CONSTANTINESCU

Key words: Analog circuits, Fault diagnosis, Approximate symbolic pole/zero expressions, Simulation after test.

This paper deals with diagnosis of soft faults for analog circuits in simulation after test approach. Simple approximate pole/zero expressions, depending only on the most significant parameters, are computed. The parameters values for the soft faults in linear circuits are identified employing these expressions and gain vs. frequency characteristics. This approach uses simpler equations and proves a higher power to identify the faulty components than other approaches.

1. INTRODUCTION

Modern analog circuits are characterised by high complexity and circuit elements embedded inaccessibly within circuit chips. This is why a significant effort has been directed to develop fault diagnosis techniques that no longer need direct access to the circuit components of analog circuits.

Simulation after test (SAT) approach assumes that the nominal components values are given and the faulty network topology is known. Next, some measurements are performed at externally accessible test terminals. The measurement data are then supplied to a computer program which uses them along with information concerning the circuit topology and nominal components values, in order to compute the parameters values corresponding to the faulty elements. Finally, the faulty components are identified by determining whether the parameters values are inside or outside the design tolerance margins.

For linear time-invariant circuits, the fault diagnosis equations could be constituted by the symbolic expressions of circuit functions corresponding to selected test points [1, 2, 3, 4]. Usually some voltages are measured using sinusoidal excitations in an appropriate frequency range. In [1, 2, 3] symbolic methods are employed for the test frequencies selection in multifrequency fault diagnosis of analog linear circuits. In [4] is proposed an approach based on the
evaluation of the condition number. A test error index is defined and the ranges of suitable frequencies for performing the measurements are identified.

A new approach for analog fault analysis is proposed in this paper. This approach relies on the approximate symbolic pole/zero expressions in terms of circuit parameters [5]. The gain vs. frequency characteristic points out the pole and zero values. Equating these values to the symbolic pole/zero expressions, fault diagnosis equations are obtained. This method uses simpler equations and proves a higher power to identify the faults than other approaches.

Section 2 presents the testability matrix and the ambiguity groups for the proposed approach. The algorithm for fault diagnosis using approximate pole-zero expressions is described in Section 3. The method is illustrated in Section 4 by the fault diagnosis for a lumped equivalent circuit of an isolation transformer.

2. TESTABILITY MATRIX AND AMBIGUITY GROUPS

The SAT approach to analog fault diagnosis consists in two important steps: testability analysis and fault location [4]. Testability analysis consists at its turn in two main steps: testability evaluation and ambiguity group determination. In the fault location phase, the faulty components have to be identified. Testability quantifies the degree of solvability of fault diagnosis problem, once given the circuit topology, selected test points and the set of unknown components. If the fault of a component produces the same measurement results as the fault of another component, it is not possible to discern which component is faulty. In this case, the two faults constitute an ambiguity group. An ambiguity group of higher order contains more than two faults. The presence of ambiguity groups does not permit the unique identification of the faulty components.

The testability matrix \( B \) is defined in [4]. The entries of the matrix are the derivatives of the fault diagnosis equations with respect to the potentially faulty circuit parameters. As in this paper these equations are \( p/z \) expression = value, where the pole/zero symbolic expressions are equated to their values deduced from the frequency characteristics, the testability matrix is defined by

