Rev. Roum. Sci. Techn.– Électrotechn. et Énerg.
Vol. 64, 1, pp. 63–68, Bucarest, 2019

LEVEL CROSSING RATE OF MACRO-DIVERSITY SYSTEM IN THE PRESENCE OF FADING AND CO-CHANNEL INTERFERENCE

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Key words: Gamma large scale fading, Rayleigh fading, Nakagami-*m* co-channel interference, Level crossing rate, Selection combining (SC).

In this paper, the macro-diversity system consisting of macro-diversity (MAD) selection combining (SC) receiver and two micro-diversity (MID) SC receivers, is considered. This MAD system alleviates the impact of short term fading, long term fading and co-channel interference. The desired signal suffers Rayleigh short term fading and correlated Gamma long term fading. Cochannel interference is affected by Nakagami-*m* short term fading and independent Gamma long term fading. The expression for the level crossing rate (LCR) of MAD system under such conditions is derived in closed form and presented graphically. The influence of Gamma long term fading severity parameter, Gamma long term fading correlation coefficient and Nakagami-*m* short term fading severity parameter on the LCR are analyzed and discussed. This work is of great importance in the design of wireless communication systems in the presence of combined effect of fading and interference. Also, the LCR is important systems' performance because, for example, the LCR influences the error correction codes performance.

1. INTRODUCTION

Co-channel interference, long term fading and short term fading cause degradation of the wireless communication system performance. Refraction, diffraction, absorption and reflection of radio wave cause multipath propagation resulting in signal envelope variation [1]. Large obstacles between transmitter and receiver cause shadowing which results in signal envelope average power variation. There are different distributions which can be used to describe signal envelope variation and signal envelope average power variation in fading channels, such as Rician, Nakagami-*m*, Rayleigh, Weibull, α-μ, Gamma and log-normal [1].

Rayleigh distribution can be used to describe small scale signal envelope variation in non line-of-sight multipath fading channels. Nakagami-*m* distribution also can be used to describe short term fading and has parameter *m* known as severity parameter. It takes values from zero to one. Nakagami-*m* distribution is general distribution and shows better fitting with experimental data.

The system performance measures of the first order are: the outage probability (OP), bit error probability (ABEP), channel capacity and amount of fading. The OP is an important performance measure of wireless communication system defined as the probability that signal envelope falls below the specified threshold. In [2], wireless propagation over κ-μ fading channels with Gamma distributed ratio of dominant and scattering random line-of-sight components is considered, and OP and average BEP of such wireless transmission are derived.

The second order performance measures are the level crossing rate (LCR) and the average fade duration (AFD). LCR is defined as the number of crossing at determined level in positive or negative direction and can be evaluated as average value of the first time derivative of random process (RP). AFD is deffined as average time that signal envelope is below the determined threshold and can be evaluated as the ratio of the OP and the LCR. The OP can be evaluated from cumulative distribution function (CDF).

There are many works in open technical literature considering performance analysis of wireless communication system which uses macrodiversity (MAD) technique to reduce short term fading, long term fading and co-channel interference (CCI) effects on the system performance. For example, AFD for dual selection diversity in correlated Rician fading with Rayleigh co-channel interference is determined in [3]. In [4], macro-diversity system with MAD SC receiver and two micro-diversity (MID) maximal ratio combining (MRC) receivers operating over shadowed Nakagami-*m* short term fading in the presence of CCI is analyzed. LCR and AFD of the proposed MAD are evaluated.

LCR of MAD SC system with two MID SC receivers over correlated Gamma shadowed α–μ multipath fading channel is studied in [5]. The paper [6] handles with MAD system consisting of MAD SC receiver and two MID SC receivers working in Gamma shadowed Weibull short term fading channel in the presence of CCI exposed to Weibull multipath fading. The closed form expression for the LCR at the output of MAD SC receiver is obtained.

Selection diversity receiver over correlated Rayleigh fading channels in the presence of multiple interferers is observed in [7]. Wireless system working over Gamma shadowed Nakagami-*m* multipath fading channel in the presence of CCI exposed to Nakagami-*m* short term fading and Gamma long term fading is taken into consideration in [8]. In [9], scenario where multiple interferences affect wireless system is analyzed. Dual-branch SC diversity receiver over Rician fading channel under the influence of multiple Nakagami-*m* faded CCIs is considered based on signal to interference ratio (SIR). The closed form expressions for first-order and second-order performance measures, such as OP, LCR, and AFD, of the SC output signal are obtained. MAD SC receiver with two MID SC receivers in the presence of Gamma long term fading, Rayleigh short term fading and CCI impacted by Rician short term fading is observed in [10] and OP of considered MAD system is calculated in closed form.

In [11], SC MAD receiver with two dual-branch SC MID parts in the presence of CCI is investigated. The received signal propagated over Gamma shadowed Rayleigh fading channel while CCI propagated over Gamma shadowed Nakagami-*m* multipath fading channel. The OP at the output of MAD system is derived.

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The analytical expressions for the OP of different diversity systems (such as SC, MRC and equal gain combining (EGC)), exposed to CCI and various fading influence are mostly derived in the literature. But, the respective expressions for the LCR and the AFD are not available for all known scenarios, although twenty years ago LCR is introduced for derivation the error correction codes performance [12]. There, packet loss is observed as a level crossing problem and the effect of Doppler frequency, fade margin, and burst error correction coding on packet error rate for data transmission over Rayleigh fading channels are studied. In [13] is shown importance of the second order system performance. The authors supposed the useful signal and CCI envelopes subjected to Rayleigh fading. In [14], the expressions for the LCR and AFD of dual EGC diversity system in the presence of CCI in Rayleigh fading channel are obtained in closed forms.

We improved here this scenario by introducing CCI experiencing Nakagami-*m* fading and independent Gamma fading, because Nakagami-*m* distribution is general distribution and shows better fitting with experimental data. So, the main advantage of this paper compared to the previously published ones is the treatment of the CCI under the influence of both, fast and slow fading. Here, unlike the paper [11], the characteristics of the second order are considered for the wireless MAD SC receiver with two MID SC receivers, where desired signal is subjected to Rayleigh fading and Gamma shadowing and CCI is exposed to Gamma long term fading and Nakagami-*m* short term fading resulting in system performance degradation. The probability density function (PDF), CDF and LCR of the ratio of Rayleigh RP and Nakagami-*m* RP are evaluated. Obtained formulas are used for evaluating the LCR of the SIR at the MID SC receivers' outputs and MAD SC receiver output, in closed form. This expression for the LCR of MAD system can be used for calculation the AFD of the observed MAD system. To the best of our knowledge, the LCR of MAD system in the presence of Gamma long term fading, Rayleigh short term fading and Gamma shadowed Nakagami-*m* co-channel interference is not reported in open technical literature.

2. LEVEL CROSSING RATE OF SC RECEIVER OUTPUT SIGNAL ENVELOPE

Macro-diversity system has MAD SC receiver and two MID SC receivers. At the inputs of MID SC receivers, desired signal and co-channel interference are present. Desired signal experiences Rayleigh small scale fading and Gamma large scale fading. Co-channel interference is subjected to Nakagami-*m* fading and Gamma large scale fading. The model of such MAD is shown in Fig. 1. Desired signal envelopes at inputs of the first MID are denoted with x_{11} and x_{12}, with x_{21} and x_{22} at the inputs of the second MID, with x_1 and x_2, at the outputs of MIDs. Finally, desired signal envelope is denoted with x at the output of MAD. Co-channel signal envelopes at the inputs of the first MID are denoted with y_{11} and y_{12}, at inputs of the second MID with y_{21} and y_{22}, at the output of MID with y_1 and y_2, and CCI envelope at output of MAD SC receiver is denoted with y. Desired signal envelopes x_{ij} have Rayleigh distribution [2]:

$$p_{x_{ij}}\left(x_{ij}\right)=\left(2x_{ij}/\Omega_i\right)\mathrm{e}^{-x_{ij}^2/\Omega_i},\ x_{ij}\geq 0, i=1,2; j=1,2. \quad (1)$$

CCI envelopes follow Nakagami-*m* distribution [15]:

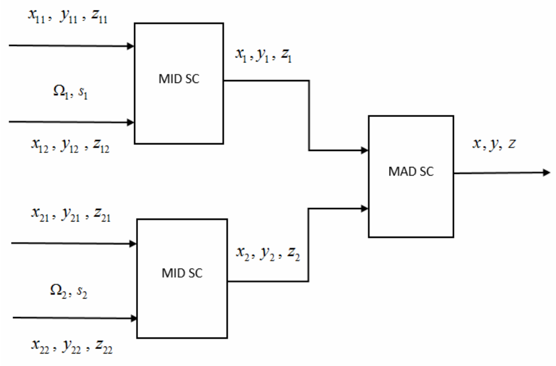

Fig. 1 – Macro-diversity system with macro-diversity SC receiver and two micro-diversity SC receivers.

$$p_{y_{ij}}\left(y_{ij}\right)=\frac{2}{\Gamma(m)}\left(\frac{m}{s_i}\right)^m y_{ij}^{2m-1}\mathrm{e}^{-\frac{m}{s_i}y_{ij}^2},\ y_{ij}\geq 0, \quad (2)$$

where i=1, 2; j=1, 2; $\Gamma(m)$ is the Gamma function, m is Nakagami-*m* short term fading severity parameter and $\Omega_i=\overline{x_{ij}^2}, s_i=\overline{y_{ij}^2}$. Ω_i and s_i are powers of desired signal and interference, respectively. The ratio of x_{ij} and y_{ij} is:

$$z_{ij}=x_{ij}/y_{ij}, x_{ij}=z_{ij}y_{ij}. \quad (3)$$

The PDF of z_{ij} is [16]:

$$p_{z_{ij}}\left(z_{ij}\right)=\int_0^\infty \mathrm{d}y_{ij}\, y_{ij}\, p_{x_{ij}}\left(z_{ij}y_{ij}\right)p_{y_{ij}}\left(y_{ij}\right)= \\ =2s_i z_{ij}\Omega_i^m m^{m+1}/\left(\Omega_i m+s_i z_{ij}^2\right)^{m+1}. \quad (4)$$