\[
B = \begin{bmatrix}
\frac{\partial s_{p_1}}{\partial p_1} & \frac{\partial s_{p_1}}{\partial p_2} & \ldots & & \frac{\partial s_{p_1}}{\partial p_y} \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
\frac{\partial s_{p_n}}{\partial p_1} & \frac{\partial s_{p_n}}{\partial p_2} & \ldots & \ldots & \frac{\partial s_{p_n}}{\partial p_y} \\
\frac{\partial s_{z_1}}{\partial p_1} & \frac{\partial s_{z_1}}{\partial p_2} & \ldots & \ldots & \frac{\partial s_{z_1}}{\partial p_y} \\
\ldots & \ldots & \ldots & \ldots & \ldots \\
\frac{\partial s_{z_m}}{\partial p_1} & \frac{\partial s_{z_m}}{\partial p_2} & \ldots & \ldots & \frac{\partial s_{z_m}}{\partial p_y}
\end{bmatrix}
\] (1)
\( s_{p_1} \ldots s_{p_p} \) and \( s_{z_1} \ldots s_{z_m} \) are the symbolic pole/zero expressions and \( p_1, \ldots, p_V \) are the potentially faulty circuit parameters. An ambiguity group can be identified as a circuit parameter set whose corresponding columns in the testability matrix \( B \) are linearly dependent [4]. The rank of the testability matrix defines the testability. The testability is a measure of the the unique solvability of the fault diagnosis problem. If the testability is smaller than the number of the potentially faulty circuit parameters, then more test points or more test frequencies must be considered in order to reach the unique solvability of the fault diagnosis problem. Otherwise, a smaller number of potentially faulty circuit parameters must be taken into account.

3. FAULT DIAGNOSIS ALGORITHM

The SAT approach to the fault diagnosis problem in analog circuits employs the following main steps:

- set-up of fault diagnosis equations;
- testability analysis;
- identification of testable components;
- calculus of the unknown parameters;
- identification and location of the faulty components.

**Set-up of fault diagnosis equations.** In this paper the fault diagnosis equations are the symbolic expressions of the poles \( s_{p_k} \) and zeros \( s_{z_k} \) of the network function:

\[
H(s) = K \frac{(s - s_{z_1})(s - s_{z_2}) \ldots (s - s_{z_m})}{(s - s_{p_1})(s - s_{p_2}) \ldots (s - s_{p_m})}.
\] (2)

The set-up of fault diagnosis equations contains the following steps:

- measurement of the frequency characteristics for the non-faulty circuit and for the faulty one;
- parameter ranking (according to the sensitivities of each pole/zero value) by design space exploration;
- computation of the approximate symbolic pole/zero expressions;
- identification of the numerical pole/zero values for the non-faulty circuit and for the faulty one;
- equating the approximate symbolic pole/zero expressions to corresponding numerical values for the faulty circuit.

Any pole or zero value depends on all circuit parameters. Simple expressions, depending on the most significant parameters only are very useful for our purpose.

The pole/zero sensitivities \( S_{x_i}^{M_j} \)
are used for the design space exploration in order to compute the significant parameters for each pole/zero. In (3) \( M_j \) is the value of a certain pole/zero, \( \Delta M_j \) is the variation of \( M_j \) corresponding to the variation \( \Delta x_i \) of the circuit parameter \( x_i \). This exploration leads to a parameter ranking for each pole/zero expression. Significant parameters have sensitivities \( S^j_{x_i} \) included in the range \( [S_{\text{max}} \cdot 0.1, S_{\text{max}}] \) where \( S_{\text{max}} \) is the greatest modulus of \( S^j_{x_i} \) in the design space, less significant parameters have \( S^j_{x_i} \in [0.1S_{\text{max}}, 0.05S_{\text{max}}] \) and non-significant parameters correspond to \( S^j_{x_i} \in [0.05S_{\text{max}}, 0] \).

The symbolic pole expressions are computed taking into account the significant parameters only or the significant and less significant parameters. The algorithm for the computation of the approximate symbolic pole/zero expressions is described in [5] and employs symbolic LR iterations. Due to the computational complexity, even for a relatively simple circuit, simplifications are made during these iterations. The algorithm checks all simplification possibilities. The simplification is validated by measuring its influence on eigenvalue modules. The simplification is accepted if the relative error for any eigenvalue is less than an imposed value \( \varepsilon_A \). A term in a sum may be neglected during symbolic LR decomposition if this operation produces a maximum relative error \( \varepsilon_2 \) on this sum. The simplification of too intricate pole/zero expressions is considered too. To this end the validity range in the design space is tested using comparison with numerical eigenvalues. The simplification of an eigenvalue expression within an error margin \( \varepsilon_p \) consists in successive testing of the elimination of the less significant term in the numerator and in the denominator of the expression. Testing is performed in some points of the validity range.

The frequency characteristics \( H(j\omega) \) of the faulty circuit and the non-faulty are given by measurements. Using classical asymptote intersection the pole and zero modules can be determined. A slope change of 12 dB/oct. points out a pole or zero complex pair. The numerical values of poles and zeros are equated to the corresponding symbolic expressions. Solving these equations the circuit parameters are identified.

**Testability analysis.** The testability matrix \( B \) (1) of the circuit under test (CUT) is built, its measure \( T \) being given by the rank of \( B \):

\[
T = \text{rank}(B).
\]
If $T$ is equal with the number of unknown parameters $V$, then all parameters could be determined by solving the system of diagnosis equations considering the set of measurements. If $T < V$ the system of diagnosis equations could be solved only if $V - T$ parameters are considered known, i.e. not faulty. At the same time, the linearly dependent columns of the matrix $B$ point out the ambiguity groups.

**Identification of testable components.** The number of unknown parameters must be equal to $T$ and should be chosen according to a specific criterion corresponding with the goals of the problem.

**Calculus of the unknown parameters.** The fault diagnosis equations represent a set of nonlinear equations with respect to the unknown parameters. The Newton-Raphson method is employed to solve this system:

$$x^{(j+1)} = x^{(j)} - J(x^{(j)})^{-1} \cdot f(x^{(j)}) .$$

(5)

The Jacobian matrix is a non-singular square submatrix of $B$.

**Identification and location of the faulty components.** This paper deals with soft faults. Soft or parametric faults are defined as a variation of parameter values of components that lead to abnormal functioning of the circuit. For example, in the case of a filter, the criterion for soft faults is considered as the variation of components values that produces a variation of the cut-off frequency, from the nominal value, by more than 20%. Once the actual values of the components computed, the faulty components are identified and located by comparison with the nominal ones.

4. EXAMPLE

The fault diagnosis for the equivalent circuit of an isolation transformer (Fig. 1) is performed. This circuit is described in [6] and its parameters can be computed using the method developed in [7]. The shields of the isolation transformer are used to change the frequency characteristic in order to reject some high frequency perturbation signals. These shields are built as separate windings exhibiting a distributed capacitive effect with respect to primary and secondary windings.

The values of the circuit parameters are: $C_1 = 53.7 \ \text{pF}$, $C_2 = 2.54 \ \text{nF}$, $C_3 = 0.9 \ \text{nF}$, $L_1 = 0.28 \ \text{mH}$, $L_2 = 0.28 \ \text{mH}$, $L_3 = 60 \ \text{mH}$, $R_1 = 0.35 \ \Omega$, $R_2 = 0.424 \ \Omega$, $R_3 = 465 \ \Omega$.

The frequency characteristic of the gain with both shields connected to ground (the no fault case) is measured. The pole and zero modules (determined as
it is was pointed out in Section 3) are given in Table 1. The same frequency characteristic with the first shield not grounded (Fault 1) is measured, its pole/zero modules being given in Table 1.

The following two faults are simulated:
- The value of $L_1$ is modified to 3.28 mH (Fault 2).
- The value of $C_2$ is set to 2.27 nF, as it was for Fault 1, and the value of $L_1$ is modified to 3.28 mH (Fault 3).