Cumulative distribution function of z_{ij} is [17]:

$$p_{z_{ij}}\left(z_{ij}\right)=\int_0^\infty \mathrm{d}y_{ij}\, y_{ij}\, p_{x_{ij}}\left(z_{ij}y_{ij}\right)\cdot \\ \cdot p_{y_{ij}}\left(y_{ij}\right)=\frac{2s_i z_{ij}\Omega_i^m m^{m+1}}{\left(\Omega_i m+s_i z_{ij}^2\right)^{m+1}}. \quad (5)$$

The integral in (5) can be solved by the formula [18]:

$$\int_0^\lambda \frac{x^m}{\left(a+bx^n\right)^p}\mathrm{d}x=\frac{a^{-p}}{n}\left(\frac{a}{b}\right)^{\frac{m+1}{n}}B_z\left(\frac{m+1}{n}, p-\frac{m+1}{n}\right); \\ z=\frac{b\lambda^n}{a+b\lambda^n}, a>0, b>0, n>0, 0<\frac{m+1}{n}<p, \quad (6)$$

where $B_z(a, b)$ is incomplete Beta function [18; 8.38]. Using the expression (6), previously written expression for CDF of the ratio of two variables becomes:

$$F_{z_{ij}}\left(z_{ij}\right)=mB_{s_i x_{ij}^2/\left(m\Omega_i+s_i z_{ij}^2\right)}(1,m). \quad (7)$$

The first time derivative of z_{ij} is:

$$\dot{z}_{ij}=\dot{x}_{ij}/y_{ij}-x_{ij}\dot{y}_{ij}/y_{ij}^2. \quad (8)$$

The first time derivative of Rayleigh RP is Gaussian RP. Also, the first time derivative of Nakagami-*m* RP is Gaussian RP $\dot{z}_{ij}$. The mean of $\dot{z}_{ij}$ is zero, and variance is:

$$\sigma_{\dot{z}_{ij}}^2=\sigma_{\dot{x}_{ij}}^2/y_{ij}^2+x_{ij}^2\sigma_{\dot{y}_{ij}}^2/y_{ij}^4, \\ \sigma_{\dot{x}_{ij}}^2=\pi^2 f_m^2\Omega_i, \sigma_{\dot{y}_{ij}}^2=\pi^2 f_m^2 s_i/m \quad (9)$$

Here, f_m is the maximum Doppler frequency. After substituting, the variance of $\dot{z}_{ij}$ becomes:

$$\sigma_{\dot{z}_{ij}}^2=\left(\pi^2 f_m^2/y_{ij}^2\right)\left(\Omega_i+\left(s_i/m\right)z_{ij}^2\right). \quad (10)$$

The joint PDF of z_{ij}, $\dot{z}_{ij}$ and y_{ij} is [2]:

$$p_{z_{ij},\dot{z}_{ij}y_{ij}}\left(z_{ij},\dot{z}_{ij}y_{ij}\right)=p_{\dot{z}_{ij}}\left(\dot{z}_{ij}\middle|z_{ij}y_{ij}\right)p_{z_{ij}}\left(z_{ij}\middle|y_{ij}\right)p_{y_{ij}}\left(y_{ij}\right), \quad (11)$$

where:

$$p_{z_{ij}}\left(z_{ij}\middle|y_{ij}\right)=\left|\frac{\mathrm{d}x_{ij}}{\mathrm{d}z_{ij}}\right|p_{x_{ij}}\left(z_{ij}y_{ij}\right)=y_{ij}\,p_{x_{ij}}\left(z_{ij}y_{ij}\right),$$

$$p_{\dot{z}_{ij}}\left(\dot{z}_{ij}\middle|z_{ij}y_{ij}\right)=\frac{1}{\sqrt{2\pi}\,\sigma_{\dot{z}}}\mathrm{e}^{-\dot{z}^2/2\sigma_{\dot{z}}^2}. \quad (12)$$

The joint PDF of z_{ij}, and $\dot{z}_{ij}$ is:

$$p_{z_{ij},\dot{z}_{ij}}\left(z_{ij},\dot{z}_{ij}\right)=\int_0^\infty \mathrm{d}y_{ij}\,p_{z_{ij}\dot{z}_{ij}y_{ij}}\left(z_{ij}\dot{z}_{ij}y_{ij}\right). \quad (13)$$

The level crossing rate of random process z_{ij} is [19]:

$$N_{z_{ij}}\left(z_{ij}\right)=\int_0^\infty \mathrm{d}\dot{z}_{ij}\,\dot{z}_{ij}\,p_{z_{ij}\dot{z}_{ij}}\left(z_{ij}\dot{z}_{ij}\right)= =\frac{\sqrt{2\pi}\,f_m\left(m\Omega_i\right)^{m-1/2}s_i^{1/2}z_{ij}\Gamma\left(m+1/2\right)}{\Gamma\left(m\right)\left(s_i z_{ij}^2+m\Omega_i\right)^m}. \quad (14)$$