Table 1

<table>
<thead>
<tr>
<th>Pole/zero modules</th>
<th>No fault</th>
<th>Fault 1</th>
<th>Fault 2</th>
<th>Fault 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>p_1</td>
<td>$</td>
<td>$2.08 \times 10^7$</td>
<td>$2.08 \times 10^7$</td>
</tr>
<tr>
<td>$</td>
<td>p_2</td>
<td>$</td>
<td>$7.45 \times 10^6$</td>
<td>$8.1 \times 10^6$</td>
</tr>
<tr>
<td>$</td>
<td>p_3</td>
<td>$</td>
<td>$3.6 \times 10^6$</td>
<td>$3.6 \times 10^6$</td>
</tr>
<tr>
<td>$</td>
<td>p_4</td>
<td>$</td>
<td>$1.83 \times 10^5$</td>
<td>$1.83 \times 10^5$</td>
</tr>
<tr>
<td>$</td>
<td>p_5</td>
<td>$</td>
<td>$397.5$</td>
<td>$397.5$</td>
</tr>
<tr>
<td>$</td>
<td>z_1</td>
<td>$</td>
<td>$6.2 \times 10^5$</td>
<td>$6.2 \times 10^5$</td>
</tr>
<tr>
<td>$</td>
<td>z_2</td>
<td>$</td>
<td>$4.22 \times 10^5$</td>
<td>$4.22 \times 10^5$</td>
</tr>
<tr>
<td>$</td>
<td>z_3</td>
<td>$</td>
<td>$4.22 \times 10^5$</td>
<td>$4.22 \times 10^5$</td>
</tr>
<tr>
<td>$</td>
<td>z_4</td>
<td>$</td>
<td>$0$</td>
<td>$0$</td>
</tr>
</tbody>
</table>

The software packages SAPWIN and MAPLE are employed for circuit analysis and advanced mathematic calculus as the computation of the approximate pole/zero expressions.
The design space is considered as:

\[
P_{10,251} = P_{FC}, \quad n_{F_{8.3},n_{F_{25.12}}} = C, \quad n_{F_{35.1},n_{F_{45.03}}} = C,
\]
\[
m_{H_{042},m_{H_{14.01}}} = L, \quad m_{H_{42.0},m_{H_{14.02}}} = L, \quad H_{09.0},H_{03.03} = L,
\]
\[
\Omega \Omega \in R, \quad \Omega \Omega \in R, \quad \Omega \Omega \in R.
\]

The design space exploration using the sensitivity computation leads to the significant and less significant parameters for each pole or zero. It results:

\[p_1 = \{C_3, C_2, C_1\}, \quad p_2 = \{C_1, C_2\}, \quad p_3 = \{L_1, L_2, R_3\}, \quad p_4 = \{L_1, L_2\}, \quad p_5 = \{L_3, R_3\},\]
\[z_1 = \{C_1, R_2, L_1, R_1, R_2\}, \quad z_2 = \{C_1, L_1, R_3\}, \quad z_3 = \{C_1, L_1, R_3\}, \quad z_4 = \{C_1, R_3, L_1, L_2, R_1, R_2\}\]

The analysis of the pole/zero patterns in Table 1 and of the significant parameters resulting from the design space exploration lead to the conclusion that the diagnosis of all considered faults can be done using the symbolic expressions of the first four poles, which are given in Table 2.

### Table 2

<table>
<thead>
<tr>
<th>Pole/zero expressions</th>
</tr>
</thead>
<tbody>
<tr>
<td>[p_1:= (-1.7<em>10^{(-11)})<em>c3-3.0</em>10^{(-12)}<em>c2-0.2e-1</em>2</em>c3)/(9.1*10^{(-10)})<em>c3^2+2.2</em>10^{(-10)}<em>c2^3+c2</em>c3^2);]</td>
</tr>
<tr>
<td>[p_2:= (-4.4733262433905076328^{10^{(-84)}})*c2^3-3.9031214835653274505^{10^{(-60)}}*c2^6+3.660181250116909^{10^{-35}}<em>c2^9+ 6.69517</em>10^{(-19)}*c2^{10}+1.171209464194488776^{10^{(-93)}}*c2^2+8.074421815120371^{10^{(-52)}}*c2^7-1.3202740111044338943^{10^{(-75)}}*c2^4-7.4267459310983659588^{10^{(-67)}}*c2^2);]</td>
</tr>
<tr>
<td>[p_3:=(-2867.82<em>l2^2</em>l1^3-589.06<em>l1^5-1705.42</em>l2<em>l1^4-833.5</em>l2^5-3061.5<em>l2^3</em>l1^2-2082.0<em>l2^4</em>l1)/(l2<em>l1^5+2.87</em>l2^4<em>l1^2+1.38</em>l2^3<em>l1^3+2.4</em>l2^2<em>l1^4+3.4</em>l2*l1^5);]</td>
</tr>
<tr>
<td>[p_4:=(0.284e-1<em>l1</em>l2^2-95.8<em>l1^4-516.7</em>l1<em>l2^3-210.29</em>l2^4-390.0<em>l1^3</em>l2-634.9<em>l2^2</em>l1^2+0.25e-2<em>l2^2</em>l1^2)/(0.359e-3<em>l2^2</em>l1^2-0.325e-3<em>l1</em>l2^3+l1^5+5.0<em>l2</em>l1^4+11.3<em>l2^3</em>l1^2+7.2<em>l2^4</em>l1-0.8e-4<em>l2^4+2.03</em>l2^5+10.1<em>l2^2</em>l1^3);]</td>
</tr>
</tbody>
</table>

The simplified expressions in Table 2 are computed using \(\varepsilon_A = 0.01\), \(\varepsilon_2 = 0.005\), \(\varepsilon_p = 0.01\).
For example, Fig. 2 represents the variation of the pole/zero formulas a) $p_1 = f(C_2,C_3)$ and b) $p_3 = f(L_1,L_2)$ in Table 2 versus circuit parameters in the design space in comparison with the numerical values.

![Graphs](image)

The Newton-Raphson method implemented in Maple is employed to solve the systems of nonlinear equations corresponding to all cases. The computed parameter values are given in Table 3.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>No fault</th>
<th>Fault 1</th>
<th>Fault 2</th>
<th>Fault 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$C_2$</td>
<td>2.54 nF</td>
<td>2.27 nF</td>
<td>2.54 nF</td>
<td>2.27 nF</td>
</tr>
<tr>
<td>$C_3$</td>
<td>0.9 nF</td>
<td>0.9 nF</td>
<td>0.9 nF</td>
<td>0.9 nF</td>
</tr>
<tr>
<td>$L_1$</td>
<td>2.8 mH</td>
<td>2.8 mH</td>
<td>3.28 mH</td>
<td>3.28 mH</td>
</tr>
<tr>
<td>$L_2$</td>
<td>2.8 mH</td>
<td>2.8 mH</td>
<td>2.8 mH</td>
<td>2.8 mH</td>
</tr>
</tbody>
</table>

5. CONCLUSIONS

A new method for analog fault diagnosis is proposed in this paper. This method relies on approximate symbolic pole/zero expressions in terms of significant circuit parameters.
A parameter ranking for each pole/zero expression is performed in order to consider only the significant parameters in symbolic form. By this way this method employs simpler equations than other approaches. The simplified expressions that depend on few parameters can decouple the equations system in smaller systems which are easier to solve.

The fault diagnosis of the equivalent circuit of an isolation transformer is performed, taking into account three soft faults which modify the parameters $C_2$, $L_1$, and $C_2$ and $L_1$, respectively. All faults have been diagnosed without belonging to an ambiguity group.

The same circuit has been diagnosed in [8] using the multi-frequency method based on the circuit voltage gain. In this case the soft fault of $L_1$ belongs to an ambiguity group containing the soft fault of $L_2$, also. This result has been obtained using a pseudo-optimal frequency range determined starting from qualitative criteria based on sensitivity computation.

The proposed method seems to give more insight into circuit properties because it relies on pole/zero values which describe better its behavior than circuit responses at arbitrary frequencies in a pseudo-optimal range.

The limited range of validity of the approximate symbolic pole/zero expressions can be a disadvantage of the proposed method. This is because the validity range in the parameter space of each pole/zero expression is tested. If the computed value of the faulty parameter lies out of this range, new expressions can be built starting around other parameter value than the nominal one. We intend to use this technique in future research.

ACKNOWLEDGMENTS

This paper was supported by CNCSIS-UEFISCSU, project code ID_1698, Contract 683/2009.

Received on 30 August 2011

REFERENCES