The LCRs of the output SIR of the first and the second MID SC receivers, using terms (7) and (14), and i=1, 2, are:

$$N_{x_i|\Omega_i s_i}\left(z_i\right)=2F_{z_{ij}}\left(z_{ij}\right)N_{z_{ij}}\left(z_{ij}\right)=\frac{\sqrt{2\pi}\,f_m m^{m+1/2}}{\left(s_i z_{ij}^2+m\Omega_i\right)^m\Gamma\left(m\right)}\cdot$$

$$\cdot\,\Omega_i^{m-1/2}s_i^{1/2}z_{ij}\Gamma\left(m+1/2\right)B_{s_i z_i^2/\left(m\Omega_i+s_i z_i^2\right)}\left(1,m\right). \quad (15)$$

Graphical analysis of the LCR of the SIR at the output of the first MID SC receiver from (15) is displayed in Figs. 2 and 3. It can be seen that LCR reduces when m becomes higher. From Fig. 2, one can notice that increasing of the desired signal power Ω causes the increasing of the LCR.

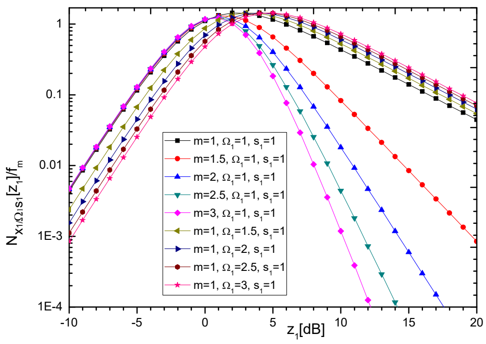

Fig. 2 – LCR at the output of MID SC combiner versus signal to interference ratio z_1, when values of parameters m and Ω_1 are changed.

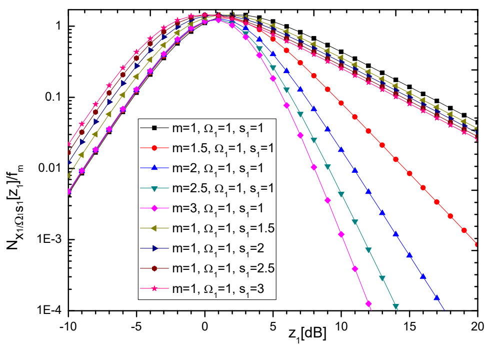

Fig. 3 –LCR at the output of MID SC combiner with variable parameters m and s_1.

On the other hand, when the interference signal power s is higher, the LCR gets smaller. The system performance is more favorable for smaller values of average LCR.

3. LEVEL CROSSING RATE OF SC RECEIVER OUTPUT SIGNAL TO INTERFERENCE RATIO

The joint PDF of the signal envelope average powers at inputs of micro-diversity SC receivers is [20]:

$$p_{\Omega_1\Omega_2}\left(\Omega_1\Omega_2\right)=\frac{\left(\Omega_1\Omega_2\right)^{(c-1)/2}}{\Gamma\left(c\right)\left(1-\rho^2\right)\rho^{c-1}\Omega_0^{c+1}}\mathrm{e}^{-\frac{\Omega_1+\Omega_2}{\Omega_0\left(1-\rho^2\right)}}\cdot$$

$$\cdot\,I_{c-1}\left(\frac{2\rho\left(\Omega_1\Omega_2\right)^{1/2}}{\Omega_0\left(1-\rho^2\right)}\right)=\frac{1}{\Gamma\left(c\right)}\mathrm{e}^{-\frac{\Omega_1+\Omega_2}{\Omega_0\left(1-\rho^2\right)}}\sum_{i=0}^{\infty}\frac{\rho^{2i}}{\Gamma\left(i+c\right)i!}\cdot$$

$$\cdot\,\frac{\left(\Omega_1\Omega_2\right)^{i+c-1}}{\Omega_0^{2i+2c}\left(1-\rho^2\right)^{2i+c}};\Omega_1\geq 0,\Omega_2\geq 0. \quad (16)$$

$In(\cdot)$ is the n-th order modified Bessel function of the first kind of order c, defined by transformation [21; (17.7.1.1)]:

$$I_v\left(x\right)=\sum_{k=0}^{\infty}\frac{x^{v+2k}}{2^{v+2k}k!\Gamma\left(v+k+1\right)}. \quad (17)$$

Co-channel interference envelopes powers s_i, i=1,2 follow Gamma distribution [22]:

$$p_{s_i}\left(s_i\right)=\frac{1}{\Gamma\left(c\right)\beta^c}s_i^{c-1}\mathrm{e}^{-\frac{1}{\beta}s_i},s_i\geq 0; \quad (18)$$

where β=E(s^2) and c describing random s-factor severity change parameter.

As shown in the Appendix, the expression for the LCR of MAD SC receiver output signal to interference ratio is:

$$N_z\left(z\right)=\frac{4\sqrt{2\pi}\,f_m m^{c+1}\Omega_0^c\left(1-\rho^2\right)^{2c}\Gamma\left(m+1/2\right)}{\Gamma^2\left(c\right)\Gamma\left(m\right)\beta^c z_1^{2c}}\cdot$$

$$\cdot\sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\sum_{i_3=0}^{\infty}\frac{\rho^{2i_1}\left(1-m\right)_{i_2}\Gamma\left(2i_1+i_3+3c\right)}{2^{2i_1+i_3+3c}i_1!\Gamma\left(i_1+c\right)\Gamma\left(i_2+2\right)\left(i_1+c\right)}\cdot$$

$$\cdot\frac{\Gamma\left(2i_1+i_3+2c+m-\frac{1}{2}\right)\Gamma\left(c+i_2+\frac{3}{2}\right)}{\left(1+i_1+c\right)_{i_3}\Gamma\left(2i_1+i_2+i_3+3c+m+1\right)}{}_2F_1\left(2i_1+i_3+3c,\right.$$

$$\left.i_2+c+\frac{3}{2},2i_1+i_2+i_3+3c+m+1,1-\frac{m\Omega_0\left(1-\rho^2\right)}{2\beta z_1^2}\right). \quad (19)$$

The level crossing rate of macro-diversity SC receiver output signal to interference ratio from the expression (19) is given in Figs. 4 and 5. In Fig. 4, the LCR is presented versus signal to interference ratio for unchangeable parameters c, Ω_0 and β. From Fig. 4, one can remarked that for bigger values of the signal to interference ratio, with increasing of parameter m, LCR decreases. At high interference, change of the parameter m has no significant impact on the LCR. From this figure one can see the influence of parameter ρ. With an increase of parameter ρ, LCR becomes lower for bigger z. For small values of z, the situation is reversed.

Figure 5 shows LCR at the output of the MAD SC combiner, due to the change of parameters c and β, when the other parameters, m, Ω_0 and ρ, are constant.

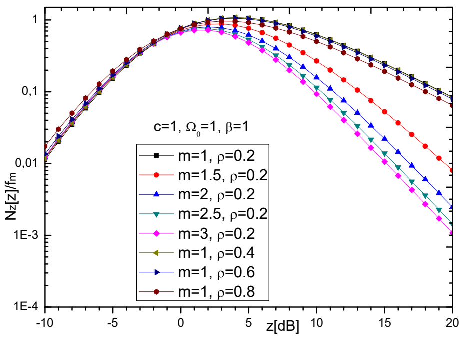

Fig. 4 – The LCR at the output of the MAD SC combiner depending on z with variable parameters m and ρ.

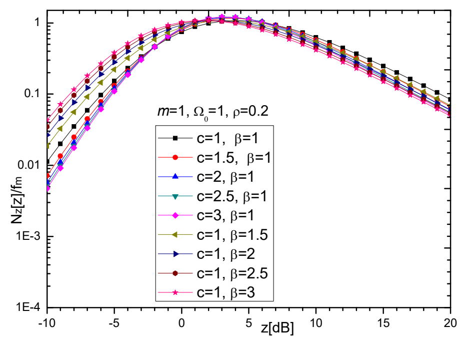

Fig. 5 –LCR at the output of MAD SC combiner depending on z with variable parameters c and β.

It is possible to see from this figure that with enlarging of parameter c (β = const), LCR decreases for negative values of z[dB], while for positive values of z [dB] the increase of parameter c causes the LRC to slightly decrease. Due to the increase of the parameter β (c = const), LCR grows for negative values, and slightly decreases for positive ones [dB]. The signal is expected to be above the level with most of the time, and the LCR is relatively low.

From the graphics, we can see that, due to the increase of the level, there is a decrease in the LCR, and the system will have better performance due to the increase of the parameter c. Generally, the best system performance is achieved due to the change of parameters m and c, and the system becomes more stable, while for the changes of the other two parameters, the ρ and β, system gives a lower performance and becomes more unstable.

4. ANALYSIS OF NUMERICAL RESULTS

In the following tables, the number of the members of the series that need to be summed to achieve the accuracy of the resulting expression at fifth significant digit, for the given values of parameters, is presented. Table 1 shows the number of members in the expression (19) that should be added to obtain accuracy at 5th decimals, when the parameters m and ρ are changed, and the other parameters are: $c=\Omega_0=\beta= 1$ (Fig.4). When the parameter m increases, the series converges slowly, while for an increasing of the parameter ρ, the series converges faster, and a higher number of members in each sum is need. For example, when the parameter $\rho= 0.2$ increases to $\rho= 0.4$ at $z= -10$ dB, the number of members in sums in the expression (19) increases from $6^3= 216$ to $7^3= 343$, in order to obtain the accuracy of (19) at 5th decimals.

Table 1

Number of terms that should be added in expression (19) for the LCR in order to reach accuracy at 5th significant digit, when parameters m and ρ change, and $c=\beta=\Omega_0=1$ (Fig.4)

	$z=-10$ dB	$z=0$ dB	$z=10$ dB
m=1, ρ=0.2	6	13	15
m=1.5, ρ=0.2	6	13	12
m=2, ρ=0.2	6	13	15
m=2.5, ρ=0.2	6	13	14
m=3, ρ=0.2	6	14	15
m=1, ρ=0.4	7	15	17
m=1, ρ=0.6	6	16	19
m=1, ρ=0.8	9	21	26

Table 2

Number of terms that should be added in expression (19) for the LCR in order to reach accuracy at 5th significant digit, when parameters c and β change, and $m=\Omega_0=1$, $\rho=0.2$ (Fig.5)

	z=-10 dB	z=0 dB	z=10 dB
c=1, β=1	6	13	15
c=1.5, β=1	6	15	18
c=2, β=1	8	16	18
c=2.5, β=1	8	17	19
c=3, β=1	9	19	20
c=1, β=1.5	6	13	16
c=1, β=2	8	15	17
c=1, β=2.5	7	15	17
c=1, β=3	7	15	17

From Table 1, one can also see that the number of the members of the series that need to be summed in the series, to achieve the same accuracy of the resulting expression, for all values of parameters is not bigger than 26. This is valid just in a few examples. Usually, it is necessary to sum 12–20 members, *i.e.*, the order is fast convergent.

Table 2 gives the values necessary for convergence of the expression (19) with respect to the variable z, when the parameters c and β are changing, and the other parameters are: $m=\Omega_0=1$, $\rho=0.2$ (Fig.5). For $z=-10$ dB, when higher values of the parameters c and β are taken, the system converges slower. On the other hand, at z=0 dB and z=10 dB, for higher values of the parameters c and β, more members need to be summed to obtain the exact value of the expression (19) at 5th decimals, and the system converges faster.

5. CONCLUSIONS

Macro-diversity reception with MAD SC receiver and two MID SC receivers, whereby desired signal experiences correlated Gamma shadowing and Rayleig short term fading and co-channell interference suffers independent Gamma long term fading and Nakagami-m short term fading is processed. MAD SC receiver mitigates Gamma long term fading effects and CCI effects on the system performance. MID SC receiver mitigates Nakagami-m short term fading effects and CCI effects on the system performance. PDF, CDF and LCR of the ratio of Rayleigh random process and Nakagami-m random process are evaluated. Then, the expressions for LCR of the SIRs at outputs of MID SC receivers and LCR of MAD SC receiver output SIR are derived. Obtained expressions rapidly converge since 10–20 terms need to be summed in order to achieve accuracy at 5th significant digit for any values of the fading channel parameters. The system performance is better for lower values of the level crossing rate. The LCR decreases when Gamma long term fading severity parameter and Nakagami-m severity parameter increase.

From our results, it is possible to find the most convenient values of the systems' parameters under the influence of the examined types of fading. By using the expresion for LCR of MAD system derived here, and by setting m=1, the LCR of MAD system in the presence of Gamma long term fading, Rayleight short term fading and Rayleigh co-chanel interference can be obtained.

ACKNOWLEDGEMENT

The paper is supported in part by the projects III44006 and TR33035 of the Ministry of Education, Science and Technological Development of Republic of Serbia.

APPENDIX

MAD SC receiver selects MID with higher signal envelope average power from inputs to provide service to user. Therefore, LCR of macro-diversity SC receiver output signal to interference ratio is [2]:

$$N_z(z)=\int_0^\infty ds_1 p_{s_1}(s_1)\int_0^\infty d\Omega_1\int_0^{\Omega_1} d\Omega_2 N_{x_1|\Omega_1 s_1}(z_1)\cdot$$
$$\cdot\, p_{\Omega_1\Omega_2}(\Omega_1\Omega_2)+\int_0^\infty ds_2 p_{s_2}(s_2)\int_0^\infty d\Omega_2\int_0^{\Omega_2} d\Omega_1\cdot$$
$$\cdot\, N_{x_2|\Omega_2 s_2}(z_2)\, p_{\Omega_1\Omega_2}(\Omega_1\Omega_2). \quad \text{(A.1)}$$

After insertion of the equations (15), (16) and (18) into (A.1), we obtain:

$$N_z(z)=\frac{4\sqrt{2\pi}f_m m^{m+1/2} z_1\Gamma(m+1/2)}{\Gamma^2(c)\Gamma(m)\beta^c}\sum_{i_1=0}^{\infty}\frac{\rho^{2i_1}}{\left(1-\rho^2\right)^{2i_1+c} i_1!}\cdot$$
$$\cdot\frac{1}{\Omega_0^{2i_1+2c}\Gamma(i_1+c)}\int_0^\infty ds_1 s_1^{c-\frac{1}{2}} e^{-\frac{s_1}{\beta}}\int_0^\infty\frac{\Omega_1^{i_1+c+m-\frac{3}{2}}}{\left(s_1z_1^2+\Omega_1 m\right)^m}d\Omega_1\cdot$$
$$\cdot\, e^{-\frac{\Omega_1}{\Omega_0\left(1-\rho^2\right)}}\int_0^{\Omega_1} d\Omega_2\Omega_2^{i_1+c-1}B_{\frac{s_1z_1^2}{m\Omega_1+s_1z_1^2}}(1,m)\, e^{-\frac{\Omega_2}{\Omega_0\left(1-\rho^2\right)}} \quad \text{(A.2)}$$

The incomplete beta function may be presented in the form [18; 8.39]:

$$B_x(p,q)=\int_0^x t^{p-1}(1-t)^{q-1}dt=\frac{x^p}{p}\,{}_2F_1(p,1-q;p+1;x). \quad \text{(A.3)}$$

The function ${}_2F_1$ is hyper geometric function or Gauss's hyper geometric function, written as [18; 15.1.1]:

$${}_2F_1(a,b;c;z)=\sum_{j=0}^{\infty}\frac{a_j b_j}{c_j}\frac{z^j}{j!}, \quad \text{(A.4)}$$

where $(a)n$ denoting the Pochhammer symbol. Using the properties [20; 6.1.22], [20; 6.1.15], [18; 8.338]:

$$(1)_n=\frac{\Gamma(1+n)}{\Gamma(1)}=n!,\ (z)_n=\frac{\Gamma(z+n)}{\Gamma(z)},\ \Gamma(1)=\Gamma(2)=1, \quad \text{(A.5)}$$

and equations (A.3) and (A.4), the incomplete beta function from (A.3) becomes:

$$B_{\frac{s_1z_1^2}{m\Omega_1+s_1z_1^2}}(1,m)=\sum_{i_2=0}^{\infty}\frac{(1-m)_{i_2}}{\Gamma(i_2+2)}\left(\frac{s_1z_1^2}{m\Omega_1+s_1z_1^2}\right)^{i_2+1}. \quad \text{(A.6)}$$

After replacement of the expression (A.6) into (A.2), we can write the LCR as:

$$N_z(z)=\frac{4\sqrt{2\pi}f_m m^{m+1/2}\Gamma(m+1/2)}{\Gamma^2(c)\Gamma(m)\beta^c}\sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\frac{(1-m)_{i_2}\rho^{2i_1}}{\left(1-\rho^2\right)^{2i_1+c}}\cdot$$
$$\cdot\frac{z_1^{2i_2+3}}{\Omega_0^{2i_1+2c}\Gamma(i_1+c)\Gamma(i_2+2)i_1!}\int_0^\infty ds_1 s_1^{i_2+c+\frac{1}{2}} e^{-\frac{1}{\beta}s_1}\int_0^\infty d\Omega_1\cdot$$
$$\cdot\frac{\Omega_1^{i_1+c+m-\frac{3}{2}} e^{-\frac{\Omega_1}{\Omega_0\left(1-\rho^2\right)}}}{\left(\Omega_1 m+s_1z_1^2\right)^{i_2+m+1}}\int_0^{\Omega_1} d\Omega_2\Omega_2^{i_1+c-1} e^{-\frac{\Omega_2}{\Omega_0\left(1-\rho^2\right)}}. \quad \text{(A.7)}$$

Let's introduce the integral J_1 to be:

$$J_1=\int_0^{\Omega_1} d\Omega_2\Omega_2^{i_1+c-1}e^{-\Omega_2/\Omega_0\left(1-\rho^2\right)}=$$
$$=\Omega_0^{i_1+c}\cdot\left(1-\rho^2\right)^{i_1+c}\int_0^{\Omega_1/\Omega_0\left(1-\rho^2\right)} t^{i_1+c-1}e^{-t}dt. \quad \text{(A.8)}$$

Integral within (A.8) is solved according to the model:

$$\gamma(\alpha,x)=\int_0^x e^{-t}t^{\alpha-1}dt. \quad \text{(A.9)}$$

where $\gamma(\alpha,\ x)$ is lower incomplete gamma function [18; 8.350]. Lower incomplete gamma function can be developed by using the form [23]:

$$\gamma(\alpha,x)=\frac{1}{a}x^a e^{-x}\,{}_1F_1(1,\alpha+1,x)=\frac{1}{a}x^a e^{-x}\sum_{j=0}^{\infty}\frac{x^j}{(1+\alpha)_j}. \quad \text{(A.10)}$$

${}_1F_1$ is the Kumar confluent hyper geometric function [21]:

$${}_1F_1(\alpha,\gamma,z)=\sum_{k=0}^{\infty}\frac{a_k}{b_k}\frac{z^k}{k!}. \quad \text{(A.11)}$$

Using (A.9), (A.10) and (A.11), we find the integral J_1 as:

$$J_1=\frac{e^{-\Omega_1/\Omega_0\left(1-\rho^2\right)}}{(i_1+c)}\sum_{i_3=0}^{\infty}\frac{\Omega_1^{i_1+i_3+c}}{\Omega_0^{i_3}\left(1-\rho^2\right)^{i_3}(1+i_1+c)_{i_3}}. \quad \text{(A.12)}$$

After inserting the expression (A.12) in the expression (A.7), we find that the LCR is:

$$N_z(z)=\frac{4\sqrt{2\pi}f_m m^{m+1/2}\Gamma(m+1/2)}{\Gamma^2(c)\Gamma(m)\beta^c z_1^{2m-1}}\sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\sum_{i_3=0}^{\infty}\frac{\rho^{2i_1}}{i_1!}\cdot$$
$$\cdot\frac{(1-m)_{i_2}}{\Gamma(i_1+c)\Gamma(i_2+2)(i_1+c)(1+i_1+c)_{i_3}\Omega_0^{2i_1+i_3+2c}}\cdot$$
$$\cdot\frac{1}{\left(1-\rho^2\right)^{2i_1+i_2+c}}\int_0^\infty ds_1 s_1^{c-m-\frac{1}{2}} e^{-\frac{1}{\beta}s_1}\int_0^\infty d\Omega_1\cdot$$
$$\cdot\frac{\Omega_1^{2i_1+i_3+2c+m-\frac{1}{2}-1}}{\left(1+\left(m/s_1z_1^2\right)\Omega_1\right)^{i_2+m+1}}e^{-\frac{2}{\Omega_0\left(1-\rho^2\right)}\Omega_1}. \quad \text{(A.13)}$$

The new integral J_2 is now defined as:

$$J_2=\int_0^\infty d\Omega_1\frac{\Omega_1^{2i_1+i_3+2c+m-\frac{1}{2}-1}}{\left(1+\left(m/s_1z_1^2\right)\Omega_1\right)^{i_2+m+1}}e^{-\frac{2}{\Omega_0\left(1-\rho^2\right)}\Omega_1}. \quad \text{(A.14)}$$

Using the following identity [18; 3.383]:

$$\int_0^\infty\frac{x^{q-1}e^{-px}}{(1+ax)^v}dx=a^{-q}\Gamma(q)\Psi\left(q,q+1-v,\frac{p}{a}\right), \quad \text{(A.15)}$$

where: $\psi(s,\ d,\ t)$ is confluent hyper geometric function [18; 9.210], the integral J_2 can be written in the following form:

$$J_2=\left(\frac{s_1 z_1^2}{m}\right)^{2i_1+i_3+2c+m-\frac{1}{2}}\Gamma\left(2i_1+i_3+2c+m-\frac{1}{2}\right)\Psi\left(2i_1+\right.$$

$$\left.+i_3+2c+m-\frac{1}{2},2i_1-i_2+i_3+2c-\frac{1}{2},\frac{2s_1z_1^2}{m\Omega_0\left(1-\rho^2\right)}\right). \quad (A.16)$$

After putting this solution in (A.13), and use the replacement $n=2s_1z_1^2/m\Omega_0(1-\rho^2)$, the LCR becomes:

$$N_z(z)=\frac{4\sqrt{2\pi}f_m m^{c+1}\Omega_0^c\left(1-\rho^2\right)^{2c}\Gamma(m+1/2)}{\Gamma^2(c)\Gamma(m)\beta^c z_1^{2c}}\cdot$$

$$\sum_{i_1=0}^{\infty}\sum_{i_2=0}^{\infty}\sum_{i_3=0}^{\infty}\frac{\rho^{2i_1}\Gamma(2i_1+i_3+2c+m-1/2)}{i_1!2^{2i_1+i_3+3c}(1+i_1+c)_{i_3}\Gamma(i_1+c)}\cdot$$

$$\cdot\frac{(1-m)_{i_2}}{\Gamma(i_2+c)(i_1+c)}\int_0^{\infty}\mathrm{d}n\,n^{2i_1+i_3+3c-1}\mathrm{e}^{-m\Omega_0\left(1-\rho^2\right)n/2\beta z_1^2}\cdot$$

$$\cdot\Psi\left(2i_1+i_3+2c+m-1/2,2i_1-i_2+i_3+2c-1/2,n\right). \quad (A.17)$$

The integral J_3 from the expression (A.17) is:

$$J_3=\int_0^{\infty}\mathrm{d}n\,n^{2i_1+i_3+3c-1}\mathrm{e}^{-m\Omega_0\left(1-\rho^2\right)n/2\beta z_1^2}\cdot$$

$$\cdot\Psi\left(2i_1+i_3+2c+m-\frac{1}{2},2i_1-i_2+i_3+2c-\frac{1}{2},n\right). \quad (A.18)$$

We can solve this integral if we use [18, Eq. 7.621]:

$$\int_0^{\infty}t^{b-1}\mathrm{e}^{-st}\Psi(a,d,t)\mathrm{d}t=\frac{\Gamma(b)\Gamma(b-d+1)}{\Gamma(a+b-d+1)}\cdot$$

$$\cdot{}_2F_1(b,b-d+1,a+b-d+1,1-s). \quad (A.19)$$

At present, we can write the integral J_3 as:

$$J_3=\frac{\Gamma(2i_1+i_3+3c)\Gamma(c+i_2+3/2)}{\Gamma(2i_1+i_2+i_3+3c+m+1)}{}_2F_1\left(2i_1+i_3+3c,\right.$$

$$\left.c+i_2+\frac{3}{2},2i_1+i_2+i_3+3c+m+1,1-\frac{m\Omega_0\left(1-\rho^2\right)}{2\beta z_1^2}\right). \quad (A.20)$$

Finally, the expression (A.17) for the LCR can be written as (19).

Received on January 20, 2018

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